Assignment 2

Total Points = 28

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1. (4 points) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by giving a containment proof (that is, prove that the left side is a subset of the right side and that the right side is a subset of the left side).

2. (4 points) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by giving a proof using logical equivalence.

3. (3 points) Suppose $U = \{1, 2, \ldots, 9\}$, $A =$ all multiples of 2, $B =$ all multiples of 3, and $C = \{3, 4, 5, 6, 7\}$. Find $C - (B - A)$.

4. (2+2=4 points) Suppose $f\colon \mathbf{N} \to \mathbf{N}$ has the rule $f(n) = 4n + 1$.

   (i) Determine whether f is 1-1. Justify your answer.

   (ii) Determine whether f is onto $\mathbf{N}$. Justify your answer.

5. (4 points) Suppose $f\colon \mathbf{R} \to \mathbf{Z}$ where $f(x) = \lceil 2x - 1 \rceil$.

   (i) If $A = \{x \mid 1 \leq x \leq 4\}$, find $f(A)$.

   (ii) If $B = \{3, 4, 5, 6, 7\}$, find $f(B)$.

   (iii) If $C = \{-9, -8\}$, find $f^{-1}(C)$.

   (iv) If $D = \{0.4, 0.5, 0.6\}$, find $f^{-1}(D)$.

6. (9 points) Let $A = \{0, 1\}$. List the following relations:

   (i) List all the binary relations on A.

   (ii) List the reflexive relations on A.

   (iii) List the irreflexive relations on A.

   (iv) List the symmetric relations on A.

   (v) List the transitive relations on A.

   (vi) List the antisymmetric relations on A.

   (vii) List the asymmetric relations on A.

   (viii) List the relations on A that are reflexive and symmetric.

   (ix) List the relations on A that are neither reflexive nor irreflexive.