① a) P(1)

$1\cdot 1! = (1+1)! - 1$
$1 = 2! - 1$
$1 = 2 - 1$
$1 = 1$

b) P(5)

$1\cdot 1! + 2\cdot 2! + 3\cdot 3! + 4\cdot 4! + 5\cdot 5! = (5+1)! - 1$
$1\cdot 1 + 2\cdot 2 + 3\cdot 6 + 4\cdot 24 + 5\cdot 120 = 6! - 1$
$1 + 4 + 18 + 96 + 600 = 720 - 1$
$719 = 719$

c) P(k)

$1\cdot 1! + 2\cdot 2! \ldots + k\cdot k! = (k+1)! - 1$

d)

$1\cdot 1! + 2\cdot 2! \ldots + k\cdot k! + (k+1)\cdot(k+1)! = (k+2)! - 1$

e) base case: $1\cdot 1! = (1+1)! - 1$
$1 = 1$

assume f(k) is true

$1\cdot 1! + 2\cdot 2! \ldots + k\cdot k! = (k+1)! - 1$

Induction hypothesis

$LS = ((k+1)! - 1) + ((k+1)(k+1)!)$

$= [(k+1)! + [(k+1)(k+1)!]] - 1$
$= (k+1)!\,[1 + (k+1)] - 1$
$= (k+1)!\,[k+2] - 1$
$= (k+2)! - 1$

② base case

$3^0 = \frac{3^{0+1} - 1}{2}$
$1 = \frac{3-1}{2}$
$1 = 1$

assume f(k) is true
$1 + 3 + \ldots + 3^k = \frac{3^{k+1} - 1}{2}$

Induction hypothesis

$LS = \frac{3^{k+1} - 1}{2} + 3^{k+1}$
$= \frac{3^{k+1} - 1}{2} + \frac{(2)3^{k+1}}{2}$
$= \frac{(3^{k+1} + (2)3^{k+1}) - 1}{2}$
$= \frac{[3^{k+1}(1 + 2)] - 1}{2}$
$= \frac{3^{k+1}(3^1) - 1}{2}$
$= \frac{3^{k+2} - 1}{2}$

③ base case: $\boxed{n^3 + 3n^2 + 2n = n(n+1)(n+2)}$

$f(1) = n(n+1)(n+2)$
$= 1(1+1)(1+2)$
$= 1(2)(3)$
$3 \mid f(1)$

assume:
$3 \mid k(k+1)(k+2)$

Inductive hypothesis:
$3 \mid k+1[(k+1)+1][(k+1)+2]$
$3 \mid (k+1)(k+2)(k+3)$
$3 \mid k(k+1)(k+2) + 3(k+1)(k+2)$

(4) $N = 4m + 7n \ \forall N \geq 18$

base cases:

$18 = 4(1) + 7(2)$
$= 4 + 14$
$= 18$ $N = 4m + 7n$

$19 = 4(3) + 7(1)$
$= 12 + 7$
$= 19$
$N = 4(m+2) + 7(n-1)$

$20 = 4(5) + 7(0)$
$= 20 + 0$
$= 20$
$N = 4(m+4) + 7(n-2)$

$21 = 4(0) + 7(3)$
$= 0 + 21$
$= 21$
$N = 4(m-5) + 7(n+1)$

Assume:
$N = 4m + 7n \ \forall N \geq 18$

Induction hypothesis:
$N+4 = 4(m+1) + 7(n)$ for all base cases
$N+4 = 4m + 7n + 4$

(5) a) $f(n) = 2^n$
$f(1) = 2^1 = 2$ $f(n+1) = f(n) \cdot f(1)$
$f(3) = f(2) \cdot 2$
$= f(1) \cdot 2 \cdot 2$
$= 2 \cdot 2 \cdot 2$
$= 8$

b) $f(n) = 5n + 2$
$f(1) = 5(1) + 2$
$= 7$
$f(n+1) = f(n) + 5$
$f(3) = f(2) + 5$
$= f(1) + 5 + 5$
$= 7 + 5 + 5$

(6) a) $a_n = 2^n$
$a_1 = 2^1$
$= 2$
$a_n = 2 \cdot 2^{n-1}$

b) $a_n = 3n - 5$
$a_1 = 3 - 5$
$= -2$
$a_n = a_{n-1} + 3$

(7) a) $S = \{3, 7, 11, 15, 19, 23\}$
$3 \in S$
$(x+4) \in S$ if $(x \in S)$

b) $1 \in S$
$(2x+1) \in S$, $(-x) \in S$ if $x \in S$

(8)
```
def foo(n,a):
  if(n==1):
    return a
  else:
    return foo(n-1,a)+a
```