HOMEWORK-1
M.I.S. - COT 5615

PROBLEM - 1.11 ⇒

(a) = word count vector (n vector ω) ⇒

$$\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix}$$

Here $\omega_1, \omega_2 \ldots \omega_n$ elements represent the word count of $1^{st}, 2^{nd} \ldots n^{th}$ words from dictionary respectively in the document.

What is $1^T\omega$ = ?

$$1^T\omega = [1, 1 \text{ ---- } n \text{ times}] \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix}$$

$= 1\cdot\omega_1 + 1\cdot\omega_2 \text{ ------ } 1\cdot\omega_n$

$= \omega_1 + \omega_2 \text{ ---- } \omega_n$

= Total number of words in the document.

(b) = what does $\omega_{282} = 0$ mean?

It means that word count of 282 th word from the dictionary in the document is zero. In other words, word number 282 from the dictionary is not present in the document.

(C) = n-vector h represents the histogram of the word counts. i.e. fraction of the words in the document that are word i.

$$h = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \end{bmatrix} \quad \text{and word count vector } \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix}$$

word fraction (h_i) for any ith word could be represented as $= \dfrac{\text{word count of ith word}}{\text{Total number of words in the document.}}$

$$h_i = \frac{w_i}{w_1 + w_2 + w_3 \cdots w_n}$$ [where $i = 1, 2, \cdots n$]

$$\boxed{h_i = \frac{e_i^T w}{1^T w}}$$

[$e_i^T w$ = word count (occurrence) of ith word from dictionary in the document.
$1^T w$ = Total number of words in the document]

PROBLEM - 1.12 ⇒

currency vector (5-vector c) = $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix}$

where c_1 = amount in USD currency
c_2 = amount in RMB (chinese yuan) currency
c_3 = amount in EUR (euro) currency
c_4 = amount in GBP (british pound) currency
c_5 = amount in JPY (Japanese yen) currency.

lets consider a 5-vector r, which represents the currency exchange rates from a particular currency to US dollars.

currency exchange rate vector = r = $\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix}$

where r_1 = Exchange rate USD to USD (i.e. 1)
r_2 = Exchange rate from (RMB to USD) (1 RMB = 0.14 USD)
r_3 = Exchange rate from (EUR to USD) (1 EUR = 1.11 USD)
r_4 = Exchange rate from (GBP to USD) (1 GBP = 1.22 USD)
r_5 = Exchange rate from (JPY to USD) (1 JPY = 0.0095 USD)

considered rates are current exchange rates.

c_1, c_2, c_3, c_4, c_5 → amounts could be Positive or negative (if its the owed amount)

Total Value of CASH in (USD) $= r^T C$

$$= \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix}^T \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix}$$

$$\Rightarrow \cancel{\begin{bmatrix} r_1, & r_2, & r_3, & r_4, & r_5 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix}$$

$$\Rightarrow r_1 c_1 + r_2 c_2 + r_3 c_3 + r_4 c_4 + r_5 c_5$$

$$\boxed{\text{So Total value of cash (USD)} = r_1 c_1 + r_2 c_2 + r_3 c_3 + r_4 c_4 + r_5 c_5}$$

if we substitute current exchange rates then-

$$\boxed{\text{Total cash value (USD)} = 1 \cdot c_1 + (0.14) c_2 + (1.11) c_3 + (1.22) c_4 + (0.0095) c_5}$$

Here values of c_1, c_2, c_3, c_4, c_5 amounts could be positive or negative (if the amount is liability)

PROBLEM-1.13 $\Rightarrow$

Distribution of ages represented by 100-vector x.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{100} \end{bmatrix}$$

Here x_i = number of people of age $(i-1)$
x_1 = number of people of age $(1-1)=0$
x_2 = number of people of age 1 yr.
$\vdots$
x_{100} = no. of people of age 99 yrs.

(a) = Total number of people in the population =
Sum of number of people in all age groups.

Total number of people = $x_1 + x_2 + x_3$ ---- x_{100}

$$\boxed{\text{Total number of people} = \mathbf{1}^T x}$$

(b) = Total number of people in population age 65 & over?

Subvector $x_{66:100}$ will represent people in population of age 65 and over.

$$x_{66:100} = \begin{bmatrix} x_{66} \\ x_{67} \\ \vdots \\ x_{100} \end{bmatrix}$$

where x_{66} = no. of people of age 65 yrs.
x_{67} = no. of people of age 66 yrs.
$\vdots$
x_{100} = no. of people of age 99 years.

Hence Total no. of people in population age 65 and over =

$= x_{66} + x_{67} + x_{68}$ ---- x_{100}

$$\boxed{\text{no. of people of age 65 \& over} = \mathbf{1}^T x_{66:100}}$$

(C) = average age of Population could be calculated as →

$$\text{average age of population} = \frac{\sum_{i=1}^{100} (i-1)\, x_i}{\sum_{i=1}^{100} x_i}$$

$$\Rightarrow \frac{0 \times x_1 + 1 \times x_2 + 2 \times x_3 - - - - - 99 \times x_{100}}{x_1 + x_2 + x_3 - - - - + x_{100}}$$

Lets assume a is a 100-vector which represents age group from 0 to 99 yrs.

$$a = \begin{bmatrix} 0 \\ 1 \\ 2 \\ \vdots \\ 99 \end{bmatrix}$$

Then we can write average age of population as ⇒

$$\Rightarrow \frac{a^T x}{1^T x}$$

$$\boxed{\text{average age of population} = \frac{a^T x}{1^T x}}$$

where $a = \begin{bmatrix} 0 \\ 1 \\ 2 \\ \vdots \\ 99 \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{100} \end{bmatrix}$

Problem-1.19 =

$$\hat{z}_{t+1} = (z_t, z_{t-1} \cdots z_{t-M+1})^T \beta, \quad \text{wher } t = M, M+1 \cdots$$

Time Period = daily , M = 10

Then $\hat{z}_{t+1}$ = represents AR model prediction.

Since we want to predict tomorrows value lets put

$t = M = 10$

Then

$$\hat{z}_{10+1} = (z_{10}, z_9, z_8 \cdots z_{10-10+1})^T \beta$$

$$\hat{z}_{11} = (z_{10}, z_9, z_8 \cdots z_1)^T \beta$$

Here z_{11} = represent AR model prediction for tommorrow i.e. for 11th day.

$(z_{10}, z_9 \cdots z_1)$ = values for respective days.

(a) = $\beta \approx e_1$

Then $\hat{z}_{11} = \begin{bmatrix} z_{10} \\ z_9 \\ z_8 \\ \vdots \\ z_1 \end{bmatrix}^T \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ (both are 10-vectors)

$$\hat{z}_{11} = 1 \cdot z_{10} + 0 \cdot z_9 + 0 \cdot z_8 \cdots 0 \cdot z_1$$

$$\boxed{\hat{z}_{11} = z_{10}}$$

i.e. Prediction value By AR model = Todays value (which is 10th day - today)

(b) = $\beta \approx 2e_1 - e_2$

Then $\beta \approx 2 \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ (assuming e_1, e_2 of order 10)

$\beta \approx \begin{bmatrix} 2 \\ -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

Then

$\hat{z}_{11} = \begin{bmatrix} z_{10} \\ z_9 \\ \vdots \\ z_1 \end{bmatrix}^T \begin{bmatrix} 2 \\ -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ (both are 10-vectors)

$\hat{z}_{11} = 2 \cdot z_{10} - 1 \cdot z_9 + 0 \cdot z_8 - - - + 0 \cdot z_1$

$\boxed{\hat{z}_{11} = 2z_{10} - z_9}$

i.e. Tomorrows Predicted value = 2 (Todays value) − (yesterdays value)

(C) = $\beta \approx e_6$

Then $\hat{z}_{11} = \begin{bmatrix} z_{10} \\ z_9 \\ \vdots \\ z_1 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ (Both are 10-vectors)

$\hat{z}_{11} = 0 \cdot z_{10} + 0 \cdot z_9 + 0 \cdot z_8 + 0 \cdot z_7 + 0 \cdot z_6 + 1 \cdot z_5 + 0 \cdot z_4 + 0 \cdot z_3 + 0 \cdot z_2 + 0 \cdot z_1$

$\boxed{\hat{z}_{11} = z_5}$

i.e. Tomorrows Predicted value = value of the 5th day from beginning

PROBLEM - 1.20 ⇒

(1) = no. of Bytes required to store a n-vector = $8n$

Hence number of Bytes required to store 10^5-vector =

$\Rightarrow 8 \times 10^5$

Hence number of Bytes required to store 100, 10^5-vectors

$\Rightarrow 100 \times 8 \times 10^5$

100, 10^5-vectors require $\Rightarrow 8 \times 10^7$ Bytes

(2) = linear combination of 100, 10^5-vector with 100 non zero coefficients.

linear combinations $= \beta_1 v_1 + \beta_2 v_2 + \beta_3 v_3 - \cdots \beta_{100} v_{100}$

Total number of multiplications involved here =

$\Rightarrow 100 \times 10^5 = 10^7$ multiplications.

Total number of addition operations involved here =

$\Rightarrow 99 \times 10^5$ additions.

Total number of flops $= 100 \times 10^5 + 99 \times 10^5$

$= 10^5(100 + 99)$

$= 199 \times 10^5$

(3) = How long the above operation will take on computer capable of doing 1 Gflop/sec.

Speed of Computer = 10^9 flops/sec.

$$\text{Time taken to do Linear combination} = \frac{199 \times 10^5}{10^9}$$

$$= \frac{199}{10^4}$$

Time Taken = 0.0199 seconds.

= 19.9 Milli Seconds.