Assignment 2

Group 13
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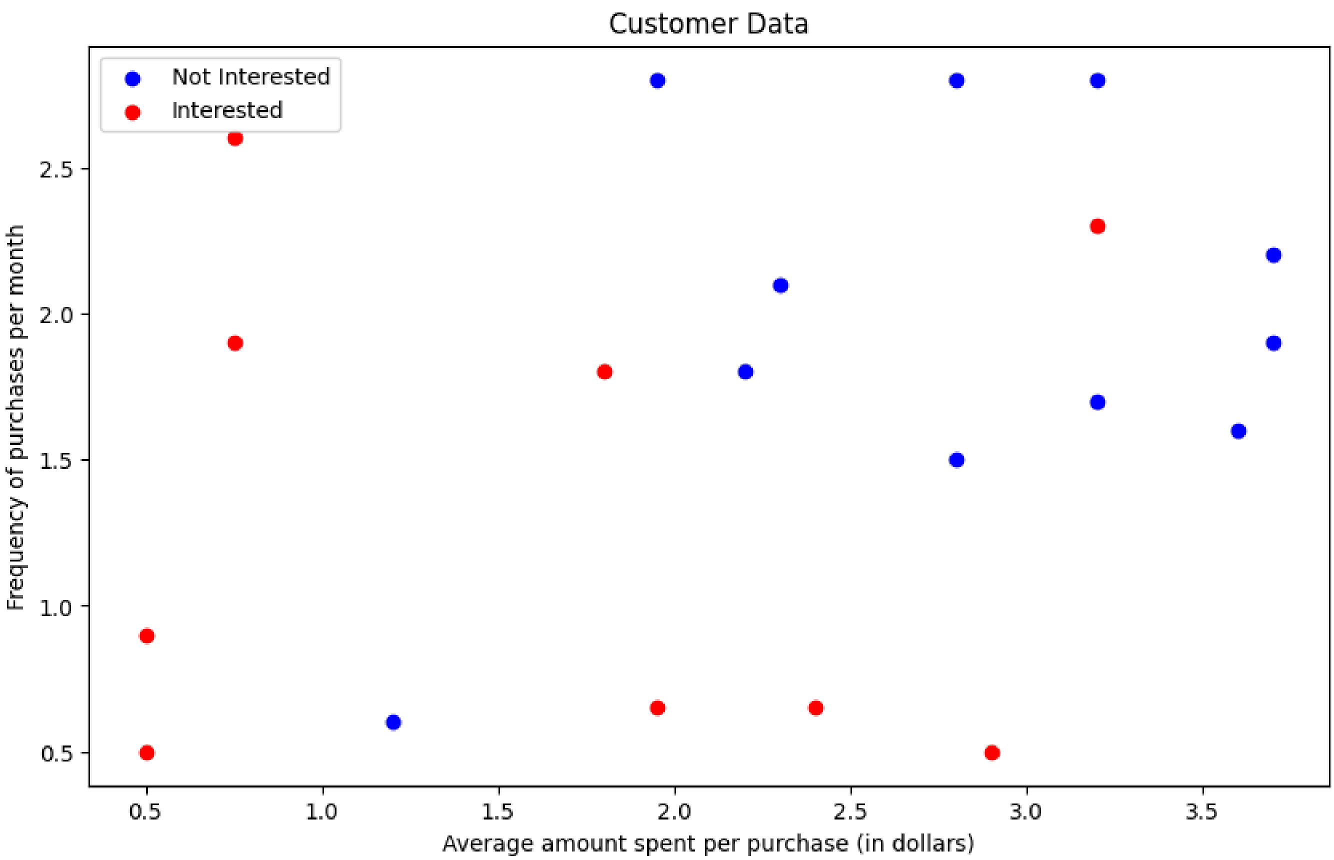
Question 1

1.A



1.B

```python
import math

#Euclidian distance
def euclidean_distance(point1, point2):
    return math.sqrt((point1[0] - point2[0])**2 + (point1[1] - point2[1])**2)



#K Nearest Neighbours
def fnKNN(dataset, new_point, k):
    distances = []
    #Find distances

    for data in dataset:
        x1, x2, c = data
        dist = euclidean_distance((x1, x2), new_point)
        distances.append((dist, c))

    #Find k nearest
    distances.sort()
    neighbors = distances[:k]

    #Get prediction
    votes = {0: 0, 1: 0}
    for neighbor in neighbors:
        votes[int(neighbor[1])] += 1

    predicted_label = 1 if votes[1] > votes[0] else 0
    return predicted_label
```

1.C

```python
import random

#assess the performance
def assess_knn(data, k, ratio):
    #Shuffle data
    random.seed(42)
    shuffled = data.copy()
    random.shuffle(shuffled)

    #Split dataset
    train_size = int(len(shuffled) * ratio)
    train_data = shuffled[:train_size]
    test_data = shuffled[train_size:]

    #Test accuracy of KNN
    correct = 0
    for point in test_data:
        x1, x2, c = point
        predicted_c = fnKNN(train_data, (x1, x2), k)
        if predicted_c == c:
            correct += 1

    accuracy = correct / len(test_data)
    return accuracy

data = df[["x1", "x2", "c"]].values.tolist()

#Test splits with k = 1
acc_80 = assess_knn(data, 1, 0.8)
acc_60 = assess_knn(data, 1, 0.6)
acc_50 = assess_knn(data, 1, 0.5)

print(f"80% train: {acc_80:.2f}")
print(f"60% train: {acc_60:.2f}")
print(f"50% train: {acc_50:.2f}")
```

80% train: 1.00
60% train: 0.75
50% train: 0.50

1.D

PYTHON

```
results = []
for k in range(1, 5):
    acc_80 = assess_knn(data, k, 0.8)
    acc_60 = assess_knn(data, k, 0.6)
    acc_50 = assess_knn(data, k, 0.5)
    results.append((k, acc_80, acc_60, acc_50))

print("K\t80%\t60%\t50%")
for result in results:
    print(f"{result[0]}\t{result[1]:.2f}\t{result[2]:.2f}\t{result[3]:.2f}")
```

K	80%	60%	50%
1	1.00	0.75	0.50
2	0.75	0.50	0.50
3	1.00	0.75	0.80
4	0.50	0.38	0.50

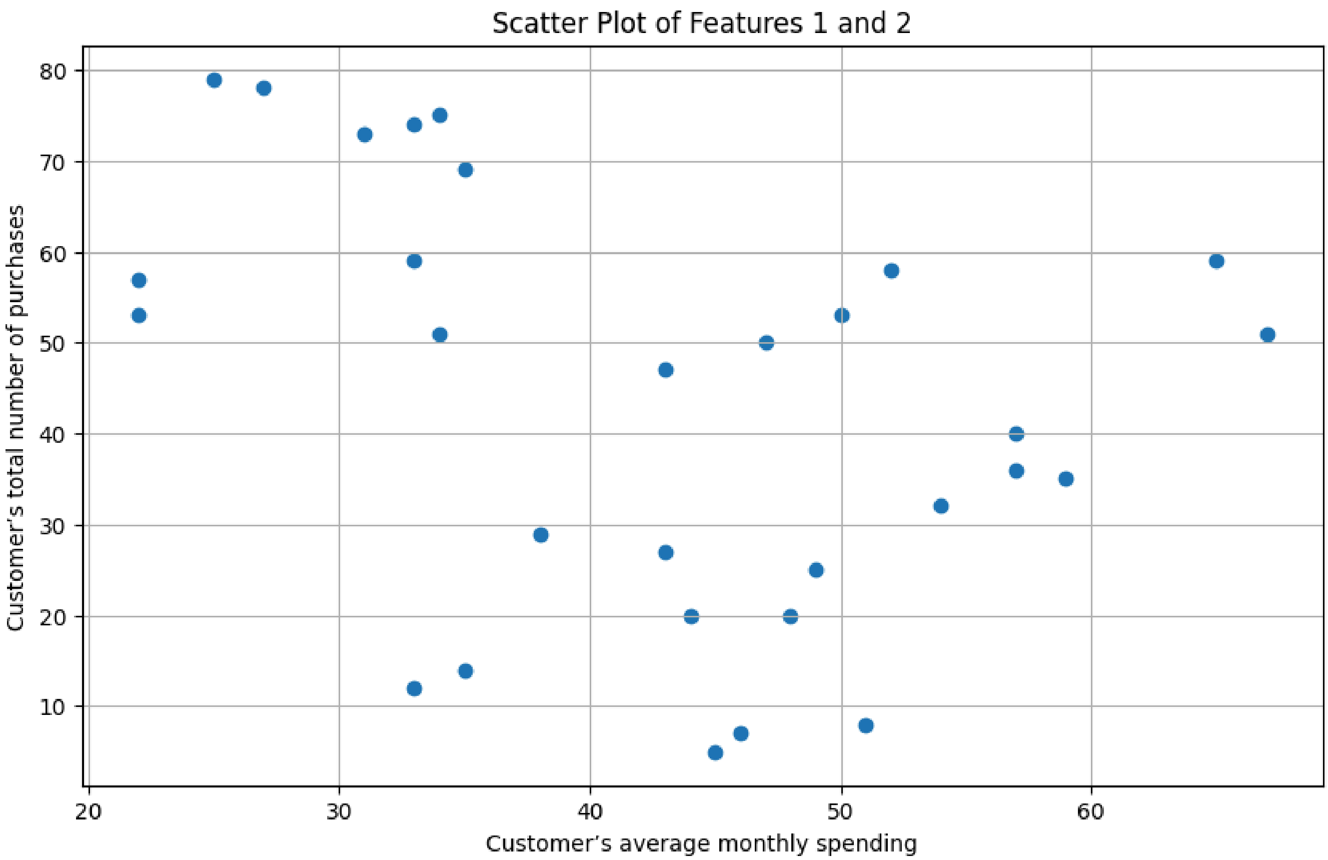
K affects performance in the way that a lower value may be influenced by noise while higher values may underfit and miss smaller trends. The training set also affects the performance in the way that larger training sets are most often better and generalize better.

1.E

The best combinations are K = 1 and k = 2 at an 80% split with both having a 100% accuracy rate.

Question 2

1.A


Scatter Plot of Features 1 and 2
Customer's total number of purchases
Customer's average monthly spending
80
70
60
50
40
30
20
10
20
30
40
50
60

1.B

PYTHON

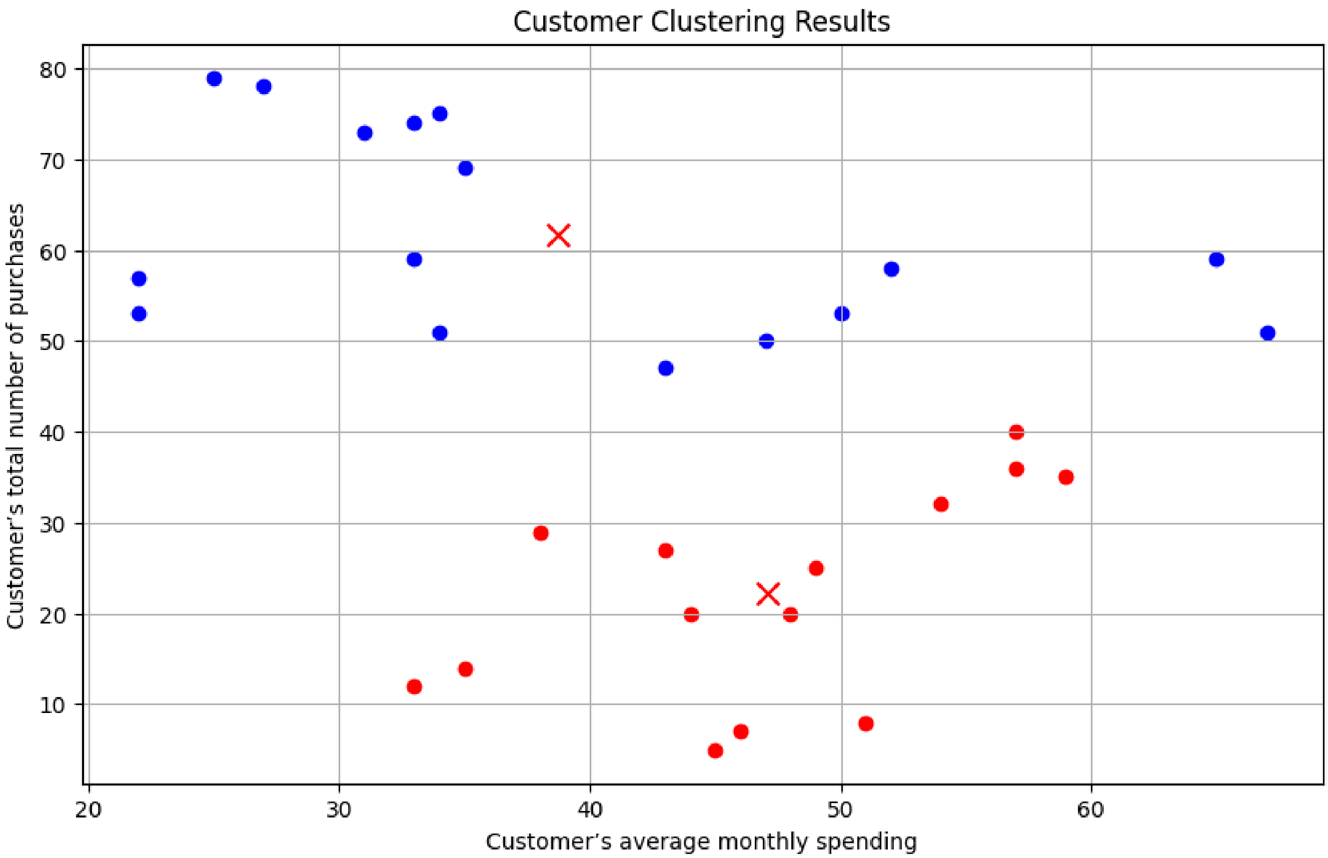
```
import random
import math
from copy import deepcopy

#Euclidian Distance
def euclidean_distance(x, y):
  return math.sqrt((x[0] - y[0])**2 + (x[1] - y[1])**2)

#K_Means
def k_means(data, k, max_iterations):
    #Initialize centroids
    centroids = deepcopy(data[:k])

    for iteration in range(max_iterations):
        #Assign clusters
        clusters = [[] for _ in range(k)]
        for point in data:
            distances = [euclidean_distance(point, centroid) for centroid in
centroids]
            cluster_index = distances.index(min(distances))
            clusters[cluster_index].append(point)

    #New centroids
    new_centroids = []
    for cluster in clusters:
        if cluster:
            x_vals = [point[0] for point in cluster]
            y_vals = [point[1] for point in cluster]
            new_centroids.append([sum(x_vals) / len(x_vals), sum(y_vals) /
len(y_vals)])
        else:
            new_centroids.append(centroids[clusters.index(cluster)])
    #Check changes
    if all(euclidean_distance(c1, c2) <0.001 for c1, c2 in zip(centroids,
new_centroids)):
      break
    centroids = new_centroids

    #Labels
    labels = []
    for point in data:
        distances = [euclidean_distance(point, centroid) for centroid in
centroids]
        labels.append(distances.index(min(distances)))
```

```
    return labels, centroids
```

2.C

```python
#Change to list
data_points = df[["f1", "f2"]].values.tolist()

#K_Means
labels, centroids = k_means(data_points, 2, 100)

#Plot
plt.figure(figsize=(10, 6))

colors = ["blue", "red"]
for i, point in enumerate(data_points):
    plt.scatter(point[0], point[1], c=colors[labels[i]])

for i, centroid in enumerate(centroids):
    plt.scatter(centroid[0], centroid[1], c="red", marker="x", s=100)

plt.xlabel("Customer’s average monthly spending")
plt.ylabel("Customer’s total number of purchases")
plt.title("Customer Clustering Results")
plt.grid(True)
plt.show()
```

Centroid 1: x1 = 38.75, x2 = 61.62
Centroid 2: x1 = 47.07, x2 = 22.14



2.D

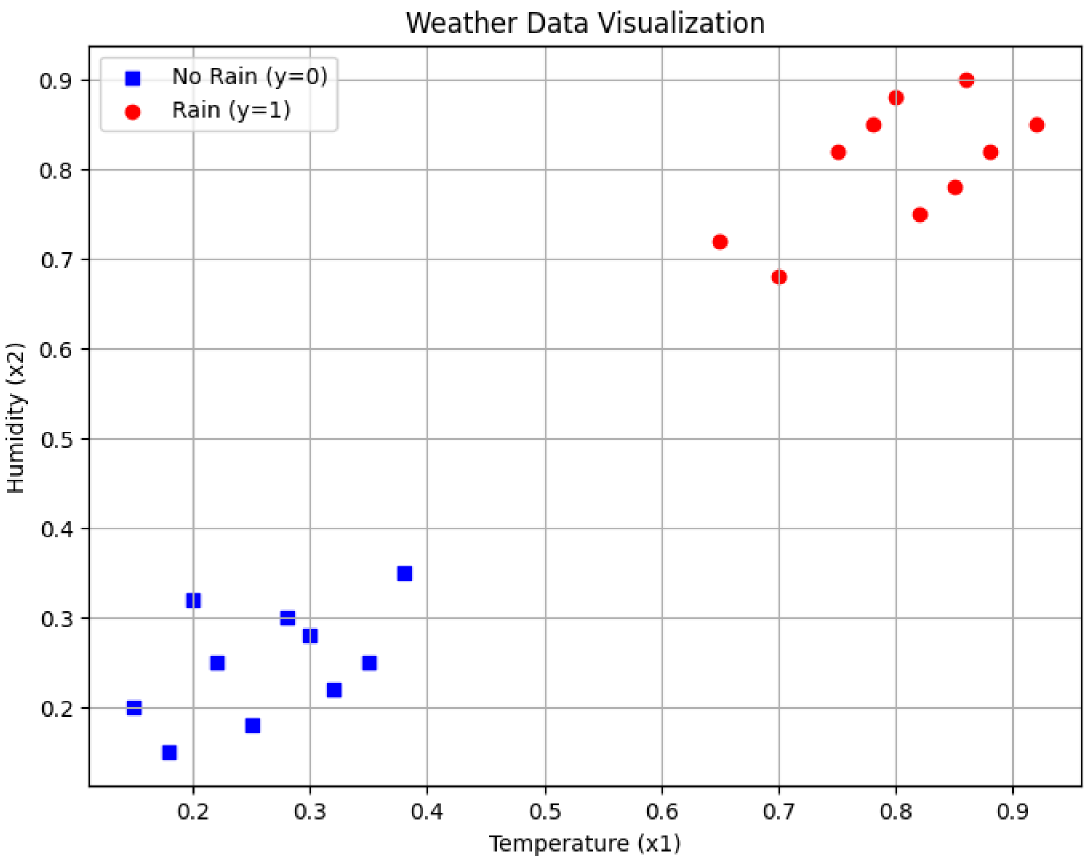
Cluster 1: 14 data points
Cluster 2: 16 data points

2.E

Centroid 1: (47.07, 22.14)
Centroid 2: (38.75, 61.62)

Question 3

3.A



Weather Data Visualization
No Rain (y=0)
Rain (y=1)
Humidity (x2)
Temperature (x1)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

3.B

```python
import pandas as pd
import numpy as np

# Load the dataset
weather_data = pd.read_csv("WeatherData_Q3.csv")  # Change to your actual
path if needed

# Prepare the features and labels
X = weather_data[['temp', 'humid']].values
y = weather_data['rain'].values

# Split into training (first 15) and test set (last 5)
X_train, y_train = X[:15], y[:15]
X_test, y_test = X[15:], y[15:]

# Initialize perceptron parameters
learning_rate = 0.1
max_iterations = 1000
np.random.seed(42)
weights = np.random.uniform(-0.5, 0.5, size=2)
bias = np.random.uniform(-0.5, 0.5)

# Step activation function
def activation(z):
    return 1 if z >= 0 else 0

# Train the perceptron
for epoch in range(max_iterations):
    errors = 0
    for xi, target in zip(X_train, y_train):
        z = np.dot(weights, xi) + bias
        prediction = activation(z)
        error = target - prediction
        if error != 0:
            weights += learning_rate * error * xi
            bias += learning_rate * error
            errors += 1
    if errors == 0:
        break

# Prediction function
def predict(X):
    return [activation(np.dot(weights, xi) + bias) for xi in X]
```

```python
# Evaluate accuracy
train_predictions = predict(X_train)
test_predictions = predict(X_test)
train_accuracy = np.mean(train_predictions == y_train)
test_accuracy = np.mean(test_predictions == y_test)

# Output results
print("Final Weights:", weights)
print("Final Bias:", bias)
print("Training Accuracy:", train_accuracy)
print("Test Accuracy:", test_accuracy)
```

Final Weights: [-0.10245988 0.45371431]
Final Bias: -0.16800605818859493
Training Accuracy: 1.0
Test Accuracy: 1.0

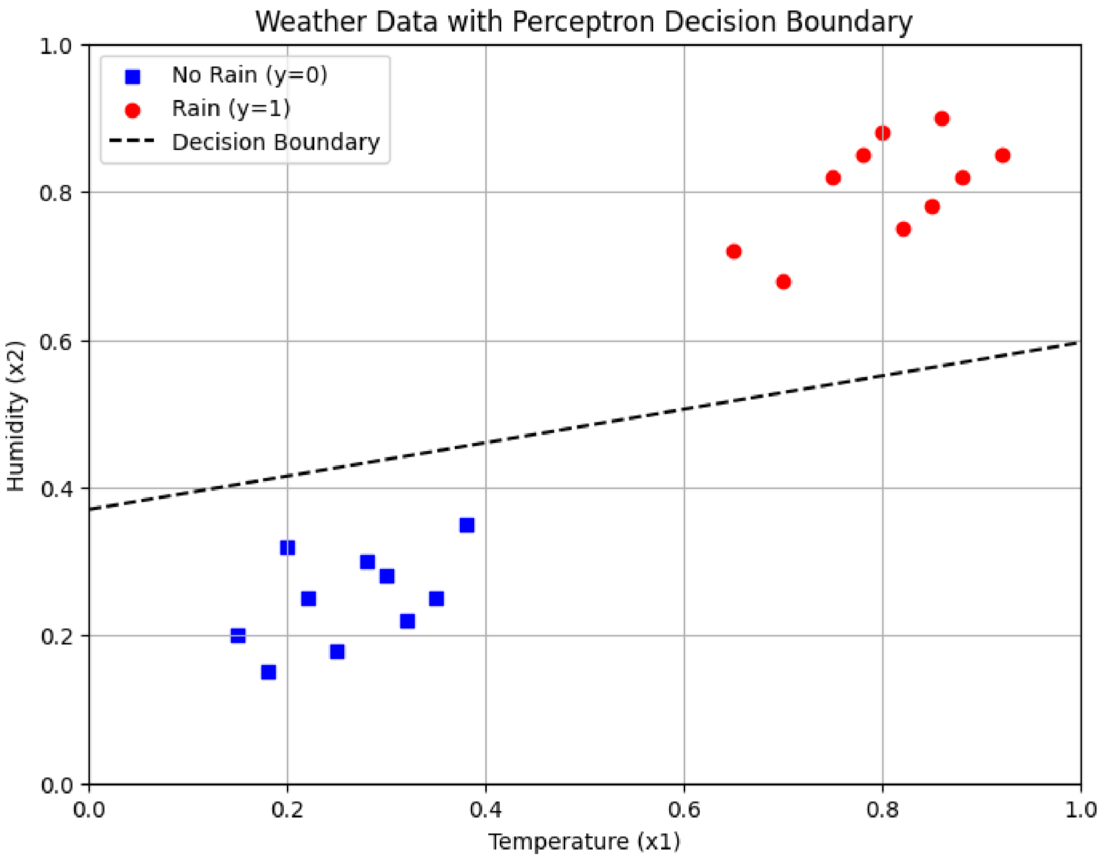
1.C

Does the Perceptron separate the two classes (Rain vs. No Rain)?:
Based on the plot and accuracy on both the training and test sets, the Perceptron does successfuly sepreate the data into the two classes. This indicates that the dataset is linearly separable, which is exactly what the Perceptron is designed for.

Based on the dataset's pattern, explain why or why not the perceptron works well (or fails):
The perceptron works well because the relationship between temperature and humidity appears to be linearly separable, meaning there exists a straight line that can divide the two classes (rain vs. no rain).Also the data points for each class (Rain = 1, No Rain = 0) form clear, non-overlapping clusters in the 2D graph. The problem is simple as it only contains 2 features and clear separation which makes it ideal for a Perceptron.

Suggestions for Improvement:
Possible improvements are that we could've added code to show how many times the model loops (epochs) before learning the pattern. This helps understand how fast the model learns. Also it could've been helpful to calculate accuracy on the test set during training to see if the model is improving or not. A different could have also been beneficial because if the data was not linearly separable, this model would fail

Decision boundary:



Testing 2 different train/test splits:
Split 1: First 15 train / Last 5 test
Training Accuracy: 1.00
Testing Accuracy : 1.00

Split 2: Random 15 train / 5 test
Training Accuracy: 1.00
Testing Accuracy : 0.80

Comparing Results:
Split 1 performed perfectly on both training and test sets while split 2 performed perfectly for training but only 80% test accuracy. Since split 2 is random, it's possible the test set had harder examples that the model didn't handle it well. While the perceptron performs well overall, the results show that its performance can vary depending on how the data is split.

Question 4

4.A

Variables and Domains:

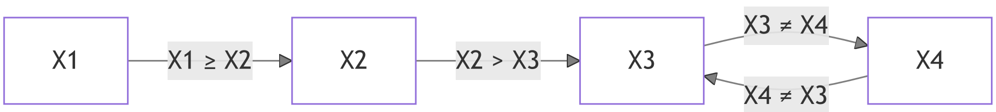
- X_1: {1, 2, 3, 4}
- X_2: {3, 4, 5, 8, 9}
- X_3: {2, 3, 5, 6, 7, 9}
- X_4: {3, 5, 7, 8, 9}

Constraints:

1. $X_1 \geq X_2$
2. $X_2 > X_3$
3. $X_3 \neq X_4$

Constraint graph:

- **Nodes**: X_1, X_2, X_3, X_4
- **Directed Arcs**:
  - $X_1 \to X_2$ (for $X_1 \geq X_2$)
  - $X_2 \to X_3$ (for $X_2 > X_3$)
  - $X_3 \to X_4$ and $X_4 \to X_3$ (for $X_3 \neq X_4$)



4.B

Initial Queue of Arcs

- $X_1 \geq X_2$: Arcs (X_1, X_2) and (X_2, X_1)
- $X_2 > X_3$: Arcs (X_2, X_3) and (X_3, X_2)
- $X_3 \neq X_4$: Arcs (X_3, X_4) and (X_4, X_3)

Initial Queue: $(X_1, X_2), (X_2, X_1), (X_2, X_3), (X_3, X_2), (X_3, X_4), (X_4, X_3)$

Initial Domains

- $D_1 = \{1, 2, 3, 4\}$
- $D_2 = \{3, 4, 5, 8, 9\}$
- $D_3 = \{2, 3, 5, 6, 7, 9\}$
- $D_4 = \{3, 5, 7, 8, 9\}$

1. **Arc** (X_1, X_2), Constraint: $X_1 \geq X_2$
   - For each $x_1 \in D_1$, check if there exists $x_2 \in D_2$ such that $x_1 \geq x_2$:
     - $x_1 = 1$: $D_2 = \{3, 4, 5, 8, 9\}$, all > 1, no support.
     - $x_1 = 2$: All $x_2 > 2$, no support.
     - $x_1 = 3$: $x_2 = 3$, $3 \geq 3$, supported.
     - $x_1 = 4$: $x_2 = 3$, $4 \geq 3$, supported.
   - Remove 1 and 2 from D_1.
   - **Revised** $D_1 = \{3, 4\}$.
   - D_1 changed, add neighbors' arcs to X_1 except X_2. Only neighbor is X_2, so no new arcs.
2. **Arc** (X_2, X_1), Constraint: $X_1 \geq X_2$
   - For each $x_2 \in D_2$, check if there exists $x_1 \in D_1$ such that $x_1 \geq x_2$:

    - $x_2 = 3$: $D_1 = \{3, 4\}$, $3 \geq 3$, supported.
    - $x_2 = 4$: $x_1 = 4$, $4 \geq 4$, supported.
    - $x_2 = 5$: $D_1 = \{3, 4\}$, both < 5, no support.
    - $x_2 = 8$: No $x_1 \geq 8$.
    - $x_2 = 9$: No $x_1 \geq 9$.
  - Remove 5, 8, 9 from D_2.
  - **Revised** $D_2 = \{3, 4\}$.
  - D_2 changed, add (X_k, X_2) for neighbors X_1, X_3 except X_1. Add (X_3, X_2).

3. **Arc** (X_2, X_3), Constraint: $X_2 > X_3$
  - $D_2 = \{3, 4\}$, $D_3 = \{2, 3, 5, 6, 7, 9\}$:
    - $x_2 = 3$: $x_3 = 2$, $3 > 2$, supported.
    - $x_2 = 4$: $x_3 = 2$, $4 > 2$, supported.
  - No revision to D_2.

4. **Arc** (X_3, X_2), Constraint: $X_2 > X_3$
  - For each $x_3 \in D_3$, check if there exists $x_2 \in D_2$ such that $x_2 > x_3$:
    - $x_3 = 2$: $x_2 = 3$, $3 > 2$, supported.
    - $x_3 = 3$: $x_2 = 4$, $4 > 3$, supported.
    - $x_3 = 5$: $D_2 = \{3, 4\}$, both < 5, no support.
    - $x_3 = 6, 7, 9$: No $x_2 > 6, 7, 9$.
  - Remove 5, 6, 7, 9 from D_3.
  - **Revised** $D_3 = \{2, 3\}$.
  - D_3 changed, add (X_4, X_3) (neighbors X_2, X_4, except X_2).

5. **Arc** (X_3, X_4), Constraint: $X_3 \neq X_4$
  - $D_3 = \{2, 3\}$, $D_4 = \{3, 5, 7, 8, 9\}$:
    - $x_3 = 2$: $x_4 = 3$, $2 \neq 3$, supported.
    - $x_3 = 3$: $x_4 = 5$, $3 \neq 5$, supported.
  - No revision.

6. **Arc** (X_4, X_3), Constraint: $X_3 \neq X_4$
  - $D_4 = \{3, 5, 7, 8, 9\}$, $D_3 = \{2, 3\}$:
    - $x_4 = 3$: $x_3 = 2$, $3 \neq 2$, supported.
    - $x_4 = 5, 7, 8, 9$: All $\neq 2, 3$, supported.
  - No revision.

7. **Arc** (X_3, X_2) (re-added), Constraint: $X_2 > X_3$
  - $D_3 = \{2, 3\}$, $D_2 = \{3, 4\}$:
    - $x_3 = 2$: $x_2 = 3$, $3 > 2$, supported.
    - $x_3 = 3$: $x_2 = 4$, $4 > 3$, supported.
  - No revision.

8. **Arc** (X_4, X_3) (re-added), Constraint: $X_3 \neq X_4$
  - Same as step 6, no revision.

The queue is now empty, and no further revisions occur.

Final Domains

- X_1: {3, 4}
- X_2: {3, 4}
- X_3: {2, 3}
- X_4: {3, 5, 7, 8, 9}

4.C

A network is arc-consistent if, after applying AC-3, all domains are non-empty, and every arc is consistent. Since no domain became empty and all arcs were processed without further revisions, the scheduling network is **arc-consistent**.

4.D

Since the network is arc-consistent, we assign values satisfying all constraints:

- Choose $X_3 = 2$ ($D_3 = \{2, 3\}$).
- $X_2 > X_3 = 2$, $D_2 = \{3, 4\}$, pick $X_2 = 3$.
- $X_1 \geq X_2 = 3$, $D_1 = \{3, 4\}$, pick $X_1 = 3$.
- $X_3 \neq X_4$, $X_3 = 2$, $D_4 = \{3, 5, 7, 8, 9\}$, pick $X_4 = 3$.

Schedule: $X_1 = 3, X_2 = 3, X_3 = 2, X_4 = 3$

Verification:

1. $X_1 \geq X_2$: $3 \geq 3$, true.
2. $X_2 > X_3$: $3 > 2$, true.
3. $X_3 \neq X_4$: $2 \neq 3$, true.

4.E

Revise Domains
Starting from part (b)'s domains:

- $D_1 = \{3, 4\}$
- $D_2 = \{3, 4\}$
- $D_3 = \{2, 3\}$
- $D_4 = \{3, 5, 7, 8, 9\}$

New constraint: $X_1 \neq X_4$. Add arcs (X_1, X_4) and (X_4, X_1) to the queue.

1. **Arc** (X_1, X_4), Constraint: $X_1 \neq X_4$
   - $x_1 = 3$: $x_4 = 5$, $3 \neq 5$, supported.
   - $x_1 = 4$: $x_4 = 3$, $4 \neq 3$, supported.
   - No revision to D_1.
2. **Arc** (X_4, X_1), Constraint: $X_1 \neq X_4$
   - $x_4 = 3$: $x_1 = 4$, $3 \neq 4$, supported.
   - $x_4 = 5, 7, 8, 9$: All $\neq 3, 4$, supported.
   - No revision to D_4.

No domains change, so they remain:

- X_1: {3, 4}
- X_2: {3, 4}
- X_3: {2, 3}
- X_4: {3, 5, 7, 8, 9}

The original network was arc-consistent, and the new arcs are consistent with no domain reductions. All domains are non-empty, so the network **remains arc-consistent**. Solutions like $X_1 = 4, X_2 = 4, X_3 = 3, X_4 = 5$ still exist, satisfying all constraints, including $X_1 \neq X_4$.