Report submitted for Assignment of **19ECE301 Control Theory**

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Title: Roll Angle Control of an Aircraft

Marks (to be filled by the faculty)

S.no	Description	Excellent (10)	Good (5)	Not acceptable (0)
1	Design and analysis using CAD tools (especially the control system designer app)			
2	Understanding of the problem (To be decided on seeing the quality of the report and if necessary, by interacting with the students)			
3	Relationship of the problem to the topics taught in the syllabus			
4	Report structure (figures included should have proper title and labelled axes)			
	Total marks (40)			

Problem statement:

Design a control system to regulate the roll angle of an aircraft by converting its roll dynamics from state-space to a transfer function. The system must be adjusted to meet specific performance criteria, including a settling time of less than 5.8 seconds, minimal overshoot, and overall stability. Implement and test the system in MATLAB, validating its performance through step response, root locus, and stability analysis to ensure reliable and effective roll angle control.

1. Roll Angle Control of an Aircraft:

As you can see from Figure.1:

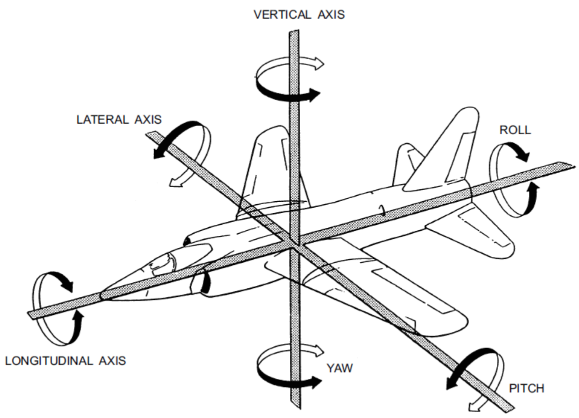


Figure.1

By adjusting the roll angle, pilots can execute smooth turns and maintain coordinated flight. This control is crucial not only for optimizing performance and fuel efficiency but also for ensuring passenger comfort and safety during flight. A well-designed roll control system enhances the aircraft's responsiveness to pilot inputs and minimizes overshoot and settling time. Effective roll control contributes significantly to the overall performance and safety of modern aviation.
The State-Space Model of the aircraft system is given as [1]:

State-Space Matrices

Matrix A

	Col 1	Col 2	Col 3	Col 4
Row 1	-0.254	0.0	-1.0	0.182
Row 2	-16.02	-8.4	2.19	0.0
Row 3	4.488	-0.35	-0.76	0.0
Row 4	0.0	1.0	0.0	0.0

Matrix B

	Col 1
Row 1	0.0
Row 2	-28.916
Row 3	-0.244
Row 4	0.0

Matrix C

	Col 1	Col 2	Col 3	Col 4
Row 1	0	0	0	1

Matrix D

	Col 1
Row 1	0

The matrices A, B, C, and D are essential components in the state-space representation of dynamic systems. The matrix A is called the state matrix, and it shows how the current state of the system affects how that state changes over time. The matrix B is the input matrix, which describes how the control input influences the state of the system. Together, these two matrices are used in the state equation: $\dot{x}(t)=Ax(t)+Bu(t)$

On the output side, matrix C is known as the output matrix. It maps the current state of the system to the output, while matrix D is the feedforward matrix, which represents any direct effect of the input on the output. This relationship is captured in the output equation: $y(t)=Cx(t)+Du(t)$. Overall, these matrices work together to describe how a dynamic system responds to inputs and produces outputs based on its current state.

2.SOLUTION:

Modelling:

The continuous time transfer function that represents the aileron deflection roll angel is given by

$$\frac{\Delta\,\phi(s)}{\Delta\sigma_{\alpha\ (s)}} = G_{(s)} \quad \text{................(1)}$$

In MATLAB:

- Use the command *System = ss(A, B, C, D);* to create a state-space system.
- Use the command *G = tf(System);* to convert the state-space representation to a transfer function.

```
G =

         -28.92 s^2 - 29.86 s - 139.4
  ----------------------------------------------
  s^4 + 9.414 s^3 + 13.97 s^2 + 48.04 s + 0.4271
```

.............(2)

The poles and zero of the open- loop transfer code is given by the code:

Zeros = zero(G);
Poles = pole(G);
disp('Zeros:');
disp(Zeros);
disp('Poles:');
disp(Poles)

Zeros and Poles of the open loop transfer function are:

```
Zeros:
  -0.5162 + 2.1341i
  -0.5162 - 2.1341i
Poles:
  -8.4328 + 0.0000i
  -0.4862 + 2.3336i
  -0.4862 - 2.3336i
  -0.0089 + 0.0000i
```

Analysis:

$$s^4 + 9.414\,s^3 + 13.97s^2 + 48.04s + 0.42 \quad \text{.....................(3)}$$

Since all the roots of the characteristic equation (3) have negative real parts, thus, the system is said to be dynamically stable. Also, the Routh Hurwitz stability criterion helps us to determine if all the roots of the characteristic equation given by lie in the left half of the s-plane.

1	13.975	0.4271
9.414	48.04	0
8.87196197153176	0.42710000000000004	0
47.58680606015877	0	0
0.4271000000000001	0	0

Routh-Hurwitz Table

Performance and Robustness

	Tuned
Rise time	103 seconds
Settling time	357 seconds
Overshoot	6.02 %
Peak	1.06
Gain margin	Inf dB @ Inf rad/s
Phase margin	73.8 deg @ 0.0157 rad/s
Closed-loop stability	Stable

Figure.2

The characteristic equation satisfies the Routh Hurwitz criterion by inspection because the equation has no missing terms, and the coefficients are all the same sign.

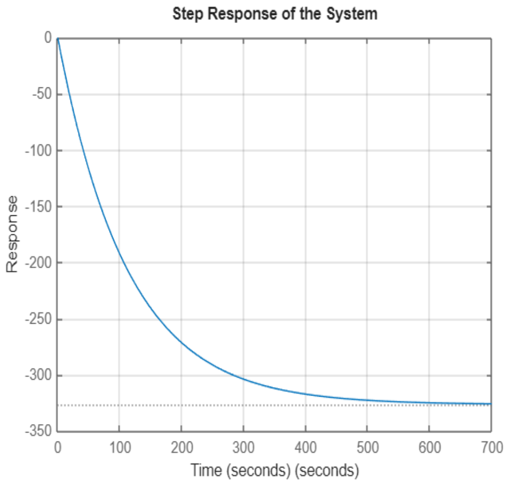


Figure.3

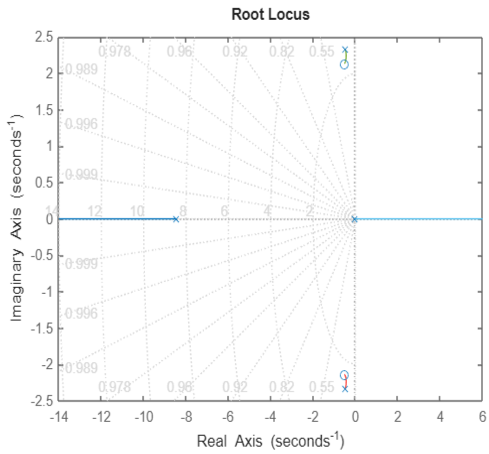


Figure.4

It is seen from Figure.3 that the dynamical characteristics of aircraft not is acceptable as there is an inversion in the loop. Also, the overshoot, rise and settling time must be modified using feedback control as we can see from Figure.2 that they are not in the right way.

To design a controller for the plant, we must first determine if the system is controllable.

MATLAB Code:

u = ctrb(A, B);
n = length(A);
rang = rank(u);
if rang == n
disp('The object is controllable.');
else
disp('The object is not controllable.');
End

Our Requirement:

Settling Time < 5.8 seconds

Using the PID tuner application in Matlab, the plant model in equation is imported and tuned till our requirements are met. Figure.5 shows the configuration of the simple feedback system used in this design.

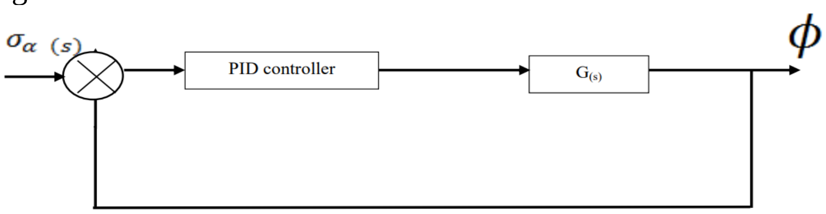


Figure.5

Using the PID tuner application on Matlab, our values for PID controller to be used in this design are:
Kp (Proportional Gain) = -0.68572
Ki (Integral Gain) = -0.34403
Kd (Derivative Gain) = -0.13793

Design:
General form of a PID Controller:

$$G_{(c)} = K_p + K_i x \frac{1}{s} + K_d * s \quad \text{......................(4)}$$

And the closed loop transfer function can be written as:

$$\frac{G_{(c)} * G_{(s)}}{1 + G_{(s)} G_{(c)}} \quad \text{................................(5)}$$

Implementing this in MATLAB we get;

```
New Closed-Loop Transfer Function:
G_CLOSED =

          19.83 s^3 + 30.42 s^2 + 105.9 s + 47.96
  --------------------------------------------------------
  s^5 + 9.414 s^4 + 33.8 s^3 + 78.46 s^2 + 106.3 s + 47.96
```
..................(6)

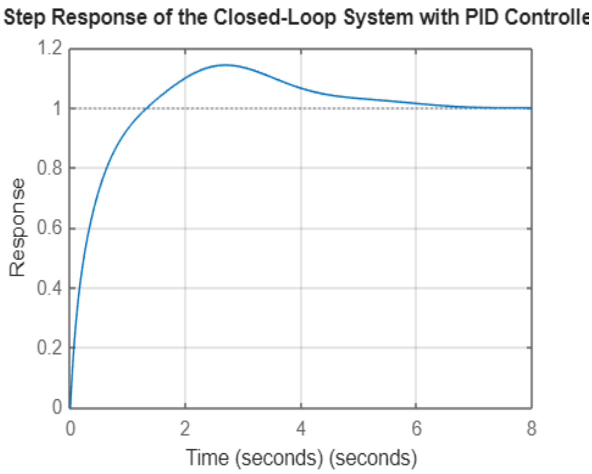


Figure.6

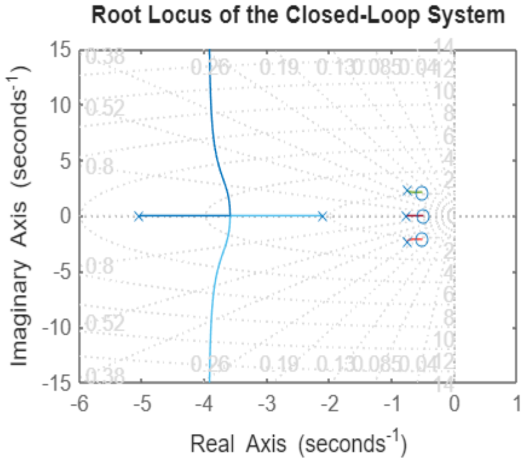


Figure.7

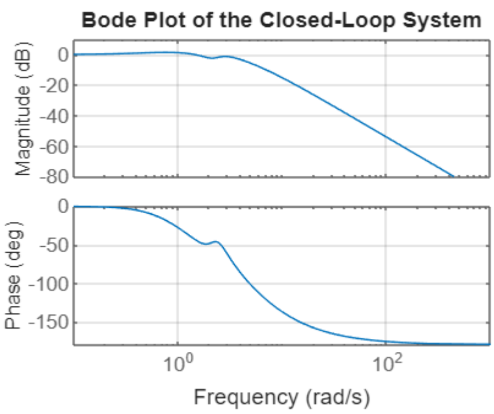
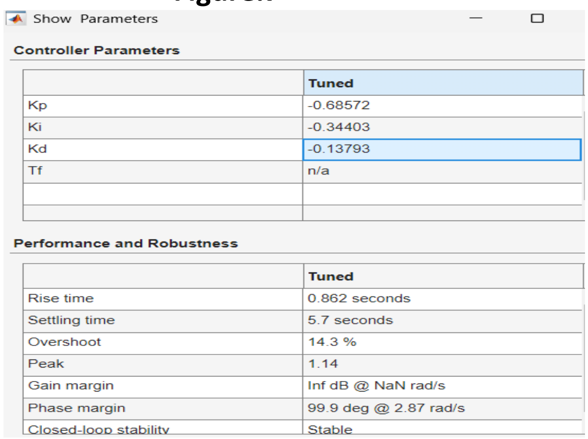


Figure.8


Show Parameters

Controller Parameters

	Tuned
Kp	-0.68572
Ki	-0.34403
Kd	-0.13793
Tf	n/a

Performance and Robustness

	Tuned
Rise time	0.862 seconds
Settling time	5.7 seconds
Overshoot	14.3 %
Peak	1.14
Gain margin	Inf dB @ NaN rad/s
Phase margin	99.9 deg @ 2.87 rad/s
Closed-loop stability	Stable

Figure.9

INFERENCE:

1. The step characteristics of our closed loop system are as shown in Figure.6 above. From the step plot, we can see that our designed controller compensated for the Inversion earlier encountered in the initial system design.
2. The root locus of the closed loop feedback system is as shown in Figure.7. It specifies a stable system as it satisfies the requirement of stability as the entire zeros lie in the left-hand part of the S-plane.
3. We finally can see from Figure.9 that our desired specifications met (i.e.) Settling Time < 5.8 seconds. We got Ts = 5.7 seconds

3.RESULTS:

Description	Theory	Figure.1
Design specification	Ts <5.8 s	
Modelling	Given	Equation. (1), (2)
Before compensation achieved parameters	Ts = 357 s Phase margin=73.8deg Overshoot % = 6.02%	Figure.2
Analysis	Step Response,Root Locus Plot	Equation. (3) Figure.3,4
Design (Type of compensator)	PID Controller	Compensator Equation. (4) General CL Equation. (5) Overall TF Equation. (6) Figure.5
After compensation achieved parameters	Ts = 5.7 s Phase margin=99.9deg Overshoot % = 14.3%	Figure.9
Results	Step Response, Root Locus, Bode Plot	Fig. 6-8

References:

[1] U. Singh and N. S. Pal, "Roll Angle Control of an Aircraft using Adaptive Controllers," 2019 International Conference on Automation, Computational and Technology Management (ICACTM), London, UK, 2019, pp. 143-147, doi: 10.1109/ICACTM.2019.8776731. keywords: {Aircraft;Aerospace control;Adaptation models;Computational modeling;Aircraft propulsion;Atmospheric modeling;Mathematical model;Aircraft roll control;Classical PID controller;Model Reference adaptive controller;Self-tuning Controller;MATLAB},

Work Done by

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