CP312 Algorithm Design and Analysis I **Winter 2024**

Assignment 5

Instructor: Dariush Ebrahimi *Due Date: 10-Apr-2024*

Instructions: You must submit your solutions as a single PDF file to MyLS. Make your solutions as detailed as possible by clearly stating every step in your answers. The assignment must be done individually. Any **COPYING** of solutions from external sources will result in a **ZERO** grade.

Problems

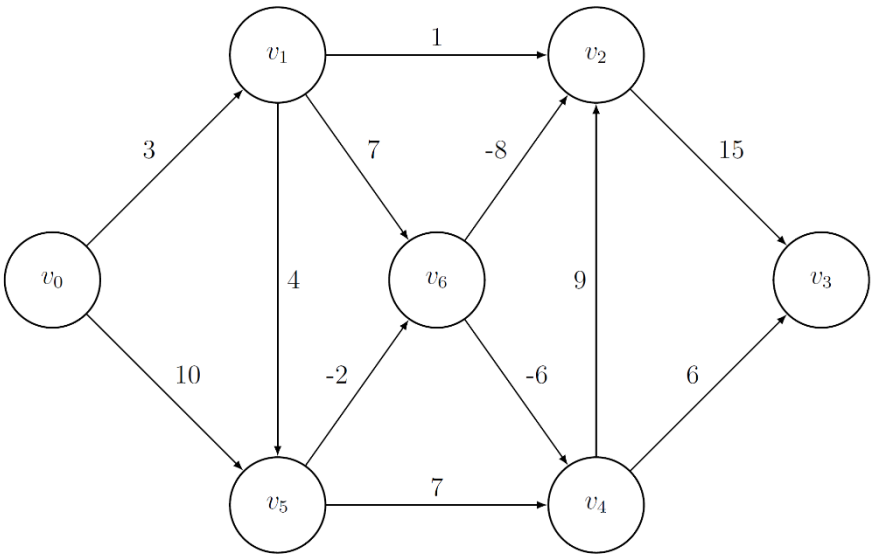
1. **Shortest Path.** In this question, you will examine the shortest path algorithms more closely and apply them to a sample graph.

(a) (5 points) The algorithms that we saw in class for finding the shortest path only keep track of the *length* of the shortest path from the source vertex to every other node.

i Explain how you can modify Dijsktra's algorithm to output the shortest path itself, not just its length; that is to show how to track the shortest path from the source to the destination.

ii Explain how you can modify the Bellman-Ford algorithm to output the shortest path itself, not just its length.

(b) You are given the following directed weighted graph $G = (V, E)$:

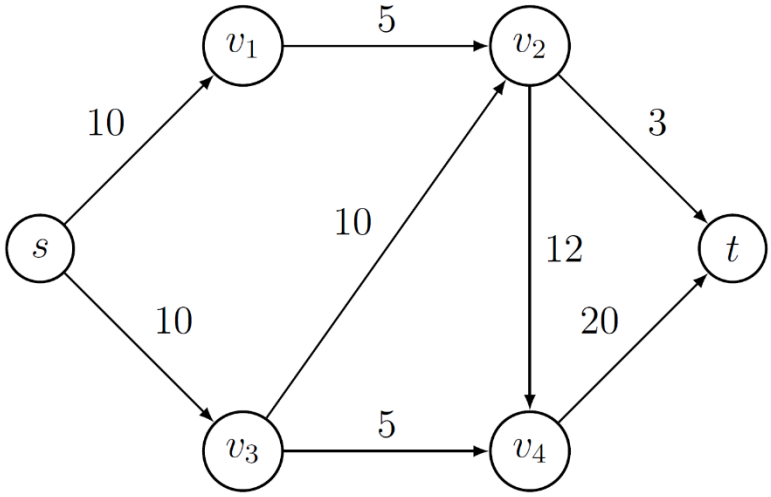


i. (10 points) Use the Bellman-Ford algorithm to determine the shortest path from node v_0 to node v_2. You may use any ordering of the edges. Show all your work including the distance values at the end of every pass.

ii. (8 points) Apply Dijkstra's algorithm on graph *G* with the source vertex as v_0 to find the shortest path to every other node. Show all your work.

iii. (2 points) State whether the shortest path from v_0 to v_2 that you obtained in part (ii) is the same as part (i). Explain your answer.

2. **Max-flow and Min-cut.** You are given the following flow graph $G = (V, E, s, t)$ representing different cities in a county with the indicated edge capacities representing the roads' traffic load capabilities.



a.) (15 points) Use the Ford-Fulkerson algorithm to find the maximum flow for graph *G*. In every iteration, show the residual network, the augmented path you chose, and the updated flows.

b.) (5 points) Determine the cut (by specifying the edges in the cut) with the capacity equal to the maximum flow you found in part (a). Explain why this is the minimum cut.

3. (5 points) **Strongly Connected.** Consider the graph below. Identify the strongly connected components of the graph.