Microsoft

Programming assignment 1
Superposition and entanglement

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General notes

- Make sure your code compiles!
- Try to test your solutions; DumpMachine is your ally
- Q# documentation: https://docs.microsoft.com/quantum
- DumpUnitary tool in Quantum Katas allows to dump the unitary you're implementing (up to relative phases)
  - https://github.com/microsoft/QuantumKatas/tree/master/utilities/DumpUnitary
- Marking scheme: 11 per task, 6 if partially correct, 8 if correct but not demonstrated. Rounded up to nearsest multiple of 5.
- There is no unique way to generate the correct answer.

Task 1.1: Amplitude change

Prepare state $|\psi\rangle = 0.8|0\rangle + 0.6|1\rangle$.

- Superposition kata, task 3:
  to prepare $\cos\alpha|0\rangle + \sin\alpha\,|1\rangle$,
  `Ry(2.0 * alpha, q);`

$$R_y(\theta) \quad \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

- In our case, $\cos\alpha = 0.8, \sin\alpha = 0.6, \alpha = ArcCos\ 0.8$.
- Q# library `Microsoft.Quantum.Math: ArcCos`

```
let alpha = ArcCos(0.8);
Ry(2.0 * alpha, q);
```

Task 1.2: Eigenvectors of Y gate

Prepare state $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + (-1)^{index} i|1\rangle)$.

- Basic gates, task 1.2: to convert $|0\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, `H(q);`
- Basic gates, task 1.5: to convert this to $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$, `S(q);`
- Basic gates, task 1.3: to convert this to $\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$, `Z(q);`

```
H(q);
S(q);
if (index == 1) {
    Z(q);
}
```

Task 1.3: SQRT(NOT)

Prepare state $|\psi\rangle = \frac{1}{2}((1+i)|0\rangle + (1-i)|1\rangle)$.

- Represent $|\psi\rangle$ as $\frac{1}{\sqrt{2}}(|+\rangle + i|-\rangle)$
- Basic gates, task 1.2: to convert $|0\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, `H(q);`
- Basic gates, task 1.5: to convert this to $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$, `S(q);`
- Basic gates, task 1.2: to convert this to $\frac{1}{\sqrt{2}}(|+\rangle + i|-\rangle)$, `H(q);`

```
H(q);
S(q);
H(q);
```

Task 1.4: Separable state

Prepare state $|\psi\rangle = \frac{1}{2}(|00\rangle + i|01\rangle - i|10\rangle + |11\rangle)$.

- Represent $|\psi\rangle$ as $\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$
- Basic gates, task 1.2: to convert $|0\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, `H(q);`
- Basic gates, task 1.5: to convert this to $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$, `S(q);`
- Basic gates, task 1.3: to convert this to $\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$, `S(q);`

```
ApplyToEachA(H, qs);
ApplyToEachA(S, qs);
Z(qs[0]);
```

Task 1.5: Non-separable state - 1

Prepare state $|\psi\rangle = \frac{1}{2}(|00\rangle + i|01\rangle + |10\rangle - |11\rangle)$.

- Start by converting $|00\rangle$ to $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$
- Add -1 phase to $|11\rangle$ term
- Add i phase to $|01\rangle$ term

```
ApplyToEachA(H, qs);
Controlled Z([qs[0]], qs[1]);
(ControlledOnInt(0, S))([qs[0]], qs[1]);
```

Task 1.6: Non-separable state - 2

Prepare state $|\psi\rangle = \frac{1}{2}(|000\rangle + |001\rangle + |010\rangle + |100\rangle)$.

- Start by converting $|000\rangle$ to $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \otimes |0\rangle$
- Need to fix the last term without affecting others: $|110\rangle \rightarrow |001\rangle$
- Use controlled NOT gates: $|110\rangle \rightarrow |111\rangle \rightarrow |101\rangle \rightarrow |001\rangle$

```
ApplyToEachA(H, qs[0..1]);
CCNOT(qs[0], qs[1], qs[2]);
CNOT(qs[2], qs[0]);
CNOT(qs[2], qs[1]);
```

Task 1.7: Non-separable state - 3

Prepare state $|\psi\rangle = \frac{1}{\sqrt{2}}(|001100\ldots\rangle + |110011\ldots\rangle)$.

- Superposition kata, task 8: prepare $\frac{1}{\sqrt{2}}(|000000\ldots\rangle + |111111\ldots\rangle)$
- Need to fix the state of qubits 3, 4, 7, 8, ...: flip them using X gate

```
H(qs[0]);
for (i in 1 .. Length(qs) - 1) {
    CNOT(qs[0], qs[i]);
}
for (i in 0 .. Length(qs) - 1) {
    if (i % 4 > 1) { X(qs[i]); }
}
```

Task 1.8: Separable state

$$|\psi\rangle = |0000\ldots\rangle + |0011\ldots\rangle + |1100\ldots\rangle + |1111\ldots\rangle + \cdots$$

- Can be represented as tensor product of 2-qubit states

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \otimes \cdots$$

- Prepare Bell state on each pair of qubits

```
for (i in 0 .. 2 .. Length(qs) - 1) {
    H(qs[i]);
    CNOT(qs[i], qs[i+1]);
}
```

Task 2.1: Are qubits entangled?

Given the amplitudes of basis states of 2-qubit system, figure out whether the qubits are entangled.

- “Multi-qubit systems” tutorial, exercises 1 and 2: state $\begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix}$ can be represented as a tensor product if and only if $c_0 c_3 = c_1 c_2$
- Deal with floating-point numbers with epsilons!

```
return AbsD(amp[0] * amp[3] - amp[1] * amp[2]) > 1E-6;
```