# Assignment 3

# 1

## 1.A

### Longest duration

For this counter example assume that we are scheduling tasks

| Task ID | Start Time | End Time | Duration |
|---|---|---|---|
| 1 | 1 | 4 | 3 |
| 2 | 1 | 3 | 2 |
| 3 | 3 | 5 | 2 |

If tasks are scheduled by longest duration only task 1 would be scheduled where as by using earliest finish time both tasks 2, and 3 can be scheduled

### Lowest task ID

| Task ID | Start Time | End Time | Duration |
|---|---|---|---|
| 1 | 1 | 4 | 3 |
| 2 | 1 | 3 | 2 |
| 3 | 3 | 5 | 2 |

If tasks are scheduled by lowest task ID only task 1 would be scheduled where as by using earliest finish time both tasks 2, and 3 can be scheduled

### Latest finish time

| Task ID | Start Time | End Time | Duration |
|---|---|---|---|
| 1 | 1 | 4 | 3 |
| 2 | 1 | 2 | 1 |
| 3 | 2 | 3 | 1 |

If tasks are scheduled by latest finish time only task 1 would be scheduled where as by using earliest finish time both tasks 2, and 3 can be scheduled

## 1.B

Assume that $Q$ is the optimal solution (fitting the most amount of tasks possible) and $\hat{Q}$ is is the **latest start time** approach

In the first step: if the **greedy** choice is made (latest start time) the optimal solution is still possible as the start time for the task chosen in this step is $\geq$ the start time for all other tasks.

This choice reduces the problem to a smaller task scheduling problem from `Start` -> `T1_start` instead of `Start` -> `End`. Continuing with further steps leads to the optimal solution.

Inductive Hypothesis: Assume that for a problem of size $k$, the greedy strategy yields an optimal solution.

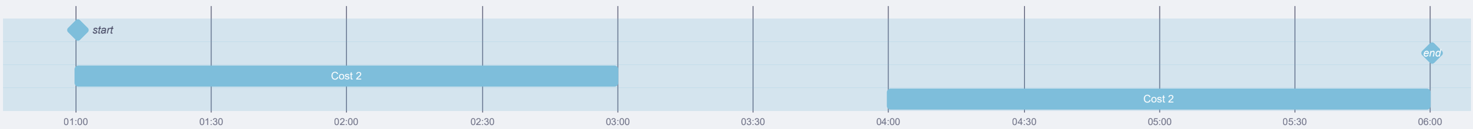
Inductive Step: Prove that the hypothesis holds for the problem of size $k+1$.
If the greedy choice is made for the first task, the problem size reduces to $k$. By the inductive hypothesis, the greedy strategy yields an optimal solution for the remaining $k$ tasks. Hence, the greedy strategy yields an optimal solution for the problem of size $k+1$.

## 1.C

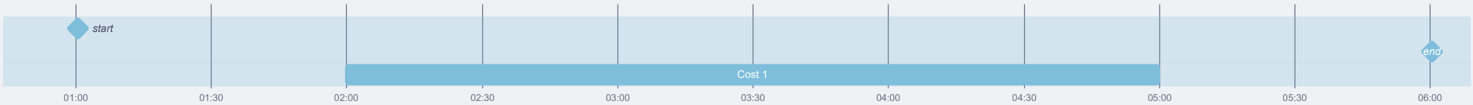
**Counter example:**
Suppose we have three activities with their start times, finish times, and costs as follows:

| Task ID | Start Time | End Time | Cost |
|---|---|---|---|
| 1 | 1 | 3 | 2 |
| 2 | 4 | 6 | 2 |
| 3 | 2 | 5 | 1 |

By utilizing the greedy choice of earliest finish time we would get:




with a total cost of 4 while the optimal solution would be:




With a total cost of 1

∴ By counter example the greedy choice of earliest finish time will not yield an optimal solution for this new problem

# 2

`YPPTPQTPYYTPTQPAYPT`

## 2A

In the string `YPPTPQTPYYTPTQPAYPT` there are 19 characters and each character is encoded with 8bits in ASCII therefore 152 bits are needed to encode this string.

## 2.B

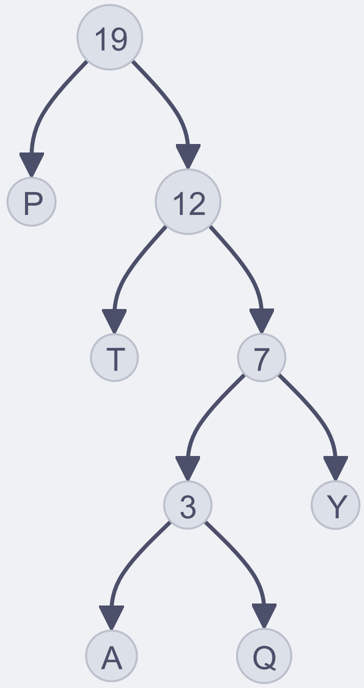
1. Find the frequency of each character in the string

| Letter | Frequency |
|---|---|
| Y | 4 |
| P | 7 |
| T | 5 |
| Q | 2 |
| A | 1 |

2. Sort the characters in increasing order of frequency

| Letter | Frequency |
|---|---|
| A | 1 |
| Q | 2 |
| Y | 4 |
| T | 5 |
| P | 7 |

3. Build a Huffman tree from the frequency data




4. Traverse the Huffman tree and build the encodings for each character found in the input file

| Letter | Encoding |
| --- | --- |
| P | 0 |
| T | 10 |
| Y | 111 |
| Q | 1101 |
| A | 1100 |

5. For each character in the input file, write the bits of the Huffman encoding
   YPPTPQTPYYTPTQPAYPT -> 111, 0, 0, 10, 0, 1101, 10, 0, 111, 111, 10, 0, 10, 1101, 0, 1100, 111, 0, 10
6. Output the codeword
   `1110010011011001111111001011010110011101 0`

## 2.C

0 | 111 | 0 | 0 | 0 | 10 | 10 | 10 | 0 | 0 | 0 | 1101
P | Y | P | P | P | T | T | T | P | P | P | Q

## 2.D

`ABCDABCDABCDABCD`
In the above string since there are only four characters and they all have the same frequency the codeword for all characters is two bits long.

# 3

| Item # | Weight | Value | Density V/W |
|---|---|---|---|
| 1 | 3 | 15 | 5 |
| 2 | 6 | 24 | 4 |
| 3 | 4 | 12 | 3 |
| 4 | 2 | 16 | 8 |

## 3.A

Using the following algorithm:

```
def knapSack(W, weights, val, n):
    K = [[0 for x in range(W + 1)]
            for x in range(n + 1)]
    for i in range(n + 1):
        for w in range(W + 1):
            if i == 0 or w == 0:
                K[i][w] = 0
            elif weights[i - 1] <= w:
                K[i][w] = max(val[i - 1]
                  + K[i - 1][w - weights[i - 1]],
                              K[i - 1][w])
            else:
                K[i][w] = K[i - 1][w]
    return K[n][W]
```

| | Capacity | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|
| Items | | | | | | | | | | |
| 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | | 0 | 0 | 0 | 15 | 15 | 15 | 15 | 15 | 15 |
| 2 | | 0 | 0 | 0 | 15 | 15 | 15 | 24 | 24 | 24 |
| 3 | | 0 | 0 | 0 | 15 | 15 | 15 | 24 | 27 | 27 |
| 4 | | 0 | 0 | 16 | 16 | 16 | 31 | 31 | 31 | **40** |

the optimal solution would be 40

## 3.B

For a bag with capacity $8$ a greedy algorithm that uses the **density** as its greedy choice would yield items [4, 1] as its choices the knapsack ends up with a value of $31$ and a weight of $5/8$ where as the optimum solution would choose items [2, 4] and have a value of $41$ and a weight of $8/8$.

### 3.C

When solving the fractional knapsack problem the greedy algorithm would output:

| items | Total weight | Value | Percentage | total |
|---|---|---|---|---|
| 4 | 2/8 | 16 | 1 | 16 |
| 1 | 5/8 | 15 | 1 | 31 |
| 2 | 8/8 | 12 | 0.5 | 43 |

The result is $43$ which is the optimum solution

## 4

### 4.A

`[1, 5, 2, 6, 3]`
1 + (5 x 2) + (6 x 3) = 29

### 4.B

A Naive solution to this problem would be to go through all possible outcomes for a sequence of numbers and return the version that had the highest value.

For a sequence of length $n$ for each pair of contiguous integers, we have 2 choices: we can either insert an addition operation or a multiplication operation. Therefore the time complexity for this problem would be $O(2^n)$

### 4.C

A counter example to a greedy approach would be:
`[1, 2, 3, 4]`

In the greedy approach the outcome would be:
1 + (2 x 3) + 4 = 11

Where as the optimum solution is:
(1 x 2) + (3 x 4) = 14

### 4.D

$$V[j] = \begin{cases} 0 & \text{if } j = 0 \\ x_1 & \text{if } j = 1 \\ max(V[j-1] + V[j], V[j-2] + V[j-1] \times V[j]) & \text{if } j \geq 2 \end{cases}$$

# 4.E

```
def max_product_sum(X):
    ln = len(X)
    V = [0]*ln
    V[0] = X[0]
    V[1] = max(X[0] + X[1], X[0] * X[1])
    for j in range(2, ln):
        V[j] = max(V[j-1] + X[j], V[j-2] + X[j-1] * X[j])
    return V[ln-1]
```

The running time for this algorithm is $O(n)$