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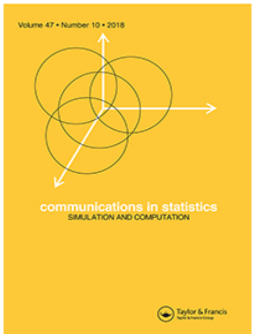

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Estimating the parameters of twofold Weibull mixture model in right-censored reliability data by using genetic algorithm

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ABSTRACT

In this article, a new method was practiced to form a model by using twofold Weibull mixture distribution in right-censored reliability data. The method depends on estimating the parameters of right-censored twofold Weibull mixture distribution in a most appropriate way to the data by using genetic algorithm techniques. The best model was tried to be found by using MSE, MAE and MAPE metrics, respectively, as fitness function in the method. To test the model, failure data of aircraft planes' windshield, which is often used in the literature, was used and the results were compared with other methods in the literature. Furthermore, the performance of the method was compared for the sample sizes, censorship ratios and mixture proportions by conducting Monte Carlo simulation study.

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KEYWORDS

Genetic algorithm; Mixture distribution; Reliability; Twofold Weibull

1. Introduction

In system reliability, systems consist of components with various structures and features. System reliability can be obtained through the reliabilities of the components that form the system. However, in many cases, while the time survival of the system can be observed, time survival of the components cannot be observed. Censorship is widespread in time survival experiments because of time and budgetary constraints (Yang, Ng, and Balakrishnan 2016). In order to estimate the parameters of time survival models that are formed by various statistical distributions in right-censored systems, best linear unbiased estimation (BLUE) (Balakrishnan, Ng, and Navarro 2011), maximum likelihood estimation (MLE) (Balakrishnan, Ng, and Navarro 2011; Ng, Navarro, and Balakrishnan 2012) and regression-based method (Zhang, Ng, and Balakrishnan 2015) were developed. These methods require complex calculations.

Mixture distributions are also used in system reliability modeling. Berchtold (2004) compared the expectation–maximization (EM), stochastic expectation–maximization (SEM) and genetic algorithm (GA) methods for the parameter estimation in mixture models. Volterman and Balakrishnan (2013) suggested an iterative method for the calculation of mixture proportions in right-censored data. Wang (2014) proposed bare bones particle swarm optimization (BBPSO) algorithm for estimating the parameters of Weibull distribution with censored data, and compared the performance of BBPSO and EM models by a Monte Carlo simulation. Wang and Huang (2014) used a particle swarm optimization (PSO) algorithm to estimate parameters of the mixture of two Weibull with complete and multiple censored data. They compared the performance of PSO and EM models by a Monte Carlo simulation. Yang, Ng, and Balakrishnan (2016) suggested SEM algorithm to be used in complete and censored systems to obtain the maximum likelihood estimations of the parameters of system's survival models. Bordes and

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Chauveau (2016) presented a few iterative methods based on EM and SEM algorithm which provide us to estimate parametric or semi-parametric mixture models, providing that they are definable, for right-censored survival data. Ruhi, Sarker, and Karim (2015) tried to estimate the parameters of the model by using EM algorithm for twofold Weibull mixture model.

The Weibull probability paper (WPP) method was widely used for estimating the parameters of Weibull mixture models. A large part of the literature is interested in the situations of a well-divided subpopulation, use various approaches in the drawings and for the characterization of asymptotes. As also mentioned by Murthy, Xie, and Jiang (2004) there are two serious disadvantages in the WPP method. The WPP method produces very rough estimations as long as iteration is not used and will be evaluated by eyesight. For this reason, they can be used as a starting point for more sophisticated statistical methods. The second disadvantage is, it does not perform statistical confidence limits for estimated parameters.

Another method for estimating the parameters of Weibull mixture models is EM. According to Ferreira and Silva (2017), EM algorithm has some advantages compared to other iterative algorithms, but it also has disadvantages. The EM algorithm may converge very slowly to the result in some datasets or datasets with too much missing information. The EM algorithm does not have an integrated process for generating an estimate of the covariance matrix of parameter estimates. However, this disadvantage can be overcome by using appropriate methods. Another disadvantage of the EM algorithm is that the prediction is highly dependent on the initial solution given to the algorithm. Therefore, the EM algorithm does not guarantee convergence to the global solution. However, the GA mutates the solution at a certain rate in each iteration to avoid local solutions. Thus, it can continue to search in other parts of the solution space.

In this article, a method based on GA was presented in order to estimate the most appropriate parameters of twofold Weibull mixture model to the right-censored reliability data. In Sec. 2, twofold Weibull mixture model was defined. Besides, maximum likelihood estimation method was defined for parameter estimation. In Sec. 3, a method based on GA was presented and the results were compared with some known methods by using windshield failure data. In Sec. 4, Monte Carlo simulation was conducted for evaluating the performance of the method in different situations. Finally, the results were given in Sec. 5.

2. Twofold Weibull mixture model

Probability density function, cumulative distribution function, time survival function and Hazard function of a random T variable, taken from Weibull distribution, is given below.

$$f(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}}, \qquad t \geq 0,\ \alpha > 0,\ \beta > 0 \tag{1}$$

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}}, \qquad t \geq 0,\ \alpha > 0,\ \beta > 0 \tag{2}$$

$$S(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}, \qquad t \geq 0,\ \alpha > 0,\ \beta > 0 \tag{3}$$

$$h(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}, \qquad t \geq 0,\ \alpha > 0,\ \beta > 0 \tag{4}$$

When the observations are divided into two clusters as D, the cluster of uncensored observations ($t_i \leq v_i$) and C as the cluster of censored data ($t_i > v_i$), maximum likelihood function is

$$L(\alpha, \beta) = \left[\prod_{i \in D} f(t_i)\right]\left[\prod_{i \in C} \left(1 - F(v_i)\right)\right], \tag{5}$$

where t_i indicates the uncensored observation time, and v_i indicates the censorship time for T_i. With $u_i = min(t_i, v_i)$, the maximum likelihood estimation of β is acquired through solving the

$$\frac{\sum_{i=1}^{n} u_i^{\hat{\beta}} \ln u_i}{\sum_{i=1}^{n} u_i^{\hat{\beta}}} - \frac{1}{\hat{\beta}} - \frac{1}{k} \sum_{i \in D} \ln u_i = 0 \tag{6}$$

equation. The maximum likelihood estimation of α is acquired with

$$\hat{\alpha} = \left[\frac{1}{n} \left(\sum_{i=1}^{n} u_i^{\hat{\beta}} \right) \right]^{1/\hat{\beta}} \tag{7}$$

equation.

The cumulative distribution function of a k-fold mixture model is

$$G(t) = \sum_{j=1}^{k} \omega_j F_j(t), \tag{8}$$

where, $F_j(t)$ is the cumulative distribution function of jth subpopulation and ω_j is the mixture distribution proportion of jth subpopulation and $\sum_{j=1}^{k} \omega_j = 1$.

Probability density function of a k-fold mixture model is

$$g(t) = \sum_{j=1}^{k} \omega_j f_j(t) \tag{9}$$

Reliability function of a k-fold mixture model is given as:

$$R_j(t) = 1 - F_j(t) \tag{10}$$

Distinctively, cumulative distribution function in twofold Weibull mixture model ($k = 2$) is

$$G(t) = \omega F_1(t) + (1 - \omega) F_2(t) \tag{11}$$

If we put Weibull(α_1, β_1) instead of $F_1(t)$ and Weibull(α_2, β_2) instead of $F_2(t)$ distributions is obtained as

$$G(t) = \left\{ 1 - \exp\left[-\left(\frac{t}{\alpha_2} \right)^{\beta_2} \right] \right\} + \omega \left\{ \exp\left[-\left(\frac{t}{\alpha_2} \right)^{\beta_2} \right] - \exp\left[-\left(\frac{t}{\alpha_1} \right)^{\beta_1} \right] \right\}, \tag{12}$$

probability density function in twofold Weibull mixture model is obtained as

$$g(t) = \omega \left\{ \frac{\beta_1}{\alpha_1} \left(\frac{t}{\alpha_1} \right)^{\beta_1 - 1} \exp\left[-\left(\frac{t}{\alpha_1} \right)^{\beta_1} \right] \right\} + (1 - \omega) \left\{ \frac{\beta_2}{\alpha_2} \left(\frac{t}{\alpha_2} \right)^{\beta_2 - 1} \exp\left[-\left(\frac{t}{\alpha_2} \right)^{\beta_2} \right] \right\}, \tag{13}$$

reliability function is obtained as

$$R(t) = \omega \left\{ \exp\left[-\left(\frac{t}{\alpha_1} \right)^{\beta_1} \right] \right\} + (1 - \omega) \left\{ \exp\left[-\left(\frac{t}{\alpha_2} \right)^{\beta_2} \right] \right\}. \tag{14}$$

Maximum likelihood estimations of model parameters

In Eq. (5), maximum likelihood estimation can be obtained with

$$\begin{aligned} \ln L = & \sum_{i \in D} \ln \left\{ \omega \left[\frac{\beta_1}{\alpha_1} \left(\frac{u_i}{\alpha_1} \right)^{\beta_1 - 1} \exp\left[-\left(\frac{u_i}{\alpha_1} \right)^{\beta_1} \right] \right] + (1 - \omega) \left[\exp\left[-\left(\frac{u_i}{\alpha_2} \right)^{\beta_2} \right] \right] \right\} \\ & + \sum_{i \in C} \ln \left\{ \omega \left[\exp\left[-\left(\frac{u_i}{\alpha_1} \right)^{\beta_1} \right] \right] + (1 - \omega) \left[\exp\left[-\left(\frac{u_i}{\alpha_2} \right)^{\beta_2} \right] \right] \right\} \end{aligned} \tag{15}$$

equation, if we replace the probability density function and cumulative distribution function of twofold Weibull distribution and take the logarithm of both sides. Maximum likelihood estimations of parameters can be found when the partial differentiation is taken and made equal to zero of (15) equation according to $\beta_1, \alpha_1, \beta_2, \alpha_2$ *and* ω parameters, respectively. Since closed-form solutions cannot be obtained from the obtained equations, iterative methods will be necessary to use in order to find the maximum likelihood estimations.

Murthy, Xie, and Jiang (2004), created a twofold Weibull mixture model for data and calculated the model parameters by applying a graphical method based on Weibull probability paper (WPP) method. The model selection has a tendency of trial and error process. WPP provides a systematical procedure for determining whether or not one of the models is appropriate for modeling a data set.

It is based on Weibull transformation as in

$$y = \ln\{-\ln[1 - F(t)]\} \text{ and } x = \ln(t) \tag{16}$$

In this transformation, the graphic of y according to x variable is called as Weibull probability graph. In the early 1970s, a special paper was developed to draw the data of this transformation and it is called Weibull probability paper (WPP). Graphics drawn with the help of this paper is named as WPP graphics. Today, many reliability software packages contain programs that can automatically produce this graphic for a given data set.

3. Proposed method and practical example

3.1. Genetic algorithm

The emergence of GA approach happened in the early 1970s. In his studies about machine learning, John Holland (1975) was influenced by the evolution and change in living beings and developed GA. GAs are often used in machine learning and optimization practices.

GA is an algorithm which tries to find the best appropriate one in many possible solutions. Sequences that include all of the information about any solution are called as chromosome. In every generation, bad solutions tend to disappear and good solutions tend to be used to create better solutions. GAs do not scan all of the solution space, but a part of it. Thus, they reach a solution in a much shorter time by making an efficient scan. An important superiority of GAs is that they get rid of the effect of local solutions by means of protecting the diversity in the population.

In order to determine how much the intended objective is fulfilled in the problem, a fitness function is determined. Usually, fitness function is intended to be maximum or minimum. Fitness function determines the quality of the chromosome. The value of the fitness function plays a very important role in whether or not the chromosome in a population transfers its data to the next generation.

Basic operators of algorithm such as mate selection, reproduction, crossover and mutation are originated from biology. Algorithm practices the survival of the fittest principle to reach more appropriate results. In every generation, by enabling individuals to live with more appropriate solutions compared to other solutions according to appropriacy in the problem area, as in natural adaptation, it leads the population to develop more and thus attempts to find the most appropriate result.

3.2. Windshield failure data set

In order to test the method, windshield failure data was used. The data set was first used in the studies of Blischke and Murthy (2000) and it contains the failure times and service times in the windshield of a certain aircraft model. Data containing the service times were not completed

Table 1. Aircraft windshield failure data.

Failure Times				Service times		
0.040	1.866	2.385	3.443	0.046	1.436	2.592
0.301	1.876	2.481	3.467	0.140	1.492	2.600
0.309	1.899	2.610	3.478	0.150	1.580	2.670
0.557	1.911	2.625	3.578	0.248	1.719	2.717
0.943	1.912	2.632	3.595	0.280	1.794	2.819
1.070	1.914	2.646	3.699	0.313	1.915	2.820
1.124	1.981	2.661	3.779	0.389	1.920	2.878
1.248	2.010	2.688	3.924	0.487	1.963	2.950
1.281	2.038	2.823	4.035	0.622	1.978	3.003
1.281	2.085	2.890	4.121	0.900	2.053	3.102
1.303	2.089	2.902	4.167	0.952	2.065	3.304
1.432	2.097	2.934	4.240	0.996	2.117	3.483
1.480	2.135	2.962	4.255	1.003	2.137	3.500
1.505	2.154	2.964	4.278	1.010	2.141	3.622
1.506	2.190	3.000	4.305	1.085	2.163	3.665
1.568	2.194	3.103	4.376	1.092	2.183	3.695
1.615	2.223	3.114	4.449	1.152	2.240	4.015
1.619	2.224	3.117	4.485	1.183	2.341	4.628
1.652	2.229	3.166	4.570	1.244	2.435	4.806
1.652	2.300	3.344	4.602	1.249	2.464	4.881
1.757	2.324	3.376	4.663	1.262	2.543	5.140
1.795	2.349	3.385	4.694	1.360	2.560	

because the failure were not observed yet. In the data set, the service times was regarded as right-censored data.

In the data set demonstrated in Table 1, a total of 153 data is included, 88 observations which describe the fault time and 65 observations which describe the service times of faulted windshields. Time unit of measurement is 1000 hours.

With the Kaplan–Meier method (Zhou and Yang 2015) which is a non-parametric method, reliability function was estimated from the observations. Let's name this estimation as $\hat{R}_{\text{KM}}$. Parameters of the model were optimized by using GA for reaching the most appropriate twofold Weibull mixture model with $\hat{R}_{\text{KM}}$ and thus $\hat{R}_{\text{GA}}$ estimations were found.

In order to reach the best model, three distance metrics below (17)–(19) are used as fitness function. Mean square error (MSE), mean absolute error (MAE) and mean absolute percentage error (MAPE) is defined with

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{R}_{\text{KM}}^{(i)} - \hat{R}_{\text{GA}}^{(i)}\right)^2 \tag{17}$$

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|\hat{R}_{\text{KM}}^{(i)} - \hat{R}_{\text{GA}}^{(i)}\right| \tag{18}$$

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\hat{R}_{\text{KM}}^{(i)} - \hat{R}_{\text{GA}}^{(i)}}{\hat{R}_{\text{KM}}^{(i)}}\right| \tag{19}$$

equations. It is aimed for used fitness functions to be minimum. Methods created by using three different fitness functions are named as GA_{MSE}, GA_{MAE} and GA_{MAPE}, respectively.

Anderson–Darling test was used in order to test the fitness of the model that is found by the data. Fischer and Kamps (2013) compared the competency of various tests used for the goodness of fit in right-censored data and Anderson–Darling test took its place as one of the best in many cases in their study. Anderson–Darling test is based on the difference between cumulative distribution function and empirical distribution function, stated in the null hypothesis. Test statistics are given with

Table 2. Parameters of GA.

Structure of Chromosome	$[\omega, \alpha_1, \beta_1, \alpha_2, \beta_2]$
Population	100
Iteration count	50
Mutation rate	0.1
Fitness function	min(MSE) min(MAE) min(MAPE)

$$A^2 = -\frac{1}{n}\sum_{i=1}^{n}(2i-1)\left\{\ln[p_{(i)}] + \ln[1-p_{(n-i+1)}]\right\} - n \tag{20}$$

equation. Here it is $p_{(i)} = F\left([x_i - \overline{x}]/s\right)$, F is the cumulative distribution function in the hypothesis and $\overline{x}$ and s are the mean of the data and standard distribution, respectively. The small Anderson–Darling value of the data (A^2), obtained as a result of the test, indicates the more fitness of the data with the model.

The proposed method is coded in R programing language (R Core Team 2018) and GA package (Scrucca 2013) is used for genetic algorithm. GA parameters selected in the software are given in Table 2.

The performance of the model obtained with the proposed GA method is compared with WPP and EM methods and the results are demonstrated in Table 3. In WPP method, the results that are obtained from the studies of Murthy, Xie, and Jiang (2004) are used. In the EM method, the software that is used in the studies of Ruhi, Sarker, and Karim (2015) is used. Initial values were taken randomly in both GA and EM methods.

The model that is obtained by using all the data may not accurately estimate in different data samples (Gigerenzer and Brighton 2009). For this reason, in order to test the estimation validity and increase the reliability of the performance comparison of the model, 10-fold cross validation is used in EM and GA methods. Cross-validation is widely accepted in data mining and machine learning fields and furthermore, it is used as a standard procedure for performance estimation and model selection (Refaeilzadeh, Tang, and Liu 2009). k-fold cross validation performs more tests in order to obtain a stable estimation of the model error and uses the mean of the performances of these tests. 10-fold cross validation ($k = 10$) is widely used in the literature (Reitermanova 2010). In the 10-fold cross-validation, the available data are randomly divided into 10 parts. 9 of the parts are used as education data to develop models, the other part was used as a test data for evaluating the performance of the available model. 10 iterations were conducted by using a different part each time for test data. The means of the fitness of the models in each iteration with training and test data are recorded by measuring with Anderson–Darling test. After ensuring the performance of the method, last model was created using all data by omitting the models that emerged during iterations.

When EM method is applied, A^2 value is obtained as 0.4093 for the training phase. Since this value is lower than the A^2 values that are found in other methods, the EM method demonstrated the best performance during the training phase. On the other hand, the lowest A^2 value was found as 0.9050 in GA_{MSE} method for the testing phase. The performance of the EM method is observed to decrease considerably the test phase. This indicates that models obtained with GA methods are more appropriate with new data.

Moreover, the models obtained in this study were compared using MSE, MAE and MAPE metrics. The model obtained with the GA-MAPE method produced the lowest MSE value, while the model obtained with the GA-MAE method produced the lowest MAE value. In contrast, the lowest MAPE value was achieved by the model obtained by the EM method.

Table 3. Comparison of GA methods and other methods in literature.

				Proposed methods		
		WPP	EM	GA_{MSE}	GA_{MAE}	GA_{MAPE}
Training	A^2	1.1910	**0.4093**	0.7112	0.6177	0.7731
	p value	0.2705	0.8415	0.5711	0.6392	0.5260
Testing	A^2	–	1.3596	**0.9050**	0.9758	0.9286
	p value	–	0.2801	0.5179	0.5098	0.5297
Model	$\hat{\omega}$	0.136	0.0188	0.0004	0.1704	0.1926
	$\hat{\alpha}_1$	8.230	0.2568	4.1248	4.2835	4.0261
	$\hat{\beta}_1$	0.429	1.2709	3.9644	3.2830	5.2992
	$\hat{\alpha}_2$	3.210	3.5138	3.3261	3.3204	3.3131
	$\hat{\beta}_2$	2.990	2.8405	2.3786	2.5441	2.2846
Performance of models	MSE	0.00133	0.00123	0.00108	0.00086	**0.00084**
	MAE	0.03074	0.02704	0.02896	**0.02207**	0.02374
	MAPE	0.11441	**0.09688**	0.10674	0.10828	0.09949

4. Simulation study

In this section, in order to evaluate the performance of the available methods by using GA, simulation study was conducted. Simulations were performed for different sample sizes, different censorship ratios and different mixture proportions. Appropriate data for twofold Weibull mixture distribution is produced according to the definitions below.

$$f_{ij} \sim \text{Weibull}(\alpha_j, \beta_j) \qquad i = 1,\ 2,\ \dots,\ n;\ j = 1,\ 2 \tag{21}$$

$$x_i = \omega f_{i1} + (1-\omega) f_{i2} \qquad i = 1,\ 2,\ \dots,\ n \tag{22}$$

Here, f_{ij} are the pseudo-random numbers that are obtained from Weibull distribution. ω is the mixture proportion in twofold Weibull mixture distribution.

Different rates of data were chosen randomly from the samples and accepted as right censored data. Models that are appropriate for the data are created with EM algorithm method and GA methods. A^2 and p values were calculated for each simulation repeats.

The parameters of the twofold Weibull mixture distribution were kept constant during the simulation study as α_1=7, β_1=3, α_2=2 and β_2=4. A total of 80 cases were examined for five different mixture proportion ($\omega = 0.1$, 0.3, 0.5, 0.7, 0.9), four different sample size ($n = 25$, 100, 250, 500) and four different censorship ratio (10%, 20%, 30%, 40%). Each case was repeated 100 times. Estimated mean square error (EMSE) and estimated bias (EBIAS) for each case are calculated according to the equation below.

$$\text{EMSE} = \frac{1}{m}\sum_{i=1}^{m}\left(\hat{\theta}_i - \theta\right)^2 \tag{23}$$

$$\text{EBIAS} = \frac{1}{m}\sum_{i=1}^{m}\hat{\theta}_i - \theta \tag{24}$$

where $\theta = (\alpha_1, \beta_1, \alpha_2, \beta_2, \omega)$ and m is count of trials in simulation.

As a result of the simulation, EBIAS and EMSE values for different mixture proportions, different sample sizes and different censorship ratios are demonstrated in Appendix A–D.

In most cases in the simulation study, GA methods found the more appropriate models from the EM method. It is observed that in the GA methods, using metrics, which measure the distance between the data and the model, as fitness function not only makes it easy to apply but also provides better results.

Among 80 cases, the proposed GA-MAPE method produced the lowest EMSE value in 30 cases, the GA-MSE method in 18 cases and the GA-MAE method in 15 cases. However, the EM

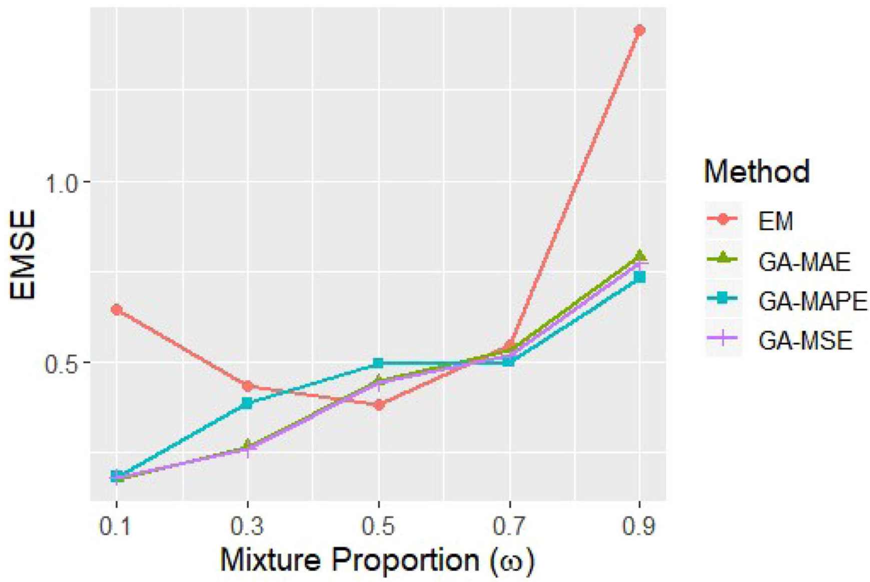

Figure 1. EMSE of the methods according to the mixture proportion.

method produced the lowest EMSE values in only 17 cases. The EM method is successful in some cases where the mixture proportion is equal to 0.5 or 0.7.

On the other hand, for the EBIAS of the β_2 parameter, the EM algorithm has the best values in 54 of 80 cases, whereas for the EBIAS of the $\alpha_1, \beta_1,$ *and* α_2 parameters, the proposed GA methods have mostly the best values. For the ω parameter, EM and GA algorithms have the best EBIAS value in similar number of cases.

In the graphic in Figure 1, the change of EMSE value according to mixture proportion can be observed in all cases. As the mixture proportion increases, EMSE value increases as well in GA methods, that is, a decrease in the fitness of the model to the data is observed. On the other hand, in the EM method, the best EMSE value was obtained when the mixture proportion is equal to 0.5. While the mixture proportion is equal to 0.5, the EM method is more successful than the proposed methods. On the contrary, GA methods are more successful in cases where the mixture proportion is different from 0.5. EM method is thought to produce worse results in cases where the mixture proportion is small or large because the EM algorithm is affected by the initial values and converges to local solutions.

In the graphic in Figure 2, the change of EMSE value according to sample size can be observed in all cases in simulation. As the sample size increases, EMSE value decreases, that is, performances of models increase. GA_{MSE} method has better performance than other methods.

In the graphic in Figure 3, the change of EMSE value according to censorship ratio can be observed in all cases in the simulation. As the censorship ratio increases, EMSE value increases, that is, performances of the models decrease. GA_{MSE} method has better performance than other methods.

5. Conclusion

As a result, a twofold Weibull mixture model was created for a data set used in the literature with an available method and the parameters of the model was estimated. When the obtained results were compared with the studies of Murthy, Xie, and Jiang (2004) and Ruhi, Sarker, and Karim (2015) on the same data set, it was determined that although EM method produces more appropriate model in the training phase, in the model created with GA_{MSE} method, the result of the Anderson–Darling goodness of fit test was found to be the smallest value in test phase. This

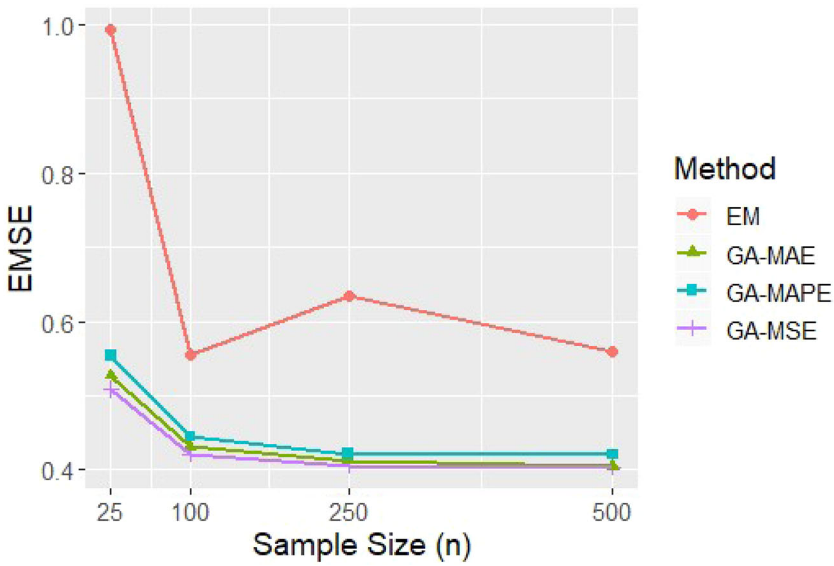

Figure 2. EMSE of the methods according to the sample size.

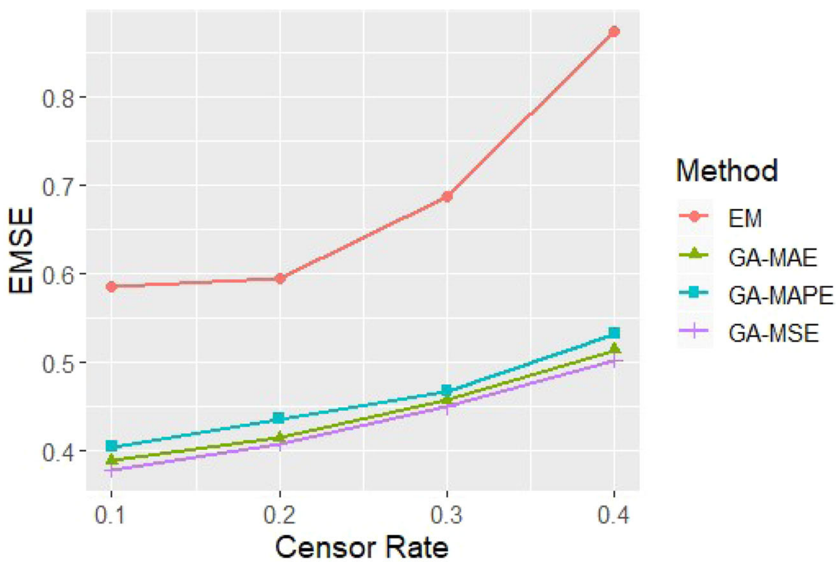

Figure 3. EMSE of the methods according to the censor rate.

indicates that the model found by GA_{MSE} method has a better estimation performance compared to other methods.

Monte Carlo simulation study was conducted in order to better examine the result obtained from the actual data. With the simulation study, the fitness of the model was tested for various sample sizes, censorship ratios and mixture proportions. Simulation study demonstrated that models found with proposed GA methods provide better fitness to the data in most cases, compared to the method where EM algorithm is used. Especially the GA_{MAPE} method has performed best in more cases in simulation. The EM method has performed better only in cases where the mixture proportion is equal to 0.5. The fact that GA is less affected by initial values and local solutions than the EM algorithm makes this difference.

Appendix A. EBIAS and EMSE for $n = 25$ in simulation.

Cases		EM					GA-MSE					GA-MAE					GA-MAPE				
ω	CR	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$
0.1	0.1	−0.752	0.214	0.081	0.012	0.092	−0.012	0.019	0.072	0.061	0.005	0.018	−0.009	0.069	0.064	0.008	0.024	−0.126	0.074	0.098	−0.001
				EMSE = 0.6489					EMSE = 0.2204					EMSE = 0.2305					**EMSE=0.1701**		
	0.2	−0.737	0.220	0.096	0.016	0.095	0.011	0.010	0.085	0.068	0.009	−0.014	−0.001	0.079	0.079	0.014	−0.023	−0.058	0.088	0.093	0.002
				EMSE = 0.6387					EMSE = 0.2251					EMSE = 0.2053					**EMSE=0.1888**		
	0.3	−0.726	0.230	0.107	0.025	0.094	−0.017	−0.044	0.096	0.090	0.012	0.010	−0.086	0.091	0.112	0.018	0.017	−0.046	0.096	0.135	0.006
				EMSE = 0.6375					**EMSE=0.1732**					EMSE = 0.2026					EMSE = 0.2216		
	0.4	−0.705	0.237	0.128	0.031	0.096	0.012	0.000	0.114	0.126	0.018	0.023	0.000	0.108	0.134	0.023	−0.008	−0.042	0.112	0.159	0.016
				EMSE = 0.6271					EMSE = 0.2594					EMSE = 0.2521					**EMSE=0.2420**		
0.3	0.1	−0.587	0.144	0.247	−0.047	0.065	−0.106	0.011	0.239	0.000	−0.027	−0.130	0.032	0.234	−0.001	−0.024	−0.139	0.045	0.245	0.016	−0.038
				EMSE = 0.4460					**EMSE=0.2384**					EMSE = 0.2387					EMSE = 0.2412		
	0.2	−0.564	0.148	0.270	−0.044	0.061	−0.111	0.052	0.257	0.025	−0.021	−0.123	0.028	0.253	0.033	−0.017	−0.134	0.045	0.263	0.032	−0.032
				EMSE = 0.4358					EMSE = 0.2868					EMSE = 0.2775					**EMSE=0.2711**		
	0.3	−0.546	0.156	0.287	−0.036	0.059	−0.100	−0.006	0.275	0.064	−0.016	−0.114	0.017	0.268	0.065	−0.012	−0.131	0.050	0.276	0.075	−0.023
				EMSE = 0.4336					**EMSE=0.2737**					EMSE = 0.3003					EMSE = 0.3222		
	0.4	−0.514	0.162	0.319	−0.031	0.058	−0.109	0.059	0.298	0.088	−0.006	−0.116	0.032	0.292	0.106	−0.001	−0.148	0.121	0.297	0.073	−0.013
				EMSE = 0.4305					**EMSE=0.3338**					EMSE = 0.3413					EMSE = 0.3418		
0.5	0.1	−0.421	0.109	0.412	−0.076	0.041	−0.212	0.325	0.351	−0.017	−0.032	−0.206	0.351	0.343	−0.012	−0.031	−0.224	0.584	0.336	0.021	−0.035
				EMSE=0.3831					EMSE = 0.4719					EMSE = 0.5062					EMSE = 0.6743		
	0.2	−0.390	0.112	0.443	−0.073	0.037	−0.185	0.286	0.371	0.010	−0.024	−0.186	0.353	0.369	0.021	−0.022	−0.205	0.561	0.348	0.020	−0.025
				EMSE = 0.3872					EMSE = 0.4512					EMSE = 0.5613					EMSE = 0.6597		
	0.3	−0.366	0.120	0.467	−0.066	0.029	−0.161	0.307	0.396	0.067	−0.019	−0.165	0.288	0.397	0.066	−0.021	−0.184	0.507	0.367	0.041	−0.020
				EMSE = 0.3965					EMSE = 0.5413					EMSE = 0.5401					EMSE = 0.6508		
	0.4	−0.323	0.125	0.511	−0.062	0.025	−0.117	0.324	0.420	0.071	−0.007	−0.114	0.295	0.430	0.101	−0.009	−0.147	0.446	0.401	0.114	−0.010
				EMSE = 0.4194					EMSE = 0.6032					EMSE = 0.6050					EMSE = 0.7154		
0.7	0.1	−0.255	0.088	0.578	−0.094	−0.025	−0.090	0.358	0.456	−0.007	−0.046	−0.106	0.353	0.436	0.007	−0.041	−0.089	0.460	0.433	0.025	−0.041
				EMSE = 0.4383					EMSE = 0.5881					EMSE = 0.5700					EMSE = 0.6059		
	0.2	−0.217	0.091	0.617	−0.091	−0.018	−0.042	0.333	0.460	0.055	−0.037	−0.058	0.357	0.488	0.056	−0.036	−0.046	0.433	0.448	0.056	−0.036
				EMSE = 0.4683					EMSE = 0.5774					EMSE = 0.6647					EMSE = 0.6296		
	0.3	−0.186	0.098	0.647	−0.085	−0.011	−0.022	0.351	0.516	0.069	−0.036	−0.014	0.303	0.541	0.084	−0.037	−0.030	0.442	0.495	0.039	−0.035
				EMSE = 0.4993					EMSE = 0.6844					EMSE = 0.7306					EMSE = 0.7134		
	0.4	−0.131	0.103	0.702	−0.081	−0.008	0.067	0.308	0.571	0.123	−0.035	0.034	0.279	0.596	0.113	−0.029	0.017	0.439	0.561	0.117	−0.029
				EMSE = 0.5661					EMSE = 0.7931					EMSE = 0.7721					EMSE = 0.8717		
0.9	0.1	−0.147	0.073	0.791	0.365	−0.054	0.075	0.321	0.552	0.045	−0.068	0.057	0.366	0.591	0.007	−0.069	0.065	0.390	0.558	0.046	−0.067
				EMSE = 1.7628					**EMSE = 0.6604**					EMSE = 0.7564					EMSE = 0.6939		
	0.2	−0.081	0.080	0.799	0.347	−0.049	0.100	0.323	0.633	0.075	−0.065	0.107	0.317	0.633	0.077	−0.065	0.089	0.362	0.626	0.082	−0.066
				EMSE = 1.6300					EMSE = 0.8326					**EMSE = 0.8266**					EMSE = 0.8574		
	0.3	−0.014	0.101	0.807	0.484	−0.047	0.151	0.348	0.661	0.108	−0.063	0.154	0.340	0.640	0.051	−0.065	0.132	0.385	0.636	0.098	−0.061
				EMSE = 2.5637					EMSE = 0.9177					**EMSE = 0.8521**					EMSE = 0.9072		
	0.4	0.075	0.146	0.916	0.732	−0.045	0.194	0.299	0.742	0.125	−0.060	0.178	0.371	0.745	0.079	−0.064	0.195	0.354	0.724	0.148	−0.059
				EMSE = 6.0531					**EMSE = 1.0425**					EMSE = 1.1017					EMSE = 1.0990		

Appendix B. EBIAS and EMSE for n =100 in simulation.

Cases		EM					GA-MSE					GA-MAE					GA-MAPE				
ω	CR	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$
0.1	0.1	−0.742	0.157	0.091	−0.036	0.097	−0.017	0.006	0.080	0.028	0.002	0.003	0.039	0.077	0.019	0.004	−0.048	−0.038	0.087	0.052	−0.007
				EMSE = 0.5975					EMSE = 0.1823					EMSE = 0.1615					**EMSE = 0.1496**		
	0.2	−0.729	0.162	0.105	−0.032	0.094	0.014	0.094	0.094	0.025	0.000	−0.044	−0.015	0.088	0.044	0.009	0.022	−0.047	0.099	0.055	−0.004
				EMSE = 0.5830					EMSE = 0.1789					**EMSE = 0.1614**					EMSE = 0.1881		
	0.3	−0.713	0.163	0.121	−0.031	0.094	0.028	0.092	0.110	0.023	0.003	−0.029	−0.051	0.103	0.052	0.013	−0.039	−0.024	0.112	0.054	−0.001
				EMSE = 0.5645					EMSE = 0.1773					**EMSE = 0.1579**					EMSE = 0.1640		
	0.4	−0.694	0.161	0.139	−0.032	0.092	0.027	−0.006	0.127	0.030	0.008	−0.003	0.004	0.118	0.049	0.017	−0.048	0.009	0.128	0.057	0.006
				EMSE = 0.5436					**EMSE = 0.1555**					EMSE = 0.1797					EMSE = 0.1921		
0.3	0.1	−0.571	0.091	0.262	−0.091	0.062	−0.132	0.026	0.254	−0.037	−0.031	−0.142	0.072	0.247	−0.032	−0.027	−0.177	0.204	0.260	−0.015	−0.040
				EMSE = 0.4173					**EMSE = 0.2050**					EMSE = 0.2217					EMSE = 0.3642		
	0.2	−0.550	0.095	0.283	−0.087	0.060	−0.151	0.188	0.269	−0.047	−0.029	−0.176	0.060	0.262	−0.029	−0.021	−0.207	0.215	0.277	−0.014	−0.036
				EMSE = 0.4064					EMSE = 0.2533					**EMSE = 0.2201**					EMSE = 0.3288		
	0.3	−0.526	0.097	0.308	−0.086	0.059	−0.134	0.084	0.293	−0.020	−0.022	−0.185	0.123	0.283	−0.030	−0.016	−0.219	0.276	0.287	−0.009	−0.026
				EMSE = 0.3949					**EMSE = 0.2596**					EMSE = 0.2743					EMSE = 0.3745		
	0.4	−0.496	0.095	0.337	−0.088	0.058	−0.149	0.078	0.314	−0.027	−0.014	−0.169	0.148	0.304	−0.019	−0.009	−0.204	0.242	0.315	−0.009	−0.020
				EMSE = 0.3849					**EMSE = 0.2627**					EMSE = 0.2951					EMSE = 0.3848		
0.5	0.1	−0.400	0.059	0.433	−0.118	0.025	−0.210	0.376	0.373	−0.086	−0.038	−0.219	0.357	0.367	−0.086	−0.033	−0.228	0.501	0.346	−0.041	−0.029
				EMSE = 0.3691					EMSE = 0.4264					EMSE = 0.4235					EMSE = 0.4871		
	0.2	−0.372	0.063	0.462	−0.115	0.028	−0.196	0.342	0.390	−0.084	−0.029	−0.208	0.378	0.381	−0.083	−0.023	−0.203	0.485	0.368	−0.055	−0.024
				EMSE = 0.3730					EMSE = 0.4157					EMSE = 0.4325					EMSE = 0.4928		
	0.3	−0.339	0.064	0.495	−0.113	0.024	−0.177	0.339	0.420	−0.077	−0.020	−0.164	0.367	0.422	−0.066	−0.025	−0.187	0.448	0.391	−0.053	−0.015
				EMSE = 0.3813					EMSE = 0.4656					EMSE = 0.4789					EMSE = 0.4818		
	0.4	−0.299	0.062	0.534	−0.115	0.025	−0.138	0.296	0.458	−0.070	−0.015	−0.135	0.329	0.440	−0.048	−0.010	−0.143	0.469	0.428	−0.072	−0.012
				EMSE = 0.3981					EMSE = 0.4420					EMSE = 0.4640					EMSE = 0.5748		
0.7	0.1	−0.229	0.039	0.604	−0.134	−0.011	−0.089	0.267	0.496	−0.127	−0.042	−0.093	0.241	0.499	−0.085	−0.042	−0.076	0.310	0.461	−0.086	−0.039
				EMSE = 0.4418					EMSE = 0.4232					EMSE = 0.4302					**EMSE = 0.3860**		
	0.2	−0.193	0.043	0.640	−0.131	−0.012	−0.039	0.278	0.538	−0.109	−0.046	−0.062	0.263	0.533	−0.083	−0.038	−0.048	0.324	0.502	−0.084	−0.038
				EMSE = 0.4713					EMSE = 0.4738					EMSE = 0.4652					**EMSE = 0.4399**		
	0.3	−0.152	0.044	0.682	−0.130	−0.009	−0.019	0.241	0.586	−0.053	−0.039	−0.010	0.270	0.577	−0.089	−0.039	−0.007	0.309	0.536	−0.059	−0.034
				EMSE = 0.5120					EMSE = 0.5404					EMSE = 0.5480					**EMSE = 0.5115**		
	0.4	−0.101	0.043	0.732	−0.131	−0.008	0.039	0.227	0.621	−0.060	−0.034	0.043	0.270	0.611	−0.059	−0.035	0.026	0.266	0.577	−0.060	−0.023
				EMSE = 0.5718					EMSE = 0.5714					EMSE = 0.6223					**EMSE = 0.5403**		
0.9	0.1	−0.120	−0.020	0.837	0.050	−0.061	0.077	0.204	0.655	−0.117	−0.071	0.061	0.171	0.670	−0.128	−0.068	0.090	0.234	0.604	−0.106	−0.066
				EMSE = 0.8098					EMSE = 0.6039					EMSE = 0.5989					**EMSE = 0.5256**		
	0.2	−0.062	−0.012	0.860	0.099	−0.059	0.123	0.195	0.694	−0.100	−0.071	0.115	0.200	0.704	−0.094	−0.070	0.111	0.231	0.694	−0.115	−0.068
				EMSE = 0.9748					**EMSE = 0.6642**					EMSE = 0.6965					EMSE = 0.6725		
	0.3	−0.003	−0.001	0.893	0.081	−0.059	0.163	0.179	0.759	−0.071	−0.065	0.164	0.198	0.758	−0.075	−0.068	0.169	0.216	0.698	−0.061	−0.061
				EMSE = 0.9383					EMSE = 0.8026					EMSE = 0.8044					**EMSE = 0.7237**		
	0.4	0.089	0.002	0.917	0.052	−0.058	0.225	0.215	0.801	−0.052	−0.057	0.219	0.213	0.812	−0.038	−0.061	0.215	0.208	0.779	−0.017	−0−0.059
				EMSE = 0.9822					EMSE = 0.9194					EMSE = 0.9852					**EMSE = 0.9068**		

Appendix C. EBIAS and EMSE for n = 250 in simulation.

Cases		EM					GA-MSE					GA-MAE					GA-MAPE				
ω	CR	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$
0.1	0.1	−0.760	0.119	0.132	0.134	0.097	0.007	0.016	0.082	0.021	0.000	−0.036	−0.026	0.075	0.035	0.008	−0.035	−0.025	0.090	0.025	−0.010
				EMSE = 0.6779					EMSE = 0.1897					**EMSE = 0.1485**					EMSE = 0.1615		
	0.2	−0.747	0.115	0.144	0.126	0.095	−0.020	0.041	0.094	0.023	0.002	−0.014	−0.018	0.088	0.029	0.009	−0.003	0.000	0.102	0.033	−0.006
				EMSE = 0.6587					EMSE = 0.1485					**EMSE = 0.1385**					EMSE = 0.1922		
	0.3	−0.734	0.120	0.163	0.226	0.093	0.006	0.087	0.110	0.017	0.003	−0.036	0.059	0.103	0.032	0.011	−0.040	−0.045	0.114	0.046	−0.001
				EMSE = 1.1044					EMSE = 0.1651					**EMSE = 0.1548**					EMSE = 0.1717		
	0.4	−0.715	0.120	0.181	0.187	0.091	0.029	0.085	0.127	0.029	0.007	−0.026	0.075	0.120	0.040	0.016	0.000	0.042	0.131	0.041	0.002
				EMSE = 0.8108					EMSE = 0.1718					**EMSE = 0.1712**					EMSE = 0.1754		
0.3	0.1	−0.601	0.045	0.314	0.024	0.052	−0.134	0.029	0.253	−0.034	−0.031	−0–0.130	0.169	0.247	−0.041	−0.028	−0.221	0.338	0.261	−0.018	−0.042
				EMSE = 0.4824					**EMSE = 0.2035**					EMSE = 0.2615					EMSE = 0.4303		
	0.2	−0.583	0.045	0.336	0.057	0.051	−0.160	0.115	0.271	−0.040	−0.028	−0.146	0.042	0.266	−0.038	−0.023	−0.223	0.269	0.276	−0.023	−0.034
				EMSE = 0.4955					EMSE = 0.2351					**EMSE = 0.2048**					EMSE = 0.3575		
	0.3	−0.554	0.043	0.356	0.064	0.051	−0.131	0.094	0.295	−0.047	−0.025	−0.185	0.173	0.282	−0.034	−0.017	−0.245	0.356	0.293	−0.028	−0.027
				EMSE = 0.5062					**EMSE = 0.2351**					EMSE = 0.2952					EMSE = 0.4339		
	0.4	−0–0.525	0.047	0.385	0.066	0.050	−0.157	0.186	0.319	−0.043	−0.020	−0.174	0.152	0.309	−0.039	−0.010	−0.202	0.346	0.319	−0.036	−0.023
				EMSE = 0.5089					EMSE = 0.2934					**EMSE = 0.2837**					EMSE = 0.4179		
0.5	0.1	−0.395	0.048	0.438	−0.126	0.027	−0.213	0.327	0.379	−0.098	−0.037	−0.224	0.338	0.373	−0.095	−0.034	−0.217	0.451	0.357	−0.056	−0.033
				EMSE = 0.3685					EMSE = 0.3758					**EMSE = 0.3683**					EMSE = 0.4146		
	0.2	−0.367	0.049	0.467	−0.126	0.022	−0.194	0.376	0.397	−0.093	−0.030	−0.205	0.334	0.396	−0.093	−0.027	−0.203	0.411	0.369	−0.057	−0.022
				EMSE = 0.3729					EMSE = 0.4410					EMSE = 0.3931					EMSE = 0.3935		
	0.3	−0.333	0.050	0.500	−0.125	0.024	−0.190	0.305	0.427	−0.092	−0.017	−0.180	0.329	0.419	−0.104	−0.020	−0.185	0.398	0.396	−0.082	−0.014
				EMSE = 0.3822					EMSE = 0.4126					EMSE = 0.4095					EMSE = 0.4001		
	0.4	−0.292	0.051	0.541	−0.124	0.025	−0.142	0.268	0.464	−0.084	−0.013	−0.134	0.308	0.454	−0.087	−0.015	−0.160	0.374	0.419	−0.066	−0.003
				EMSE = 0.3996					EMSE = 0.4105					EMSE = 0.4184					EMSE = 0.4246		
0.7	0.1	−0.274	−0.020	0.666	−0.017	−0.026	−0.092	0.209	0.522	−0.123	−0.043	−0.095	0.221	0.526	−0.133	−0.045	−0.069	0.283	0.481	−0.089	−0.043
				EMSE = 0.5586					EMSE = 0.3977					EMSE = 0.4163					**EMSE = 0.3909**		
	0.2	−0.229	−0.020	0.692	−0.001	−0.025	−0.055	0.230	0.562	−0.147	−0.047	−0.061	0.226	0.549	−0.130	−0.043	−0.042	0.246	0.507	−0.114	−0.035
				EMSE = 0.5837					EMSE = 0.4439					EMSE = 0.4493					**EMSE = 0.3754**		
	0.3	−0.177	−0.019	0.722	0.020	−0.024	−0.008	0.192	0.602	−0.121	−0.039	−0.016	0.192	0.613	−0.126	−0.042	−0.006	0.240	0.555	−0.120	−0.033
				EMSE = 0.6689					EMSE = 0.4813					EMSE = 0.5060					**EMSE = 0.4320**		
	0.4	−0.104	−0.020	0.750	0.001	−0.023	0.043	0.182	0.622	−0.086	−0.028	0.029	0.172	0.652	−0.087	−0.029	0.047	0.229	0.623	−0.100	−0.034
				EMSE = 0.6538					**EMSE = 0.5185**					EMSE = 0.5565					EMSE = 0.5395		
0.9	0.1	−0.095	−0.032	0.829	−0.033	−0.060	0.062	0.156	0.687	−0.144	−0.072	0.063	0.150	0.700	−0.143	−0.070	0.083	0.172	0.625	−0.101	−0.063
				EMSE = 0.7501					EMSE = 0.5930					EMSE = 0.6134					**EMSE = 0.4974**		
	0.2	−0.037	−0.027	0.856	−0.018	−0.058	0.104	0.133	0.746	−0.137	−0.073	0.093	0.129	0.749	−0.125	−0.065	0.117	0.160	0.686	−0.126	−0.059
				EMSE = 0.8017					EMSE = 0.6900					EMSE = 0.6982					**EMSE = 0.5989**		
	0.3	0.024	−0.023	0.893	0.007	−0.057	0.147	0.108	0.809	−0.110	−0.066	0.160	0.129	0.792	−0.115	−0.063	0.169	0.183	0.743	−0.132	−0.065
				EMSE = 0.9326					EMSE = 0.8025					EMSE = 0.7844					**EMSE = 0.7270**		
	0.4	0.117	−0.021	0.921	−0.019	−0.056	0.208	0.107	0.857	−0.099	−0.060	0.180	0.109	0.883	−0.105	−0.064	0.208	0.141	0.833	−0.096	−0.060
				EMSE = 0.9578					EMSE = 0.9124					EMSE = 0.9673					**EMSE = 0.8888**		

Appendix D. EBIAS and EMSE for n = 500 in simulation.

Cases		EM					GA-MSE					GA-MAE					GA-MAPE				
ω	CR	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_2$	$\hat{\beta}_2$	$\hat{\omega}$
0.1	0.1	−0.738	0.146	0.095	−0.045	0.092	0.008	0.045	0.082	0.037	0.001	−0.025	0.052	0.078	0.030	0.006	−0.003	−0.090	0.093	0.036	−0.010
				EMSE = 0.5865					**EMSE = 0.1693**					EMSE = 0.1747					EMSE = 0.1899		
	0.2	−0.724	0.147	0.109	−0.044	0.096	−0.001	0.040	0.096	0.035	0.002	−0.039	0.024	0.090	0.045	0.010	−0.042	0.006	0.105	0.033	−0.008
				EMSE = 0.5696					**EMSE = 0.1687**					EMSE = 0.1818					EMSE = 0.1893		
	0.3	−0.708	0.148	0.125	−0.043	0.093	0.001	0.084	0.113	0.020	0.001	−0.026	0.032	0.105	0.036	0.011	−0.040	−0.025	0.117	0.044	−0.003
				EMSE = 0.5507					EMSE = 0.1798					**EMSE = 0.1607**					EMSE = 0.1698		
	0.4	−0.689	0.148	0.144	−0.043	0.098	−0.006	0.079	0.131	0.029	0.005	−0.035	0.068	0.122	0.040	0.014	−0.069	−0.061	0.134	0.046	0.004
				EMSE = 0.5291					EMSE = 0.1635					EMSE = 0.1628					**EMSE = 0.1626**		
0.3	0.1	−0.566	0.078	0.268	−0.102	0.071	−0.168	0.154	0.253	−0.035	−0.032	−0.177	0.126	0.246	−0.028	−0.024	−0.243	0.395	0.262	−0.022	−0.041
				EMSE = 0.4120					**EMSE = 0.2478**					EMSE = 0.2680					EMSE = 0.4607		
	0.2	−0.544	0.079	0.289	−0.101	0.064	−0.172	0.169	0.270	−0.041	−0.028	−0.183	0.154	0.263	−0.033	−0.021	−0.242	0.400	0.277	−0.019	−0.035
				EMSE = 0.4004					EMSE = 0.2534					**EMSE = 0.2474**					EMSE = 0.5067		
	0.3	−0.519	0.080	0.314	−0.100	0.060	−0.195	0.185	0.291	−0.038	−0.023	−0.173	0.154	0.287	−0.032	−0.018	−0.244	0.363	0.297	−0.027	−0.030
				EMSE = 0.3890					**EMSE = 0.2851**					EMSE = 0.2978					EMSE = 0.4724		
	0.4	−0.489	0.080	0.344	−0.100	0.058	−0.178	0.197	0.320	−0.043	−0.019	−0.162	0.139	0.312	−0.032	−0.013	−0.268	0.396	0.313	−0.038	−0.018
				EMSE = 0.3788					EMSE = 0.3026					**EMSE = 0.2640**					EMSE = 0.4707		
0.5	0.1	−0.393	0.045	0.440	−0.129	0.022	−0.220	0.359	0.382	−0.098	−0.037	−0.212	0.394	0.368	−0.086	−0.034	−0.221	0.451	0.360	−0.051	−0.033
				EMSE = 0.3683					EMSE = 0.4101					EMSE = 0.4025					EMSE = 0.4179		
	0.2	−0.365	0.044	0.469	−0.130	0.024	−0.194	0.338	0.404	−0.097	−0.031	−0.181	0.343	0.400	−0.094	−0.032	−0.195	0.428	0.377	−0.069	−0.026
				EMSE = 0.3729					EMSE = 0.3970					EMSE = 0.4018					EMSE = 0.4050		
	0.3	−0.331	0.044	0.503	−0.130	0.025	−0.167	0.296	0.432	−0.104	−0.026	−0.157	0.299	0.425	−0.103	−0.025	−0.183	0.357	0.396	−0.078	−0.013
				EMSE = 0.3825					EMSE = 0.3749					EMSE = 0.3699					**EMSE = 0.3688**		
	0.4	−0.290	0.044	0.544	−0.130	0.027	−0.140	0.285	0.473	−0.078	−0.014	−0.129	0.266	0.467	−0.089	−0.018	−0.149	0.334	0.431	−0.094	−0.008
				EMSE = 0.4003					EMSE = 0.4373					EMSE = 0.4008					**EMSE = 0.3702**		
0.7	0.1	−0.258	−0.018	0.652	−0.069	−0.026	−0.081	0.239	0.511	−0.128	−0.047	−0.087	0.208	0.531	−0.129	−0.047	−0.081	0.236	0.481	−0.101	−0.037
				EMSE = 0.5080					EMSE = 0.3930					EMSE = 0.4059					**EMSE = 0.3413**		
	0.2	−0.212	−0.019	0.678	−0.065	−0.025	−0.046	0.204	0.548	−0.141	−0.043	−0.057	0.202	0.557	−0.117	−0.044	−0.051	0.231	0.524	−0.118	−0.033
				EMSE = 0.5224					EMSE = 0.4077					EMSE = 0.4261					**EMSE = 0.3894**		
	0.3	−0.164	−0.020	0.714	−0.020	−0.024	−0.006	0.201	0.581	−0.128	−0.040	−0.016	0.166	0.598	−0.139	−0.039	−0.010	0.207	0.549	−0.130	−0.030
				EMSE = 0.6312					EMSE = 0.4458					EMSE = 0.4495					**EMSE = 0.3949**		
	0.4	−0.099	−0.022	0.751	−0.017	−0.023	0.042	0.182	0.644	−0.114	−0.034	0.032	0.165	0.660	−0.130	−0.034	0.034	0.180	0.615	−0.137	−0.027
				EMSE = 0.6747					EMSE = 0.5359					EMSE = 0.5432					**EMSE = 0.4764**		
0.9	0.1	−0.075	−0.028	0.810	−0.087	−0.061	0.082	0.156	0.683	−0.155	−0.075	0.064	0.132	0.696	−0.143	−0.068	0.072	0.143	0.631	−0.139	−0.059
				EMSE = 0.6862					EMSE = 0.5788					EMSE = 0.5826					**EMSE = 0.4880**		
	0.2	−0.022	−0.028	0.845	−0.071	−0.059	0.115	0.131	0.715	−0.128	−0.065	0.105	0.105	0.744	−0.146	−0.066	0.129	0.158	0.689	−0.143	−0.067
				EMSE = 0.7526					EMSE = 0.6242					EMSE = 0.6503					**EMSE = 0.5899**		
	0.3	0.043	−0.028	0.880	−0.044	−0.057	0.141	0.085	0.808	−0.115	−0.064	0.151	0.084	0.825	−0.114	−0.064	0.162	0.134	0.754	−0.130	−0.062
				EMSE = 0.8862					EMSE = 0.7743					EMSE = 0.8198					**EMSE = 0.7051**		
	0.4	0.131	−0.028	1.009	−0.033	−0.055	0.213	0.119	0.863	−0.100	−0.064	0.211	0.082	0.857	−0.105	−0.058	0.211	0.112	0.832	−0.132	−0.059
				EMSE = 1.1772					EMSE = 0.9256					EMSE = 0.8950					**EMSE = 0.8459**		

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