SOEN6011

DELIVERABLE 1

F2: tan(x)

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Problem 1

1 Introduction

For real arguments, the Tangent function can defined as: the tangent of an angle in a right-angle triangle is the ratio of the length of the opposite leg to the length of the adjacent leg. tan(x) is a periodic tangent function. Also, Tangent function is basically defined by:

$$\tan x = \frac{\sin x}{\cos x}$$

Domain

$$(\ \Theta\ ,\ \Theta \neq k\frac{\pi}{2},\ where\ k\ is\ an\ odd\ integer)$$

Co-Domain

$$(-\infty, \infty)$$

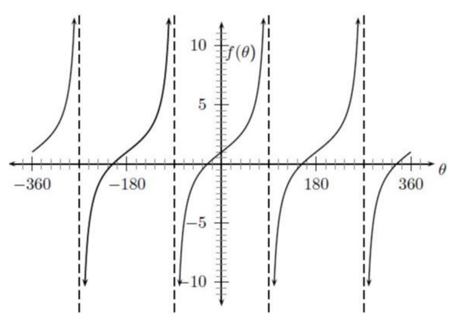

Figure 1: $\tan x$

2 Characteristics

- Period of tangent function is π.
- Vertical Asymptotes: $x = \frac{\pi}{2} + k\pi$, where k is an integer.
- Tangent is an increasing function in every interval between any of two successive vertical asymptotes, i.e $f(x1) < f(x2)\ for\ all\ x1 < x2$.
- Tangent is an odd function with mirror symmetry since $\tan(-x) = -\tan(x)$ and it's graph is symmetric with respect to origin.
- Zeroes of tangent are $n\pi$ for $n\ \in\ Z$, which are same as that of sine function because tangent function will be zero whenever sine function is zero.

Problem 2

3 Requirements

3.1 First requirement

- ID = R1
- Type= Functional Requirement
- Version= 1.0
- Priority= High
- Description= Value of x can never be $\pm 90°, \pm 270°, \pm 450°, \pm 630° \ldots$. Function will throw error message if these values are inputted by user.

3.2 Second requirement

- ID = R2
- Type= Functional Requirement
- Version= 1.0
- Priority= High
- Description= If user input anything except numeric value, function will throw error message of wrong input. For Example: If user input string, function will output Wrong Input

3.3 Third requirement

- ID = R3
- Type= Functional Requirement
- Version= 1.0
- Priority= High
- Description= The value of x should be either in degrees or radians.

4 Assumptions

User always input value of x in $°$ (degree).

Problem 3

5 Algorithms

5.1 Algorithm 1

This algorithm uses polynomial approximation to calculate the value of $\tan x$. This Algorithm assumes that user always input x in degrees. Firstly the value of x is reduced to range of $-90° < x \leq 90°$. After that Taylor series is applied to x (but before that x is converted to radians)

5.1.1 Advantages

- Complexity of this algorithm is low.
- Through this algorithm, we can find tangent of any angle using only the operations of addition, subtraction, multiplication and division.
- Value of x is reduced to small number using periodicity and symmetry of function, so that the approximations are most accurate.

5.1.2 Disadvantages

- Sometimes the calculations become very complex.

Algorithm 1 Pseudocode for calculating $\tan x$

```
procedure tan(x)
    if (−180 > x > 180 ) then
        y = |x|/180
        if (x > 0) then
            x = x − y ∗ 180
        end if
        if (x < 0) then
            x = x + y ∗ 180
        end if
    end if                    ▷ Now,x is in the range of − 180° < x ≤ 180°
    if (−90 > x > 90 ) then
        if (x > 0) then
            x = (x − 180)
        end if
        if (x < 0) then
            x = (x + 180)
        end if
    end if                    ▷ Now,x is in the range of − 90° < x ≤ 90°
    if x equals to 90 or -90 then
        print "Math ERROR"
    else if x = 45 then
        tan(x) is 1
    else if x = -45 then
        tan(x) is -1
    else
        r=x ∗ (π/180)         ▷ Converting degrees to radians
        calculate tan(x) = r + r^3/3 + 2 ∗ r^5/15 + 17 ∗ r^7/315
    end if
end procedure
```

5.2 Algorithm 2

This algorithm calculates tangent function using the formula $\tan x = \frac{\sin x}{\cos x}$. Firstly we calculates the sine of x using Taylor series , than cos of x is calculated using the trigonometric identity $\sin^2 x + \cos^2 x = 1$.

5.2.1 Advantages

- Through this algorithm, we can find sine of any angle using only the operations of addition, subtraction, multiplication and division.

5.2.2 Disadvantages

- The polynomial approximation is accurate to within ± 0.000004.
- Complexity of this algorithm is comparatively high.

Algorithm 2 Pseudocode for calculating $\tan x$

procedure $\tan(x)$
  Calculate $\sin x$
  if $(-360 > x > 360\ )$ **then**
    $y = |x|/360$
    if $(x > 0)$ **then**
      $x = x - y * 360$
    end if
    if $(x < 0)$ **then**
      $x = x + y * 360$
    end if
  end if ▷ Now,x is in the range of $-360° < x \leq 360°$
  if $(-90 > x > 90\ )$ **then**
    if $(-180 > x > 180\ )$ **then**
      if $(-270 < x < 270\ )$ **then**
        if $(x > 0)$ **then**
          $x = (180 - x)$
        end if
        if $(x < 0)$ **then**
          $x = (-x - 180)$
        end if
      else if $(-270 > x > 270\ )$ **then**
        if $(x > 0)$ **then**
          $x = (x - 360)$
        end if
        if $(x < 0)$ **then**
          $x = (360 + x)$
        end if
      end if
    else
      if $(x > 0)$ **then**
        $x = (180 - x)$
      end if
      if $(x < 0)$ **then**
        $x = (-x - 180)$
      end if
    end if ▷ Now,x is in the range of $-90° < x \leq 90°$
    if $x = 90$ **then**
      $\sin(x)$ is 1
    else if $x =$ -90 **then**
      $\sin(x)$ is -1
    else
      r=$x * (\frac{\pi}{180})$ ▷ Converting degrees to radians
      calculate $\sin(x) = r - \frac{r^3}{6} + \frac{r^5}{120}$
    end if
  end if
  Calculate $\cos x$
  $\cos x = \sqrt{1 - \sin^2 x}$
  Calculate $\tan x = \frac{\sin x}{\cos x}$
end procedure

5.3 Selection of Algorithm 1

- The Complexity of this algorithm is comparatively less.
- The result of this Algorithm is comparatively more accurate.

6 References

$http : //mathonweb.com/help_ebook/html/algorithms.htmtan$

$https : //www.siyavula.com/read/maths/grade - 11/functions/05 - functions - 08$