

# Functional Dependencies

## 1. Introduction:

- In a relational database, functional dependencies (FDs) are crucial for understanding the relationships between attributes within a table.
- A functional dependency between two attribute sets, X and Y, denoted as  $X \rightarrow Y$ , indicates that the values of X uniquely determine the values of Y.

## 2. Armstrong's Axioms:

Armstrong's axioms are fundamental rules used for reasoning about functional dependencies. These axioms guide us in deriving and understanding the relationships between attributes.

### a. Reflexivity Axiom:

- If Y is a subset of X, then  $X \rightarrow Y$ .
- This axiom reflects the idea that any subset of attributes is functionally dependent on the whole set of attributes.

Example:

Consider a relation R with attributes A, B, and C. If  $A \rightarrow B$  holds, then it's also true that  $AC \rightarrow B$ .

### b. Augmentation Axiom:

- If  $X \rightarrow Y$ , then  $XZ \rightarrow YZ$  for any attribute set Z.
- This axiom shows that adding attributes to both sides of a functional dependency maintains its validity.

Example:

If  $Name \rightarrow Age$ , then  $Name Address \rightarrow Age Address$  holds true.

### c. Transitivity Axiom:

- If  $X \rightarrow Y$  and  $Y \rightarrow Z$ , then  $X \rightarrow Z$ .
- This axiom implies that if a functional dependency can be derived indirectly, it can be inferred directly.

Example:

If Course  $\rightarrow$  Department and Department  $\rightarrow$  Faculty, then Course  $\rightarrow$  Faculty can be inferred.

### 3. Example Scenarios:

a. Student Table:

...

Roll No	Name	Age	Course
101	Alice	20	CS
102	Bob	22	ECE
103	Carol	21	CS

...

- In the above table, Roll No  $\rightarrow$  Name because each Roll No corresponds to a unique student's name.
- Roll No  $\rightarrow$  Age because each student's Roll No uniquely determines their age.
- Course  $\rightarrow$  Roll No because each course maps to multiple Roll Nos.

b. Course Enrollment Table:

...

Roll No	Course
101	DBMS
102	OS
103	DBMS

...

- In this table, Roll No  $\rightarrow$  Course because each Roll No maps to a specific course.

### 4. Inference Rules:

a. Union Rule:

- If  $X \rightarrow Y$  and  $X \rightarrow Z$ , then  $X \rightarrow YZ$ .

Example:

If Roll No  $\rightarrow$  Name and Roll No  $\rightarrow$  Age, then Roll No  $\rightarrow$  Name Age.

b. Decomposition Rule:

- If  $X \rightarrow YZ$ , then  $X \rightarrow Y$  and  $X \rightarrow Z$ .

Example:

If Roll No  $\rightarrow$  Name Age, then Roll No  $\rightarrow$  Name and Roll No  $\rightarrow$  Age.

c. Pseudo-Transitivity Rule:

- If  $X \rightarrow Y$  and  $WY \rightarrow Z$ , then  $WX \rightarrow Z$ .

Example:

If Course  $\rightarrow$  Department and CourseFaculty  $\rightarrow$  Office, then CourseFaculty  $\rightarrow$  Office can be inferred as Course  $\rightarrow$  DepartmentOffice.

## 5. Conclusion:

- Functional dependencies play a pivotal role in maintaining the accuracy and integrity of relational databases.
- Armstrong's axioms provide a systematic approach to understanding and deriving functional dependencies.
- These concepts are essential for database normalization and the efficient design of relational databases.