

$$① A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$a) A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$$

$$\therefore x \in A \cap (B \cup C)$$

$$x \in A \wedge (x \in B \vee x \in C)$$

$$\text{distribution: } (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$$

$$\text{union \& intersection: } (A \cap B) \cup (A \cap C)$$

$$\therefore A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$$

$$b) (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$$

$$x \in (A \cap B) \cup (A \cap C)$$

$$(x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$$

distributive:

$$x \in A \wedge (x \in B \vee x \in C)$$

union \& intersection:

$$A \cap (B \cup C)$$

$$\therefore (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$$

$$② A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

↓

$$\{x \mid x \in A \wedge (x \in B \vee x \in C)\} = \{x \mid (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)\}$$

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distributive law:

$$\{x \mid (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)\} = \{x \mid (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)\}$$

def. union \& intersection:

$$(A \cap B) \cup (A \cap C) = (A \cap B) \cup (A \cap C)$$

$$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$③ U = \{1, 2, \dots, 9\} \quad A = \{x \mid 2x, x \in U\} \quad B = \{x \mid 3x, x \in U\} \quad C = \{3, 4, 5, 6, 7\}$$

$$C - (B - A) \quad \begin{matrix} \downarrow \\ \{2, 4, 6, 8\} \end{matrix} \quad \begin{matrix} \downarrow \\ \{3, 6, 9\} \end{matrix}$$

$$= C - (\{3, 6, 9\} - \{2, 4, 6, 8\})$$

$$= \{1, 5, 7\} - \{7, 9\}$$

$$= \{4, 5, 6, 7\}$$

$$④ N: [0, \infty)$$

$$\text{Assume } f(n) = f(n_2)$$

$$\frac{f(n_1)}{n_1} = \frac{f(n_2)}{n_2}$$

$$n_1 = n_2$$

$$\therefore \text{by def. of 1-1 the function } f(n) = 4n + 1 \text{ is 1-1}$$

II) counter example:

$$1 = 4x + 1$$

$$0 = 4x + 1$$

$$0 = x \quad 0 \notin N$$

$\therefore$  the function is not onto

⑤ I)  $f(A) = \{1, 3, 5, 7\}$  II)  $f(B) = \{5, 7, 9, 11, 13\}$

$$\begin{cases} x = 2y - 1 \\ \frac{x+1}{2} = y \end{cases}$$

III)  $f(C) = \{-4, -3.5\}$

IV)  $f(D) = \{0.7, 0.75, 0.8\}$

⑥  $A = \{0, 1\}$   $A \times A = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$

I)  $\{(0, 0), (0, 1), (1, 0), (1, 1)\}$  II)  $\{(0, 0), (1, 1)\}$

III)  $\{(0, 1), (1, 0)\}$

IV)  $\{(0, 1), (1, 0)\}$

V)  $\{\emptyset\}$

~~VI~~  $\{(0, 0), (1, 1)\}$

~~VII~~  $\{(0, 1)\}$

~~VIII~~  $\{(0, 0), (1, 1)\}$

~~IX~~  $\{\emptyset\}$