Assignment 5

Question 1

1.A

1.A.I

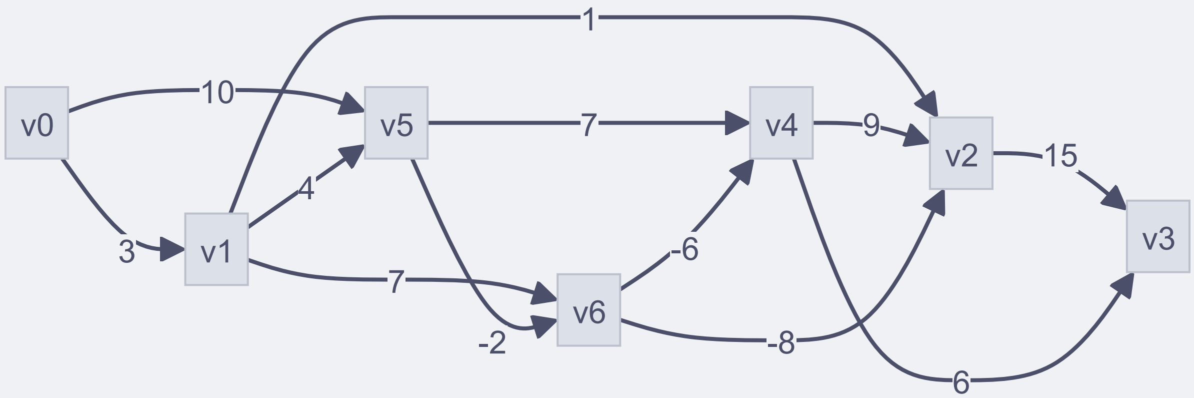
Dijsktra's algorithm can be modified to directly display the shortest path with the following modifications:

1. A HashMap data-structure called `previous` is initialized where the key is the node and the value is the nodes in which they key node is reached in the smallest cost.
2. When a shorter path to a node is found, update `previous` to the current node

1.A.II

1. A HashMap data-structure called `previous` is initialized where the key is the node and the value is the nodes in which they key node is reached in the smallest cost.
2. When a shorter path to a node is found, update `previous` to the current node

1.B



1.B.I

Initialize:

```
Distance: v0: 0, v1: ∞, v2: ∞, v3: ∞, v4: ∞, v5: ∞, v6: ∞
Predecessor: v0: None, v1: None, v2: None, v3: None, v4: None, v5: None, v6: None
```

Pass 1:
Relax all edges in the graph

```
Distance: v0: 0, v1: 3, v2: 4, v3: ∞, v4: 17, v5: 10, v6: 8
Predecessor: v0: None, v1: v0, v2: v1, v3: None, v4: v5, v5: v0, v6: v5
```

Pass 2:
Relax all edges in the graph again

```
Distance: v0: 0, v1: 3, v2: 0, v3: 15, v4: 2, v5: 10, v6: 8}
Predecessor: {v0: None, v1: v0, v2: v6, v3: v2, v4: v6, v5: v0, v6: v5
```

Pass 3:

```
Distance: v0: 0, v1: 3, v2: 0, v3: 15, v4: 2, v5: 10, v6: 8
Predecessor: v0: None, v1: v0, v2: v6, v3: v2, v4: v6, v5: v0, v6: v5
```

Because no distances were updated we know that we have reached a solution

According to the Bellman-Ford algorithm the shortest path for this graph is `v0 → v5 → v6 → v2, cost: 0`

1.B.II

Initialize:

```
Distance: v0: 0, v1: ∞, v2: ∞, v3: ∞, v4: ∞, v5: ∞, v6: ∞
Predecessor: v0: None, v1: None, v2: None, v3: None, v4: None, v5: None, v6: None
```

Step 1:
start from node `v0` there are only two adjacent nodes `v1, v5` update these nodes and distances

```
Distance: v0: 0, v1: 3, v2: ∞, v3: ∞, v4: ∞, v5: 10, v6: ∞
Predecessor: v0: None, v1: v0, v2: None, v3: None, v4: None, v5: v0, v6: None
```

Step 2:
move to `v1` which is the smallest unvisited node. Adjacent nodes are `v2`, `v5`, and `v6`. Update the nodes and distances

```
Distance: v0: 0, v1: 3, v2: 4, v3: ∞, v4: ∞, v5: 7, v6: 10
Predecessor: v0: None, v1: v0, v2: v1, v3: None, v4: None, v5: v1, v6: v1
```

Step 3:
move to `v2`. The only adjacent node is `v3`,

```
Distance: v0: 0, v1: 3, v2: 4, v3: 19, v4: ∞, v5: 7, v6: 10
Predecessor: v0: None, v1: v0, v2: v1, v3: v2, v4: None, v5: v1, v6: v1
```

Step 4:
move to `v5`. The adjacent nodes are `v4` and `v6`. Update the nodes and distances.

```
Distance: v0: 0, v1: 3, v2: 4, v3: 19, v4: 14, v5: 7, v6: 5
Predecessor: v0: None, v1: v0, v2: v1, v3: v2, v4: v5, v5: v1, v6: v5
```

Step 5:
move to`v6`. The adjacent nodes are `v2` and `v4`. Update the nodes and distances.

```
Distance: v0: 0, v1: 3, v2: -3, v3: 12, v4: -1, v5: 7, v6: 5
Predecessor: v0: None, v1: v0, v2: v6, v3: v2, v4: v6, v5: v1, v6: v5
```

Step 6:
move`v4`. The only adjacent node is `v3`. Update the nodes and distances.

```
Distance: v0: 0, v1: 3, v2: -3, v3: 5, v4: -1, v5: 7, v6: 5
Predecessor: v0: None, v1: v0, v2: v6, v3: v4, v4: v6, v5: v1, v6: v5
```

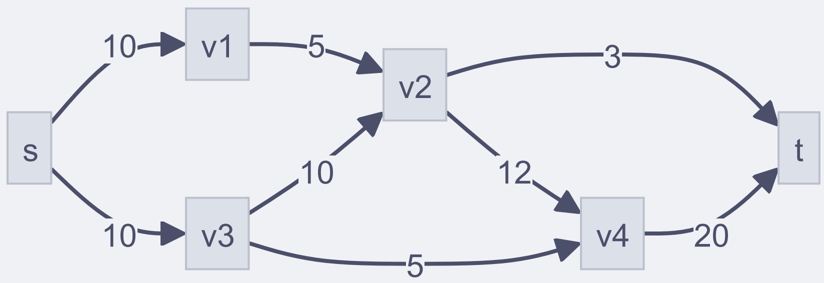
Shortest path to every node:
`v0 → v1, cost: 3`
`v0 → v1 → v5 → v6 → v2, cost: -3`
`v0 → v1 → v5 → v6 → v4 → v3, cost 5`
`v0 → v1 → v5 → v6 → v4, cost: -1`
`v0 → v1 → v5, cost: 7`
`v0 → v1 → v5 → v6, cost 5`

1.B.III

According to the Bellman-Ford algorithm the shortest path from `v0 → v2` is `v0 → v5 → v6 → v2, cost: 0`

However using Dijsktra's algorithm `v0 → v1 → v5 → v6 → v2, cost: -3` was the shortest path with Dijsktra's algorithm arriving at a better answer.

Question 2



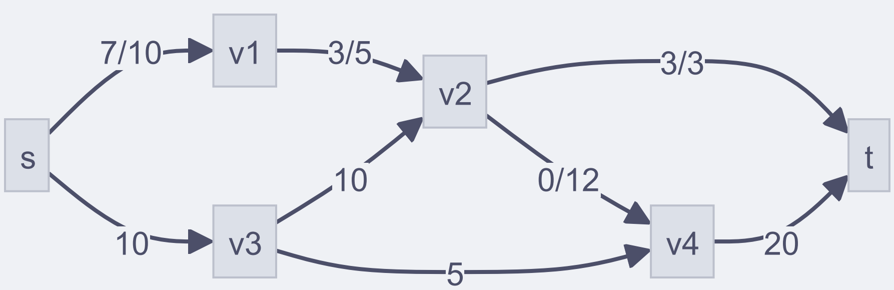
2.A

Initialize:
The flow on all edges is 0

Iteration 1:

- Augmenting Path: s -> v1 -> v2 -> t
- Residual Capacities: 10 -> 5 -> 3
- Minimum Residual Capacity (i.e., flow of this iteration): 3

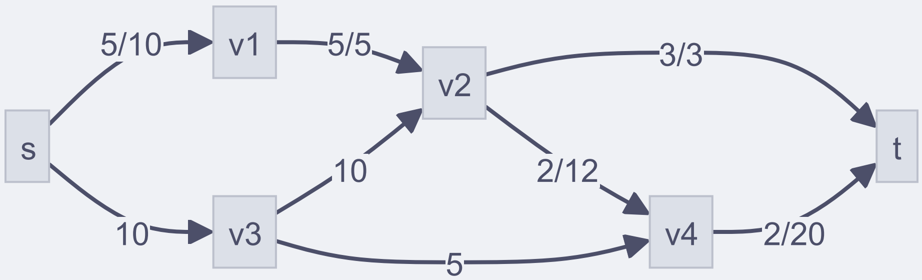
The updated flows after this iteration:



Iteration 2:

- Augmenting Path: `s → v1 → v2 → v4 → t`
- Residual Capacities: `7 → 2 → 12 → 20`
- Minimum Residual Capacity: `2`

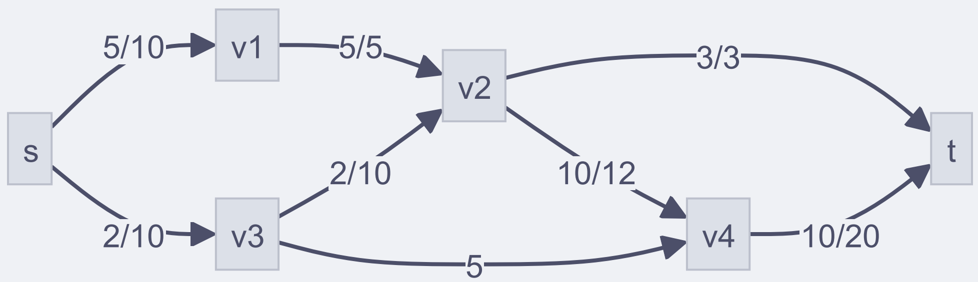
The updated flows after this iteration:



Iteration 3:

- Augmenting Path: `s → v3 → v2 → v4 → t`
- Residual Capacities: `10 → 8 → 10 → 18`
- Minimum Residual Capacity: `8`

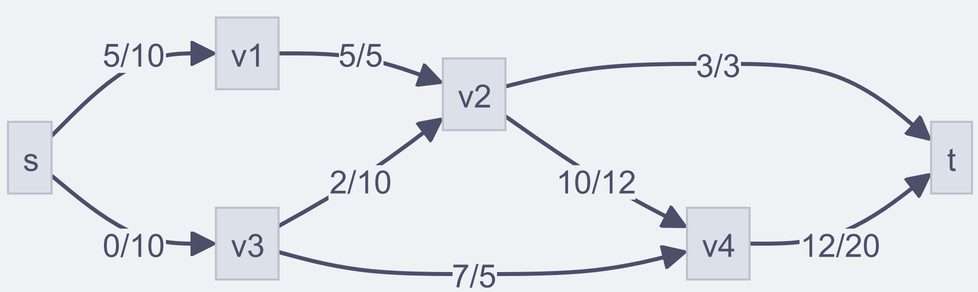
The updated flows after this iteration:



Iteration 4:

- Augmenting Path:`s → v3 → v4 → t`
- Residual Capacities: `2 → 5 → 10`
- Minimum Residual Capacity: `2`

The updated flows after this iteration:



The maximum flow of the network is the sum of the flows into the sink, which is 3 (from v2) + 12 (from v4) = **15**. So, the maximum flow for this graph is **15**.

2.B

- Source side: `s, v1, v3, v2`

- Sink side: `v4, t`
- Edges in the cut: `(v2, v4), (v2, t), (v3, v4)`
  The capacities of these edges are 12, 3, and 5 respectively. The sum of these capacities is 20, which is still greater than the maximum flow. However, the actual flow across this cut is:
- Flow of `(v2, v4)`: `10`
- Flow of `(v2, t)`: `3`
- Flow of `(v3, v4)`: `2`
  The sum of these flows is `10 + 3 + 2 = 15`, which equals the maximum flow. Therefore, this is the minimum cut of the network.

Question 3

3.A

The strongly connected components in the graph are the nodes `A, B, C, D, E, G, I, F`