EEL6825: Pattern Recognition

Homework 1

Problem 25 in Chapter 2, on Page 72 of the following textbook: Richard O. Duda, Peter E. Hart, David G. Stork, "Pattern Classification", 2nd Edition, Wiley-Interscience, October 2000.

Problem 25 is given as below:

Fill in the steps in the derivation from Eq. 59 to Eqs. 60 - 65, which are given as follows:

$$g_i(\mathbf{x}) = -\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_i)^t \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu}_i) + \ln \boldsymbol{P}(\omega_i).$$
 (59)

$$q_i(\mathbf{x}) = \mathbf{w}_i^t \mathbf{x} + w_{i0},\tag{60}$$

where

$$\mathbf{w}_i = \mathbf{\Sigma}^{-1} \boldsymbol{\mu}_i \tag{61}$$

$$w_{i0} = -\frac{1}{2}\boldsymbol{\mu}_i^t \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}_i + \ln \boldsymbol{P}(\omega_i).$$
 (62)

$$\mathbf{w}^t(\mathbf{x} - \mathbf{x}_0) = 0, (63)$$

where

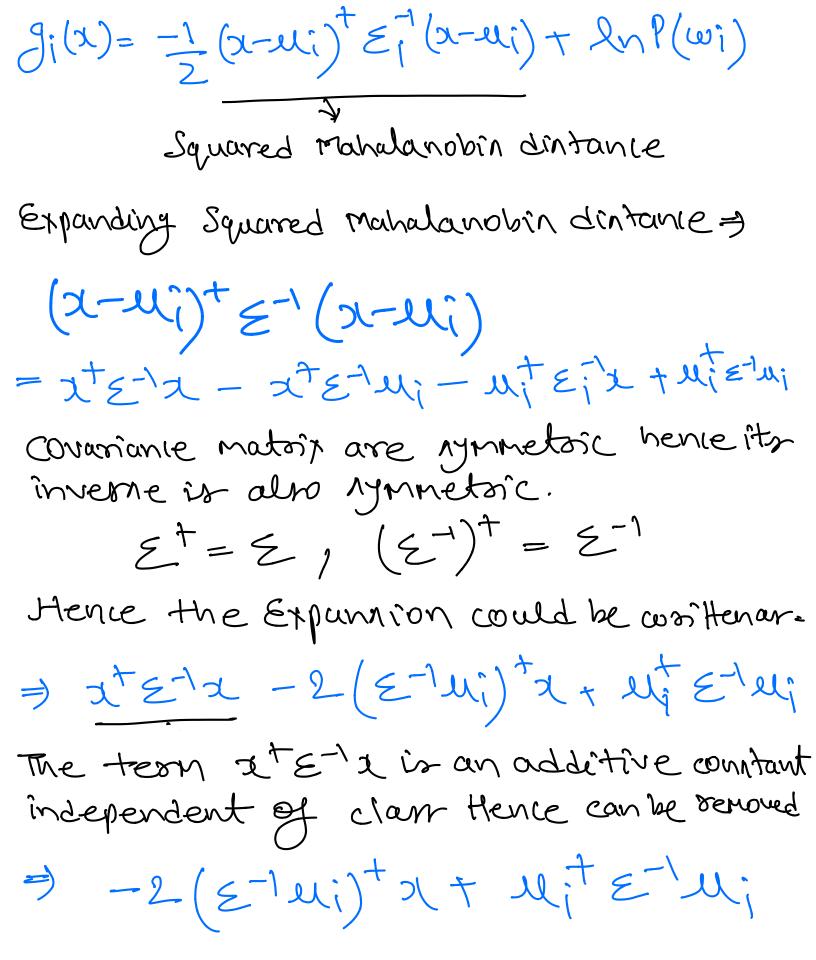
$$\mathbf{w} = \mathbf{\Sigma}^{-1}(\boldsymbol{\mu}_i - \boldsymbol{\mu}_i) \tag{64}$$

$$\mathbf{x}_0 = \frac{1}{2} (\boldsymbol{\mu}_i + \boldsymbol{\mu}_j) - \frac{\ln[\boldsymbol{P}(\omega_i)/\boldsymbol{P}(\omega_j)]}{(\boldsymbol{\mu}_i - \boldsymbol{\mu}_j)^t \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu}_i - \boldsymbol{\mu}_j)} (\boldsymbol{\mu}_i - \boldsymbol{\mu}_j). \tag{65}$$

Hwr. = The dincriminant function is given or-3;(x)= lnp(x/wi) + lnP(wi) when $P(x|w_i) \sim N(w_i, \varepsilon_i)$ and $P(x) = \frac{1}{(2\pi)^{dh}} |z|^{h} erp \left[-\frac{1}{2}(x-u)^{t}z^{-1}(x-u)\right]$ The general form of discriminant function= g;(x)= -1 (x-ui) =; (x-ui) -dln27 - 1 ln/2i/ + lnP(wi)

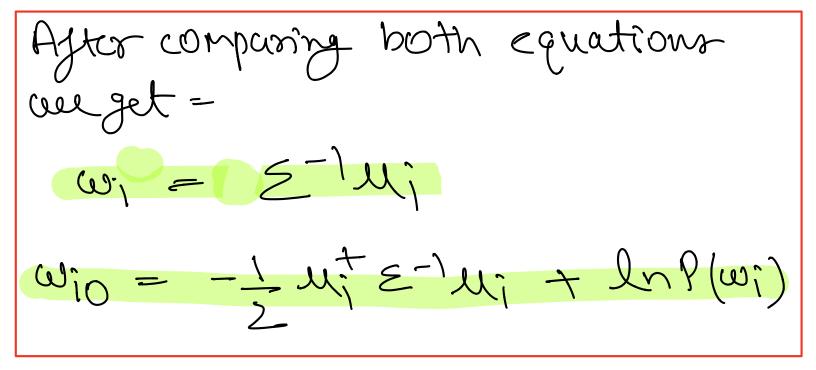
when covariances are name, i-e. $\leq_i = \leq$ Terms marked as yellow could be removed

 $g_i(x) = -\frac{1}{2}(x-u_i)^{T} E_i^{-1}(x-u_i) + \ln P(w_i)$



Thus Squared malhalanobin dintance is expanded to = $(x-u_i)^{+} z^{-1} (x-u_i) =$ -2(E-141)+2+ 41 =-14; Putting this value in J; (X) are get = gi(x)= -1 (x-ui) + E/ (x-ui) + ln P(wi) $\mathcal{G}_{i}(x) = -\frac{1}{2} \left[-2 \left(\varepsilon^{-1} u_{i} \right)^{+} x + u_{i}^{+} \varepsilon^{-1} u_{i} \right] + \ln P(w_{i})$ Jila)= (5-1 mi) ta - 1 mit = 1 mi + ln P(wi) linear dincriminant function gila) is given as = g;(x)= w; x + w;o and we obtained

gi(x)=(5-hi)ta-tuis-luit lnP(wi)



We know that Decinion boundary:
$$3(0) = 3(x)$$

$$\Rightarrow (E M_i)^{\dagger} x - \frac{1}{2} M_i^{\dagger} E^{\dagger} M_i + \ln P(w_i)$$

$$= (E M_j)^{\dagger} x - \frac{1}{2} M_j^{\dagger} E^{\dagger} M_i + \ln P(w_j)$$

$$\Rightarrow E M_i^{\dagger} x - \frac{1}{2} M_i^{\dagger} E^{\dagger} M_i + \ln P(w_i)$$

$$= E M_j^{\dagger} x - \frac{1}{2} M_j^{\dagger} E^{\dagger} M_i + \ln P(w_j)$$

$$= \frac{1}{2} \left(\frac{w_1^2 - w_2^2}{w_3} \right) = \frac{1}{2} \left(\frac{w_1^2 - w_2^2}{w_3^2} \right) = 0$$

$$= \frac{1}{2} \left(\frac{w_1^2 - w_2^2}{w_3^2} \right) + \frac{1}{2} \left(\frac{p(w_1)}{p(w_2)} \right) \left(\frac{p(w_1)}{p$$