Microsoft

Programming assignment 3
Simple algorithms

Mariia Mykhailova
Senior Software Engineer
Microsoft Quantum Systems

Task 1.1: Phase oracles for Deutsch algorithm

Covered in the lecture!

- $f_0(x) \equiv 0$: do nothing
- $f_1(x) \equiv 1$: R(PauliI, 2.0 * PI(), x);
- $f_2(x) \equiv x$: Z(x);
- $f_3(x) \equiv 1 - x$: X(x); Z(x); X(x);

Can be written shorter using just Z gate and global -1 phase:

```
if (F >= 2) {
    Z(x);
}
if (F % 2 == 1) {
    R(PauliI, 2.0 * PI(), x);
}
```

Task 1.2: Deutsch algorithm with phase oracle

Covered in the lecture!

Careful: you must reset qubit to $|0\rangle$ before returning from operation; `MResetZ` allows to measure and reset immediately.

```
using (x = Qubit()) {
    H(x);
    oracle(x);
    H(x);
    return MResetZ(x) == Zero ? 0 | 1;
}
```

Task 1.3: Phase oracle for DJ algorithm

Implement phase oracle on $\mathbf{2}$ bits for $f(x_1, x_2) = (x_1 = x_2)$.

- We can rewrite the function as $f(x_1, x_2) = x_1 \oplus x_2 \oplus 1$
- The oracle will look as follows:

$$(-1)^{x_1 \oplus x_2 \oplus 1} |x_1 x_2\rangle = -(-1)^{x_1} |x_1\rangle \otimes (-1)^{x_2} |x_2\rangle$$

- We've implemented global phase (-1) before
- And $(-1)^{x_1} |x_1\rangle$ is just the Z gate!

```
ApplyToEachCA(Z, x);
R(PauliI, 2.0 * PI(), x[0]);
```

Task 1.4: Phase oracle for BV algorithm

Implement phase oracle on $\boldsymbol{n}$ bits for $f(x) = \bigoplus s_i x_i$.

- We can rewrite the oracle as

$$(-1)^{\oplus s_i x_i} |x_1 \dots x_n\rangle = \bigotimes (-1)^{s_i x_i} |x_i\rangle$$

- Each term is just $|x_i\rangle$ if $\boldsymbol{s_i} = \boldsymbol{0}$, or $(-1)^{x_i}|x_i\rangle$ if $\boldsymbol{s_i} = 1$
- And that last one is the Z gate, similar to task 1.3

```
for (i in 0 .. Length(x)-1) {
    if (s[i]) { Z(x[i]); }
}
```

Task 2.1: Superdense coding w/ $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

- The easiest solution is for Bob to fix the state of his qubit so that the pair they share is again $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$: apply Z and X gates to his qubit
- After that, use the standard decoding procedure

```
Z(qBob);
X(qBob);
//standard decoding
CNOT(qAlice, qBob);
H(qAlice);
return [M(qAlice) == One, M(qBob) == One];
```

Task 2.2: Teleportation w/ $\frac{1}{\sqrt{2}}(|01\rangle + i|10\rangle)$

- Teleportation kata, tasks 2.1 – 2.3, except with an extra i
- Do the math diligently and do the right fixups for each scenario

```
if (not b2) {
    X(qBob);
}
if (b1 == b2) {
    Adjoint S(qBob);
} else {
    S(qBob);
}
```

Task 2.3: S-gate teleportation w/ $\frac{1}{\sqrt{2}}(|00\rangle - i|11\rangle)$

- https://quantumcomputing.stackexchange.com/questions/6397/quantum-gate-teleportation-t-gate
- https://quantumcomputing.stackexchange.com/questions/1806/what-is-quantum-gate-teleportation

- Do the math diligently and do the right fixups for each scenario, taking into account that you need to end up with $\alpha|0\rangle + i\beta|1\rangle$

```
if (b2) {
    X(qBob);
}
if (b1 == b2) {
    Z(qBob);
}
```