

CP 460 - Applied Cryptography

Stream Ciphers

Department of Physics and Computer Science
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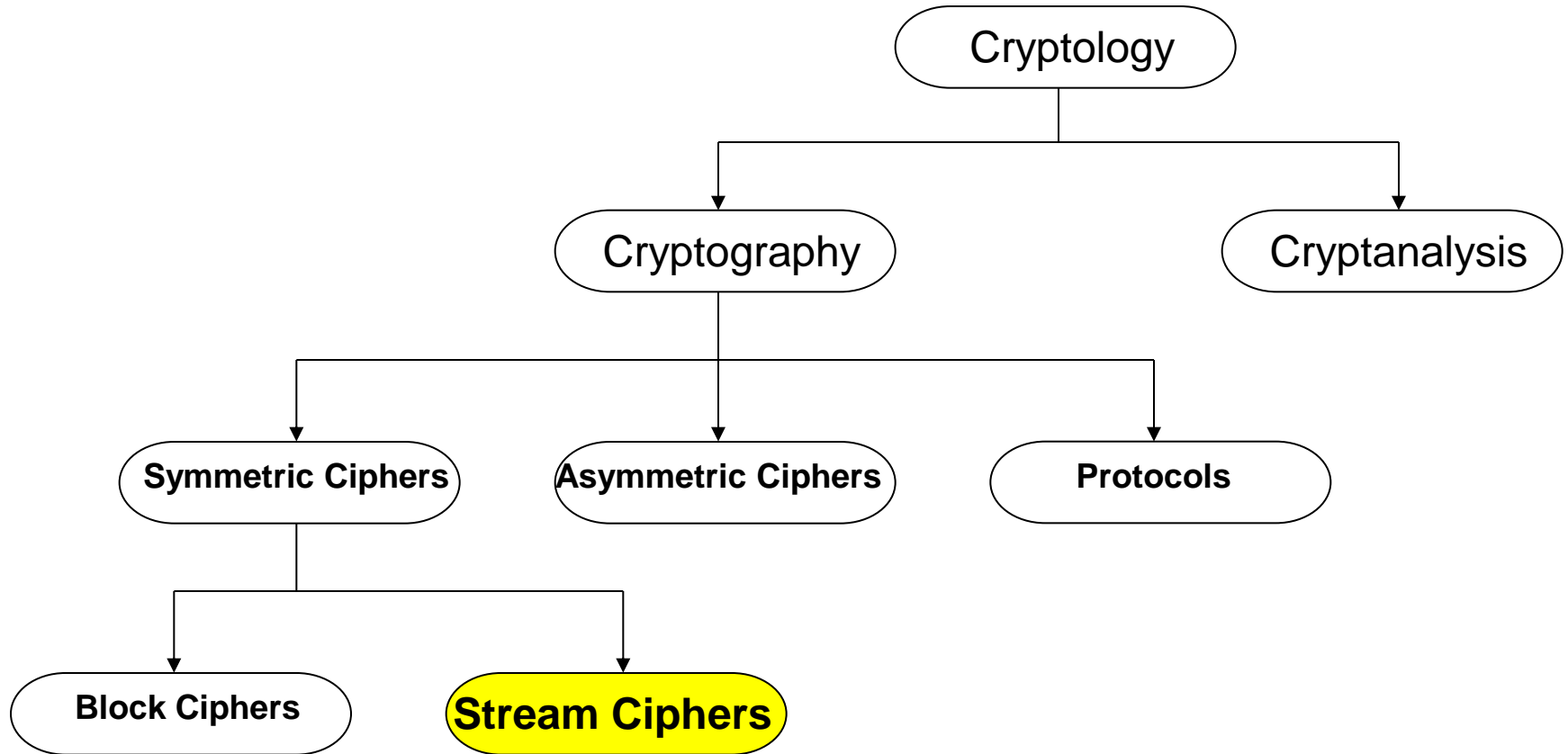
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Content of this Chapter

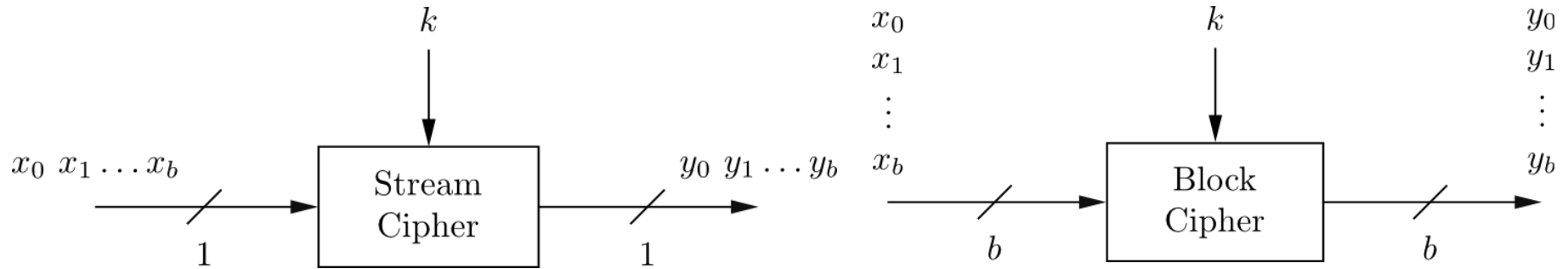
- Intro to stream ciphers
- Random number generators (RNGs)
- One-Time Pad (OTP)
- Linear feedback shift registers (LFSRs)
- Trivium: a modern stream cipher

■ Stream Ciphers in the Field of Cryptology



Stream Ciphers were invented in 1917 by Gilbert Vernam

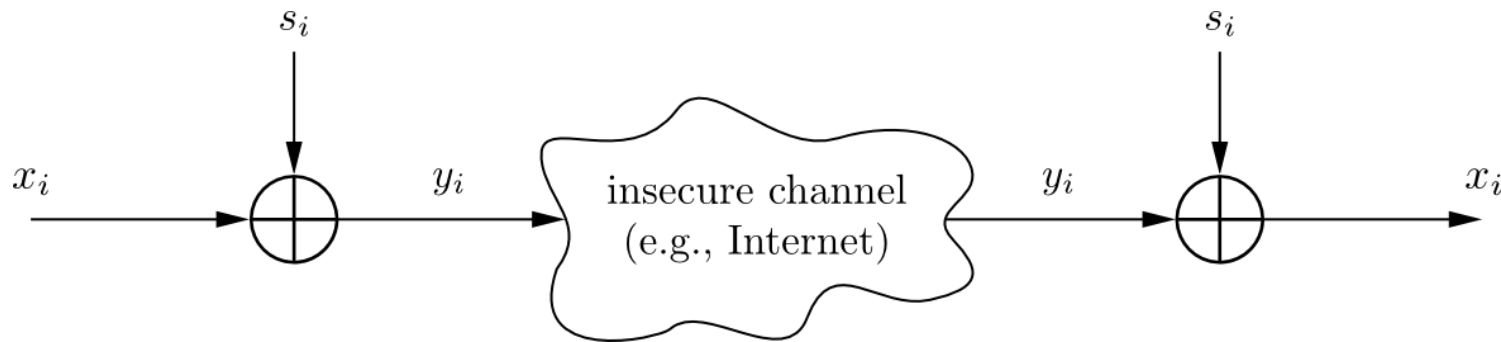
■ Stream Cipher vs. Block Cipher



- **Stream Ciphers**
 - Encrypt bits individually
 - Usually small and fast → common in embedded devices (e.g., A5/1 for GSM phones)
- **Block Ciphers:**
 - Always encrypt a full block (several bits)
 - Are common for Internet applications

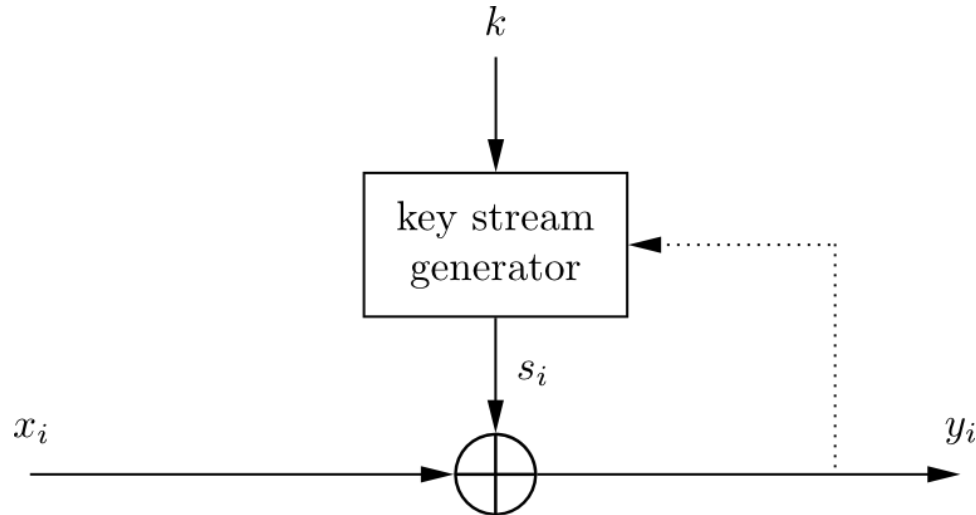
■ Encryption and Decryption with Stream Ciphers

Plaintext x_i , ciphertext y_i and key stream s_i consist of individual bits



- Encryption and decryption are simple additions modulo 2 (aka XOR)
- Encryption and decryption are the same functions
- **Encryption:** $y_i = e_{s_i}(x_i) = x_i + s_i \bmod 2$ $x_i, y_i, s_i \in \{0,1\}$
- **Decryption:** $x_i = e_{s_i}(y_i) = y_i + s_i \bmod 2$

■ Synchronous vs. Asynchronous Stream Cipher



- Security of stream cipher depends entirely on the key stream s_i :
 - Should be **random** , i.e., $\Pr(s_i = 0) = \Pr(s_i = 1) = 0.5$
 - Must be **reproducible** by sender and receiver
- **Synchronous Stream Cipher**
 - Key stream depend only on the key (and possibly an initialization vector IV)
- **Asynchronous Stream Ciphers**
 - Key stream depends also on the ciphertext (dotted feedback enabled)

■ Why is Modulo 2 Addition a Good Encryption Function?

- Modulo 2 addition is equivalent to XOR operation
- For perfectly random key stream s_i , each ciphertext output bit has a 50% chance to be 0 or 1
→ Good statistic property for ciphertext
- Inverting XOR is simple, since it is the same XOR operation

x_i	s_i	y_i
0	0	0
0	1	1
1	0	1
1	1	0

■ Stream Cipher: Throughput

Performance comparison of symmetric ciphers (Pentium4):

Cipher	Key length	Mbit/s
DES	56	36.95
3DES	112	13.32
AES	128	51.19
RC4 (stream cipher)	(choosable)	211.34

Source: Zhao et al., Anatomy and Performance of SSL Processing, ISPASS 2005

RC4 is a stream cipher, meaning it encrypts data one bit or byte at a time, rather than in blocks like DES and AES. It has an extremely high throughput in comparison to the other algorithms, indicating that it is very fast, but RC4 has been deprecated due to serious security vulnerabilities discovered over time.

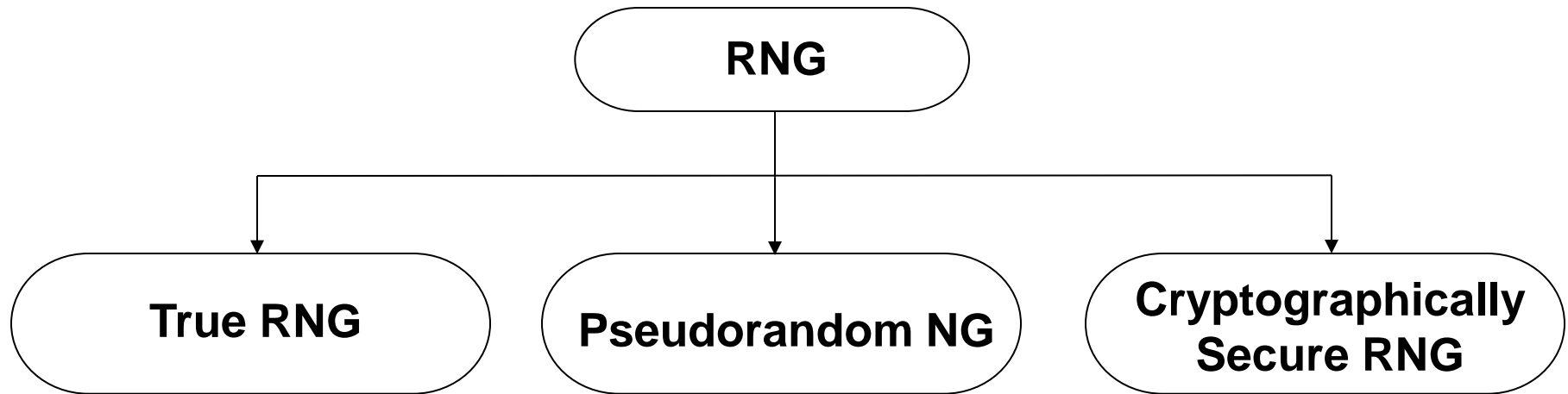
- **AES** provides a balance between speed and security, making it the best option among these ciphers for both performance and security.
- **3DES** is much slower compared to DES and AES due to its triple encryption, which also increases security but at a heavy performance cost.
- **DES**, although faster than 3DES, is no longer considered secure due to its short key length.

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■ Random number generators (RNGs)

An **RNG** is a system that generates random numbers. These numbers are used in various applications, including cryptography, where randomness is crucial for generating keys, initialization vectors, and nonces.



- **True RNGs** (TRNG) based on physical randomness.
- **Pseudorandom RNGs** (PRNG) based on algorithms.

Key Differences:

- **PRNGs** are fast and can generate long sequences of numbers from a seed but are not suitable for cryptography unless they are cryptographically secure (CSPRNGs).
- **CSPRNGs** are designed for cryptographic use and provide security guarantees, ensuring that their outputs are difficult to predict.
- **TRNGs** are based on real-world physical processes and provide true randomness, but they may be slower and more difficult to implement.

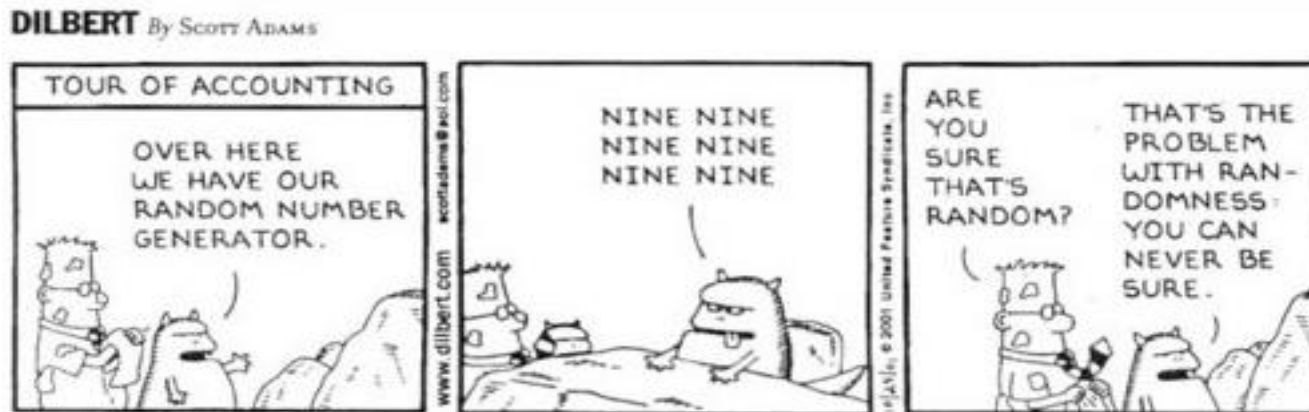
■ True Random Number Generators (TRNGs)

Coin flipping or **dice rolling**, which are classic examples of generating randomness.

TRNGs provide randomness based on physical, unpredictable processes, making their outputs truly random and non-reproducible.

- Output stream s_i should have good statistical properties:
 $\Pr(s_i = 0) = \Pr(s_i = 1) = 50\%$ (often achieved by post-processing)
- Output can neither be predicted nor be reproduced

Typically used for generation of keys, nonces (used only-once values) and for many other purposes



■ Pseudorandom Number Generator (PRNG)

- Generate sequences from initial seed value
- Typically, output stream has good statistical properties
- Output can be reproduced and can be predicted

Often computed in a recursive way:

$$s_0 = seed$$

$$s_{i+1} = f(s_i, s_{i-1}, \dots, s_{i-t})$$

Example: *rand()* function in ANSI C:

$$s_0 = 12345$$

$$s_{i+1} = 1103515245s_i + 12345 \bmod 2^{31}$$

Most PRNGs have bad cryptographic properties!

Even though they may have good statistical properties (e.g., they look random), they are **predictable** because their output can be reproduced if the seed is known.

■ Cryptanalyzing a Simple PRNG

Simple PRNG: **Linear Congruential Generator**

$$S_0 = seed$$

$$S_{i+1} = AS_i + B \bmod m$$

Assume

- unknown A , B and S_0 as key
- Size of A , B and S_i to be 100 bit
- 300 bit of output are known, i.e. S_1 , S_2 and S_3

Solving

$$S_2 = AS_1 + B \bmod m$$

$$S_3 = AS_2 + B \bmod m$$

...directly reveals A and B . All S_i can be computed easily!

Bad cryptographic properties due to the linearity of most PRNGs

■ Cryptographically Secure Pseudorandom Number Generator (CSPRNG)

- Special PRNG with additional property:
 - Output must be **unpredictable**

More precisely: Given n consecutive bits of output s_i , the following output bits s_{n+1} cannot be predicted (in polynomial time).

- Needed in cryptography, in particular for stream ciphers
- Remark: There are almost no other applications that need unpredictability, whereas many, many (technical) systems need PRNGs.

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■ One-Time Pad (OTP)

A cryptosystem is considered **unconditionally secure** if it **cannot be broken**, even if an attacker has access to **infinite computational resources**.

Unconditionally secure cryptosystem:

- A cryptosystem is unconditionally secure if it cannot be broken even with *infinite* computational resources

In OTP, the plaintext, ciphertext, and key are represented as sequences of individual bits.

One-Time Pad

- A cryptosystem developed by Mauborgne that is based on Vernam's stream cipher:
- Properties:

Let the plaintext, ciphertext and key consist of individual bits

$$x_i, y_i, k_i \in \{0, 1\}.$$

$$\text{Encryption: } e_{k_i}(x_i) = x_i \oplus k_i.$$

$$\text{Decryption: } d_{k_i}(y_i) = y_i \oplus k_i.$$

OTP is unconditionally secure if and only if the key k_i is used once!

■ One-Time Pad (OTP)

Unconditionally secure cryptosystem:

$$y_0 = x_0 \oplus k_0$$

$$y_1 = x_1 \oplus k_1$$

:

This pattern continues for every bit in the message

Plaintext x_i , random key bit k_i

Every equation is a linear equation with two unknowns

⇒ for every y_i are $x_i = 0$ and $x_i = 1$ equiprobable!

⇒ This is true iff k_0, k_1, \dots are independent, i.e., all k_i have to be generated truly random

⇒ It can be shown that this systems can *provably* not be solved.

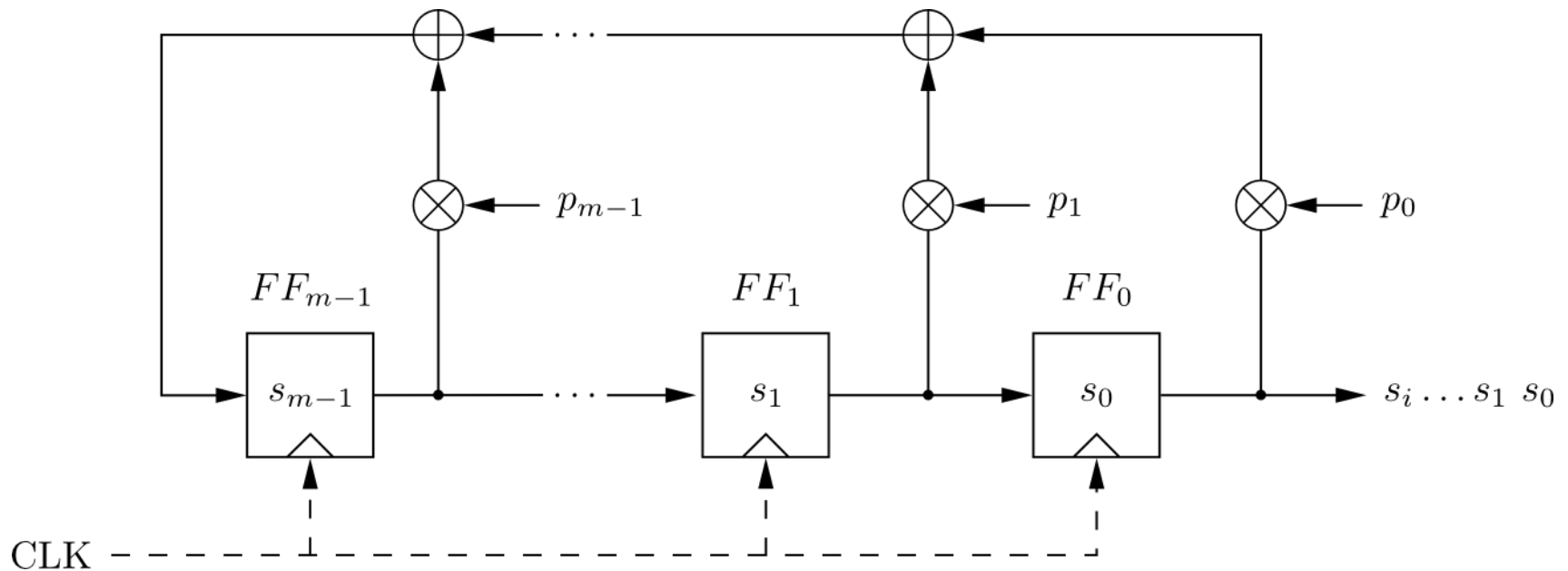
Disadvantage: For almost all applications the OTP is **impractical** since the key must be as long as the message! (Imagine you have to encrypt a 1GByte email attachment.)

This means if you want to encrypt a large file, like a **1GB email attachment**, you would need to generate and securely store a **1GB key** that is completely random.

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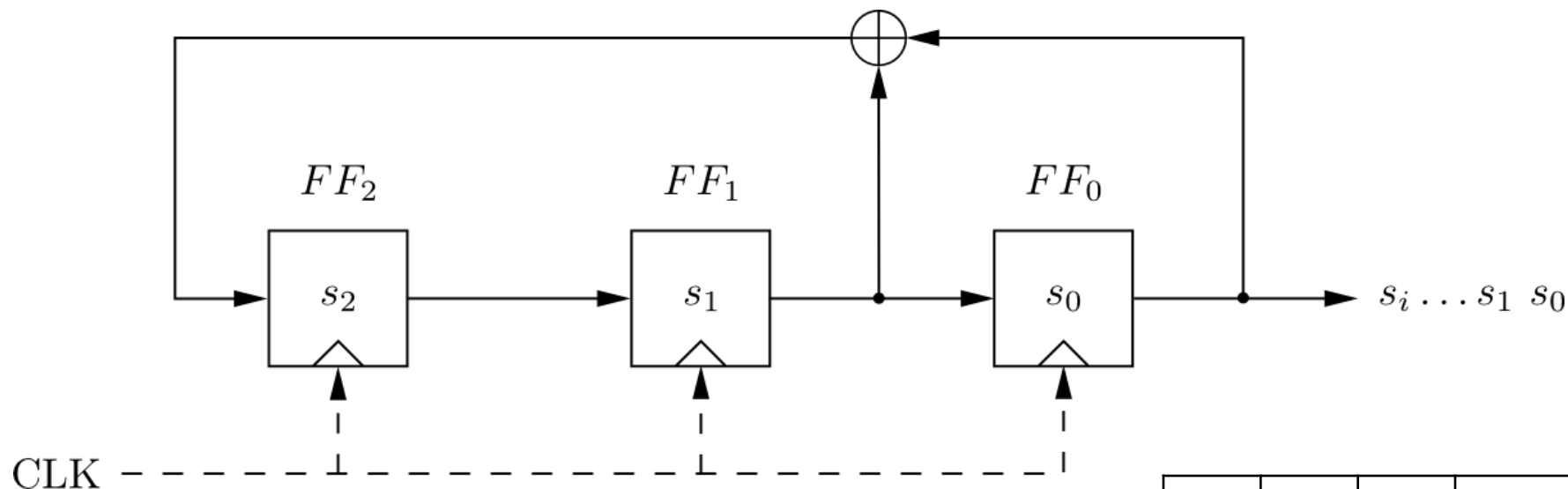
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■ Linear Feedback Shift Registers (LFSRs)



- Concatenated *flip-flops* (FF), i.e., a shift register together with a feedback path
- Feedback computes fresh input by XOR of certain state bits
- *Degree* m given by number of storage elements
- If $p_i = 1$, the feedback connection is present (“closed switch”), otherwise there is not feedback from this flip-flop (“open switch”)
- Output sequence repeats periodically
- Maximum output length: $2^m - 1$

■ Linear Feedback Shift Registers (LFSRs): Example with m=3



- LFSR output described by recursive equation:

$$s_{i+3} = s_{i+1} + s_i \mod 2$$

- Maximum output length (of $2^3-1=7$) achieved only for certain feedback configurations, .e.g., the one shown here.

<i>clk</i>	FF_2	FF_1	$FF_0=s_i$
0	1	0	0
1	0	1	0
2	1	0	1
3	1	1	0
4	1	1	1
5	0	1	1
6	0	0	1
7	1	0	0
8	0	1	0

■ Security of LFSRs

LFSRs typically described by polynomials:

$$P(x) = x^m + p_{l-1}x^{m-1} + \dots + p_1x + p_0$$

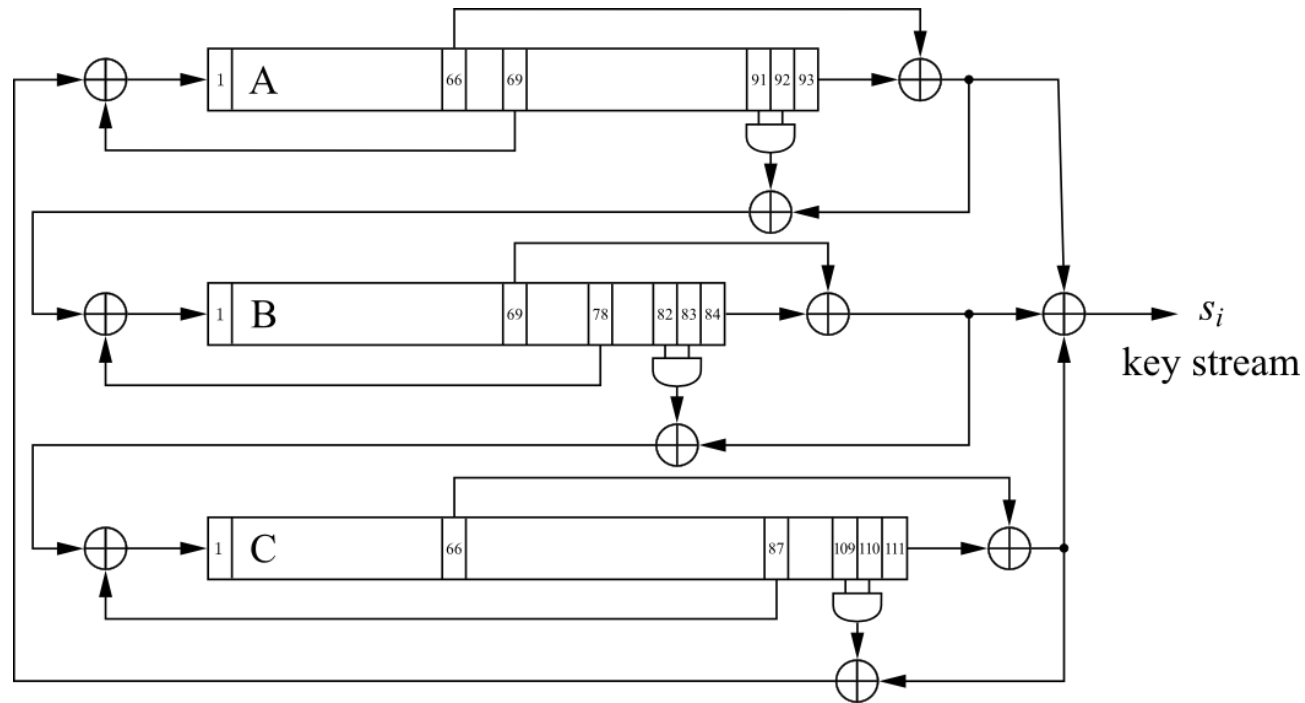
- Single LFSRs generate highly predictable output
- If $2m$ output bits of an LFSR of degree m are known, the feedback coefficients p_i of the LFSR can be found by solving a system of linear equations*
- Because of this many stream ciphers use **combinations** of LFSRs

*See Chapter 2 of *Understanding Cryptography* for further details.

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■ A Modern Stream Cipher - Trivium



- Three *nonlinear* LFSRs (NLFSR) of length 93, 84, 111
- XOR-Sum of all three NLFSR outputs generates key stream s_i
- Small in Hardware:
 - Total register count: 288
 - Non-linearity: 3 AND-Gates
 - 7 XOR-Gates (4 with three inputs)

■ Trivium

Initialization:

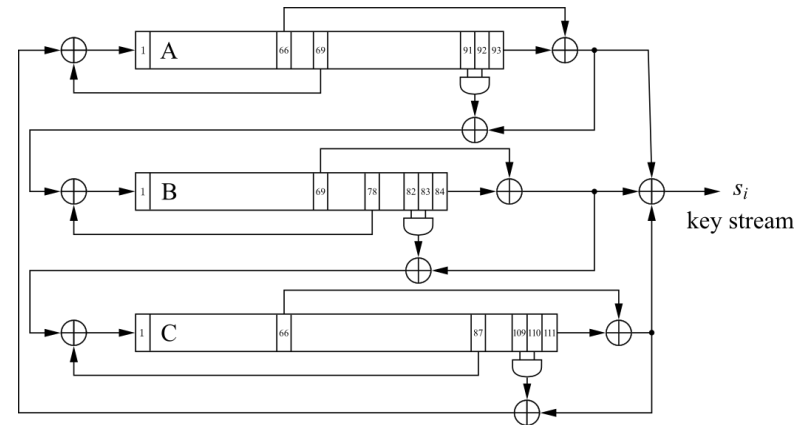
- Load 80-bit IV into A
- Load 80-bit key into B
- Set c_{109} , c_{110} , $c_{111} = 1$, all other bits 0

Warm-Up:

- Clock cipher $4 \times 288 = 1152$ times without generating output

Encryption:

- XOR-Sum of all three NLFSR outputs generates key stream s_i



Design can be parallelized to produce up to 64 bits of output per clock cycle

	Register length	Feedback bit	Feedforward bit	AND inputs
A	93	69	66	91, 92
B	84	78	69	82, 83
C	111	87	66	109, 110

■ Lessons Learned

- Stream ciphers are less popular than block ciphers in most domains such as Internet security. There are exceptions, for instance, the popular stream cipher RC4.
- Stream ciphers sometimes require fewer resources, e.g., code size or chip area, for implementation than block ciphers, and they are attractive for use in constrained environments such as cell phones.
- The requirements for a *cryptographically secure pseudorandom number generator* are far more demanding than the requirements for pseudorandom number generators used in other applications such as testing or simulation
- The One-Time Pad is a provable secure symmetric cipher. However, it is highly impractical for most applications because the key length has to equal the message length.
- Single LFSRs make poor stream ciphers despite their good statistical properties. However, careful combinations of several LFSR can yield strong ciphers.

**Thank
You**

