Finite Automata with €-Transitions (€-NFA):

Monday, April 19, 2021 12:53 PM

* An extension of the finite Automata which allows a transition ϵ, the empty string.
* It does give us some added programing convenience.

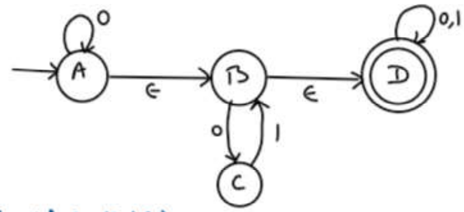
$$NFA \Rightarrow (Q, \Sigma, \delta, q_0, F)$$
where, $\delta: Q \times \Sigma \rightarrow 2^Q$

&

$$\epsilon\text{-}NFA \Rightarrow (Q, \Sigma, \delta', q_0, F)$$
where $\delta' : Q \times \Sigma \cup \{\epsilon\} \rightarrow 2^Q$

It is also called as ϵ-closure $\Rightarrow$ { transition is possible without seeing anything

ex



* ϵ-closure (q): is nothing but set of all the states which we can reach from q only on seeing ϵ.

ϵ-closure $(A) = \{A, B, D\}$

$A \xrightarrow{\epsilon} A$, $\xrightarrow{\epsilon} B \xrightarrow{\epsilon} D$

Conversion ϵ-NFA $\rightarrow$ NFA:

$$DFA \cong NFA \cong \epsilon\text{-}NFA$$

ϵNFA $\rightarrow$ NFA $\rightarrow$ DFA

	0	1
A	-	-
B		
C		
D		

* Using state-transition table

→ The method to fill the entries corresponding to given alphabet is as follows:

Lets $\delta(A, 0)$ entry has to be filled

* find the ϵ-closure $(\delta(\underbrace{\epsilon\text{-closure}(A)}, 0))$

ϵ-closure $(A) = \{A\ B\ D\}$

$$A \xrightarrow{\epsilon^*} A \xrightarrow{0} A \xrightarrow{\epsilon^*} A, B, D$$
$$\quad\ \ \rightarrow B \longrightarrow C \longrightarrow C$$
$$\quad\ \ \rightarrow D \longrightarrow D \longrightarrow D$$

	0	1
A	{A,B,C,D}	{D} ✓
B	{C,D}	{D}
C	{ }	{B,D}
D	{D}	{D}

Similarly, we can find other transition

ϵ-closure $(\delta(\epsilon\text{-closure}(A), 1))$

$$A \xrightarrow{\epsilon^*} A \xrightarrow{1} \phi \xrightarrow{\epsilon^*} \phi$$
$$\quad\ \ \rightarrow B \longrightarrow \phi \longrightarrow \phi$$
$$\quad\ \ \rightarrow D \longrightarrow D \longrightarrow D$$

ϵ-closure $(\delta(\epsilon\text{-closure}(B), 0))$

$$B \xrightarrow{\epsilon^*} B \xrightarrow{0} C \xrightarrow{\epsilon^*} C$$
$$\quad\ \ \rightarrow D \longrightarrow D \longrightarrow D$$

ϵ-closure $(\delta(\epsilon\text{-closure}(B), 1))$

$$B \xrightarrow{\epsilon^*} B \xrightarrow{1} \phi \xrightarrow{\epsilon^*} \phi$$
$$\quad\ \ \rightarrow D \longrightarrow D \longrightarrow D$$

ϵ-closure $(\delta(\epsilon\text{-closure}(C), 0))$

$$C \xrightarrow{\epsilon^*} C \xrightarrow{0} \phi \xrightarrow{\epsilon^*} \phi$$

ϵ-closure $(\delta(\epsilon\text{-closure}(C), 1))$

$$C \xrightarrow{\epsilon^*} C \xrightarrow{1} B \xrightarrow{\epsilon^*} B, D$$

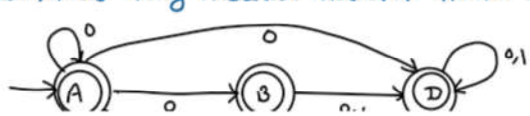
ϵ-closure $(\delta(\epsilon\text{-closure}(D), 0))$

$$D \xrightarrow{\epsilon^*} D \xrightarrow{0} D \xrightarrow{\epsilon^*} D$$

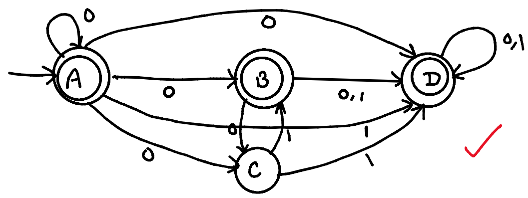
ϵ-closure $(\delta(\epsilon\text{-closure}(D), 1))$

$$D \xrightarrow{\epsilon^*} D \xrightarrow{1} D \xrightarrow{\epsilon^*} D$$

* Number of final states may increase however initial state will be same.



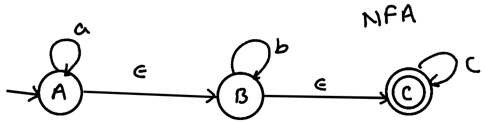
* The final state can be decided if it is reached by seeing only ϵ.



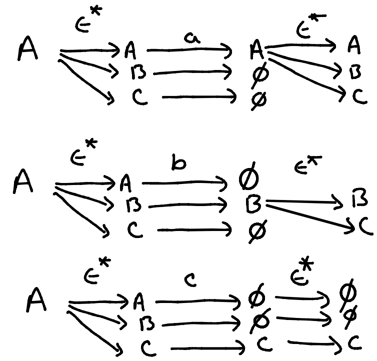
NFA

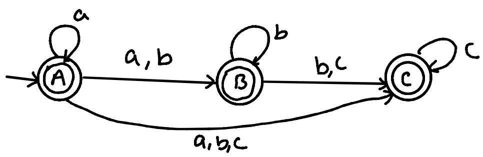
* The final state can be decided if it is reached by seeing only ϵ.

eg



	a	b	c
A	{ABC}	{BC}	{C}
B	{ }	{BC}	{C}
C	{ }	{ }	{C}





MYHILL- NERODE THEOREM:

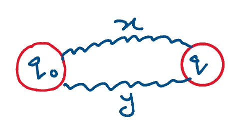
Properties of relations:

A relation R on set S is:

i) Reflexive $\Rightarrow$ if aR_a for all a in S

ii) Transitive $\Rightarrow$ if aR_b and $bR_c \Rightarrow aR_c$

iii) Symmetric $\Rightarrow$ if $aR_b \Rightarrow bR_a$



* To find equivalence of two classes:

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA.

For x & y in Σ^*, let xR_my if and only if $\delta(q_0, x) = \delta(q_0, y)$.

The relation R_m is reflexive, symmetric & transitive and thus R_m is an equivalence relation.
In addition, if xR_my, then xzR_myz for all z in Σ^*

we have already proven that

$$\delta(q_0\, xz) = \delta(\delta(q_0\, x), z) = \delta(\delta(q_0, y), z) = \delta(q_0, yz).$$

* An equivalence relation R such that xRy implies $xzRyz$ is said to be right equivalent (with respect to concatenation).

__Theorem:__ following three statements are equivalent.

i) The set $L \subseteq \Sigma^*$ is accepted by some FA.

ii) L is the union of some of equivalence classes of a right invariant equivalence relation of finite-index.

iii) Let equivalence relation R_L is defined by:

xR_Ly if and only if for all z in Σ^*, xz is in L exactly when yz is in L.

Then, R_L is of finite-index.

In Particular, the index (no. of equivalence classes) is always finite if L is a regular set.