XAVIER IYENGAR p118

Parallel Merging pb

$A_1, \ldots, A_n$ $B_1, \ldots, B_n$ two sorted arrays

$A_1 \leq \ldots \leq A_n$ $B_1 \leq \ldots \leq B_n$

merge A, B into array C $C_1 \ldots C_{2n}$ s.t.

$C_1 \leq \ldots \leq C_{2n}$

let $n = 2^k$, $\frac{n}{\log n} = r$, k, r integers

$\log n = k$

sequential merge algorithm $O(n)$ time

1. $i=1$ $j=1$ $k=1$, $A_{n+1} = +\infty$, $B_{n+1} = +\infty$
2. while $k \leq 2n$ do

   if $A_i \leq B_j$ then $C_k = A_i$ $i = i+1$

   else $C_k = B_j$ $j = j+1$

   $k = k+1$

parallel merge algorithm

1) partition the sorted array A in r groups each having $k = \log n$ elements as follows:

Group 1 : $A_1 \dots A_k$

Group 2 : $A_{k+1} \dots A_{2k}$

⋮

Group i : $A_{(i-1)k+1} \dots A_{ik}$

⋮

Group r : $A_{(r-1)k+1} \dots A_{rk}$

2) find r integers $j(1) \dots j(r)$ s.t.

$j(1)$ is the greatest index s.t. $A_k \geq B_{j(1)}$

$j(2)$ is the or greatest index s.t. $A_{2k} \geq B_{j(2)}$

$j(i)$ is the or greatest index s.t. $A_{ik} \geq B_{j(i)}$

$j(r)$ is the or greatest index s.t. $A_{rk} \geq B_{j(r)}$

(3) this induces a partition of array B, into r groups:

Group 1 : $B_1, B_2, \ldots, B_{j(1)}$

Group 2 : $B_{j(1)+1}, \ldots, B_{j(2)}$

⋮

Group i : $B_{j(i-1)+1}, \ldots, B_{j(i)}$

⋮

Group r : $B_{j(r-1)+1}, \ldots, B_{j(r)}$

4) Observation: every entry in group 1 of A is $\leq$ to every entry of group 1, 2, 3, ... of B.

assign processor i to merge group i of A & group i of B

merging group 1 of A & group 1 of B guarantees that the e_m's have reached their final position in $C(1:2n)$

parallel merge algorithm

for i from 1 to r

[find (BINARY SEARCH) $j(i) = \max\{t, \text{ s.t. } A_{iK} \geq B_t\}$

sequential merge $A_{(i-1)K+1 \ldots iK}$ & $B_{j(i-1)+1 \ldots j(i)}$]

illustration $n = 16 = 2^4$, $\log n = 4$

$K = r = 4$

A = | 1, 5, 15, 18 | 19, 21, 23, 24 | 27, 29, 30, 31 | 32, 37, 42, 44 |

B = { 2, 3, 4, 13, 15, 19, 20, 22, 28, 29, 38, 41, 42, 43, 48, 49 }

$j(1) = 5$ $j(2) = 8$ $j(3) = 10$ $j(4) = 15$

B = | 2, 3, 4, 13, 15 | 19, 20, 22 | 28, 29 | 38, 41, 42, 43, 48, 49 |

merge group 1 A & group 1 B ~> 1, 2, 3, 4, 5, 13, 15, 15, 18

" " 2 " " " 2 " ~> 19, 20, 21, 22, 23, 24

C(1:9) C(10:16) C(17:22) C(23:32)