Pumping Lemma (For Regular Languages) EXAMPLE (Part-2)

Using Pumping Lemma prove that the language $A = \{yy \mid y \in \{0,1\}^*\}$ is Not Regular

Proof:

Assume that A is Regular

Then it must have a Pumping Length = P

$S = 0^P 1 0^P 1$

x y z

$P = 7$

$\underbrace{0000}_{x}\underbrace{000}_{y}\underbrace{0100000001}_{z}$

0101

$xy^iz \Rightarrow xy^2z$

00000000000100000001

$\notin A$

$|y| > 0$

$|xy| \leq P = 7$

Pumping Lemma (For Regular Languages) EXAMPLE (Part-2)

Using Pumping Lemma prove that the language $A = \{yy \mid y \in \{0,1\}^*\}$ is Not Regular

$\underbrace{01}\underbrace{01}$

Proof :

A

$p = 7$

Case 1: The y is in the 'a' part

$$\underbrace{aa}_{x}\underbrace{aaaa}_{y}\underbrace{abbbbbbb}_{z}$$

Case 2: The Y is in the 'b' part

$$\underbrace{aaaaaaabb}_{x}\underbrace{bbbb}_{y}\underbrace{b}_{z}$$

Case 3: The y is in the 'a' and 'b' part

$$\underbrace{aaaaa}_{x}\underbrace{aabb}_{y}\underbrace{bbbbb}_{z}$$

$xy^1z \Rightarrow xy^2z$ ✗

aa aaaaaaaa a bbbbbbb

$11 \neq 7$

$xy^1z \Rightarrow xy^2z$ ✗

aaaaaaa bb bbbb bbbb b

$7 \neq 11$

$xy^1z \Rightarrow xy^2z$ ✗

aaaaa aabbaabb bbbbb

$a^n b^n$

$|xy| \leq p$ $p=7$

(1) $x y^i z \in A$ for every $i \geq 0$

(2) $|y| > 0$

(3) $|xy| \leq P$

To prove that a language is not Regular using PUMPING LEMMA, follow the below steps:

(We prove using Contradiction)

-> Assume that A is Regular

-> It has to have a Pumping Length (say P)

-> All strings longer than P can be pumped $|S| \geq P$

-> Now find a string 'S' in A such that $|S| \geq P$

-> Divide S into x y z

-> Show that $x y^i z \notin A$ for some i

-> Then consider all ways that S can be divided into x y z

-> Show that none of these can satisfy all the 3 pumping conditions at the same time

-> S cannot be Pumped == CONTRADICTION

Case 1: The Y is in the 'a' part

$\underbrace{aa}_{x}\,\underbrace{aaaa}_{y}\,\underbrace{a\,bbbbbbb}_{z}$

Case 2: The Y is in the 'b' part

$\underbrace{aaaaaaa\,bb}_{x}\,\underbrace{bbbb}_{y}\,\underbrace{b}_{z}$

Case 3: The Y is in the 'a' and 'b' part

$\underbrace{aaaaa}_{x}\,\underbrace{aabb}_{y}\,\underbrace{bbbbb}_{z}$

$xy^1z \Rightarrow xy^2z$

aa aaaa aaaa a bbbbbbb

$11 \neq 7$

$xy^1z \Rightarrow xy^2z$

aaaaaaa bb bbbb bbbb b

$7 \neq 11$

$xy^1z \Rightarrow xy^2z$

aaaaa aabbaabb bbbbb

$a^n b^n$

Pumping Lemma (For Regular Languages) | Example 1

Case 1: The y is in the 'a' part

$$\underbrace{aa}_{x}\underbrace{aaaa}_{y}\underbrace{abbbbbbb}_{z}$$

Case 2: The y is in the 'b' part

$$\underbrace{aaaaaaabb}_{x}\underbrace{bbbb}_{y}\underbrace{b}_{z}$$

Case 3: The y is in the 'a' and 'b' part

$$\underbrace{aaaaa}_{x}\underbrace{aabb}_{y}\underbrace{bbbbb}_{z}$$

7:52 / 14:15

Scroll for details

x y z

$P = 7$

Case 1: The y is in the 'a' part

$\underbrace{aa}_{x}\underbrace{aaaa}_{y}\underbrace{abbbbbbb}_{z}$

Case 2: The y is in the 'b' part

$\underbrace{aaaaaaab}_{x}\underbrace{bbbb}_{y}\underbrace{b}_{z}$

Case 3: The y is in the 'a' and 'b' part

aaaa

Pumping Lemma (For Regular Languages) | Example 1

Pumping Lemma (For Regular Languages) - EXAMPLE (Part-1)

Using Pumping Lemma prove that the language $A = \{a^n b^n \mid n \geqslant 0\}$ is Not Regular

Proof:

Assume that A is Regular

Pumping length = P

$S = a^P b^P \Rightarrow S = aaaaaaabbbbbbb$

x y z

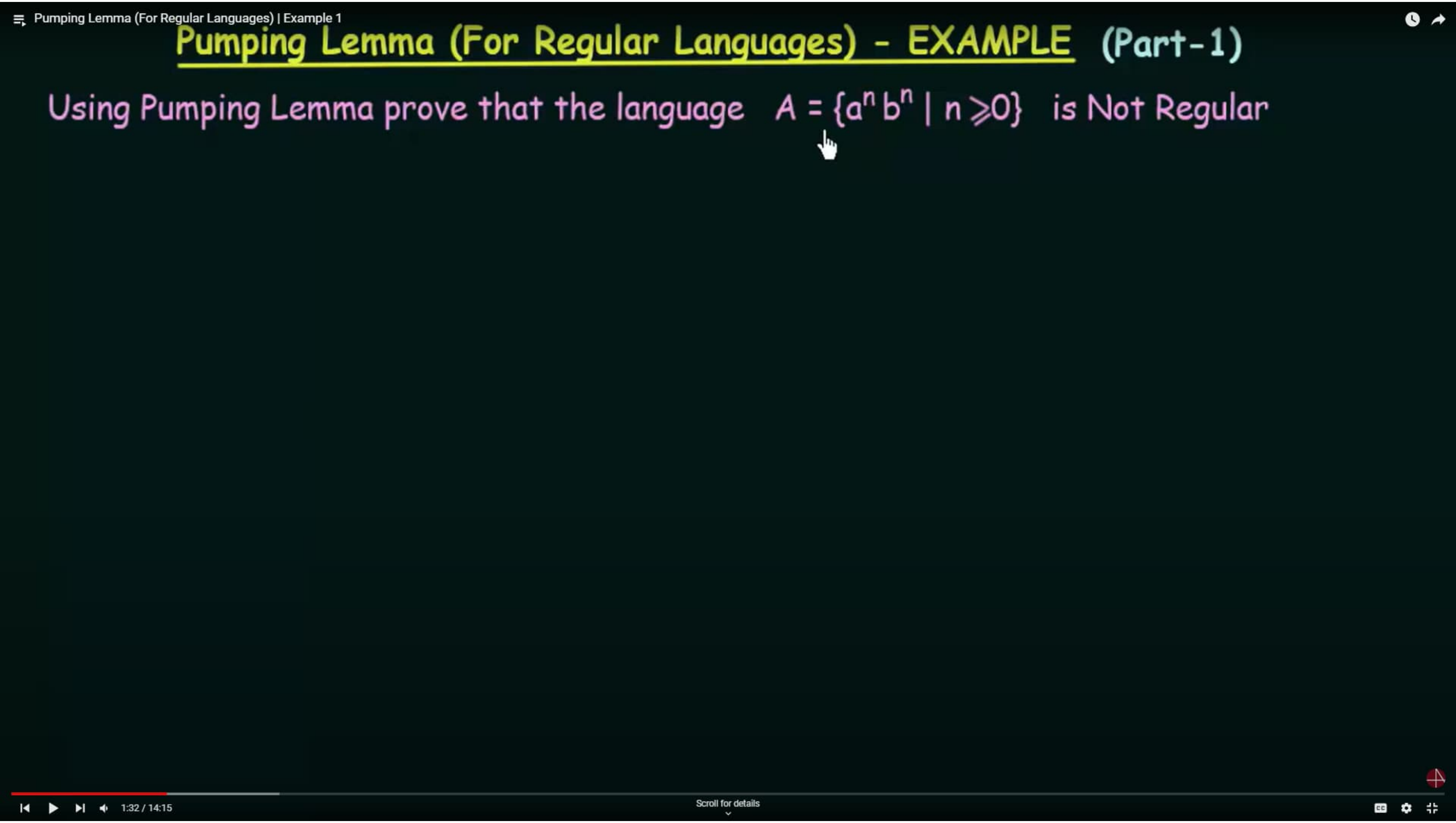
$P = 7$

Case 1: The Y is in the 'a' part

a a a a a a a b b b

5:04 / 14:15

Scroll for details


Pumping Lemma (For Regular Languages) | Example 1
Pumping Lemma (For Regular Languages) - EXAMPLE (Part-1)
Using Pumping Lemma prove that the language $A = \{a^n b^n \mid n \geq 0\}$ is Not Regular
1:32 / 14:15
Scroll for details

(1) $xy^iz \in A$ for every $i \geqslant 0$
(2) $|y| > 0$
(3) $|xy| \leq P$

To prove that a language is not Regular using PUMPING LEMMA, follow the below steps:

(We prove using Contradiction)

-> Assume that A is Regular
-> It has to have a Pumping Length (say P)
-> All strings longer than P can be pumped $|S| \geqslant P$
-> Now find a string 'S' in A such that $|S| \geqslant P$
-> Divide S into x y z
-> Show that $xy^iz \notin A$ for some i
-> Then consider all ways that S can be divided into x y z
-> Show that none of these can satisfy all the 3 pumping conditions at the same time
-> S cannot be Pumped == CONTRADICTION

Pumping Lemma (For Regular Languages)

>> Pumping Lemma is used to prove that a Language is NOT REGULAR

>> It cannot be used to prove that a Language is Regular

If A is a Regular Language, then A has a Pumping Length 'P' such that any string 'S' where $|S| \geqslant P$ may be divided into 3 parts $S = x\,y\,z$ such that the following conditions must be true:

(1) $x y^i z \in A$ for every $i \geqslant 0$

(2) $|y| > 0$

(3) $|xy| \leq P$

To prove that a language is not Regular using PUMPING LEMMA, follow the below steps:

(We prove using Contradiction)

-> Assume that A is Regular

-> It has to have a Pumping Length (say P)

Pumping Lemma (For Regular Languages)

>> Pumping Lemma is used to prove that a Language is NOT REGULAR

>> It cannot be used to prove that a Language is Regular

If A is a Regular Language, then A has a Pumping Length 'P' such that any string 'S' where $|S| \geq P$ may be divided into 3 parts $S = x\,y\,z$ such that the following conditions must be true:

(1) $x y^i z \in A$ for every $i \geq 0$

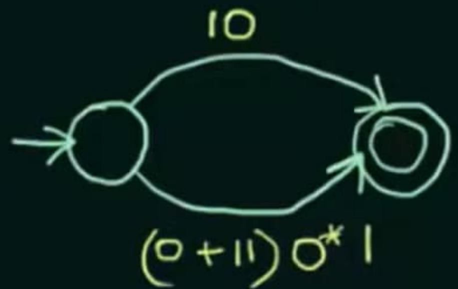
(2) $|y| > 0$

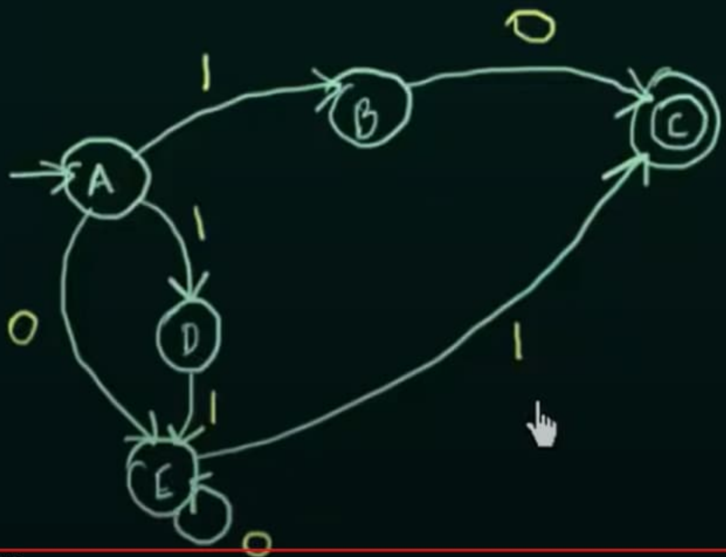
(3) $|xy| \leq P$

Conversion of Regular Expression to Finite Automata - Examples (Part-3)

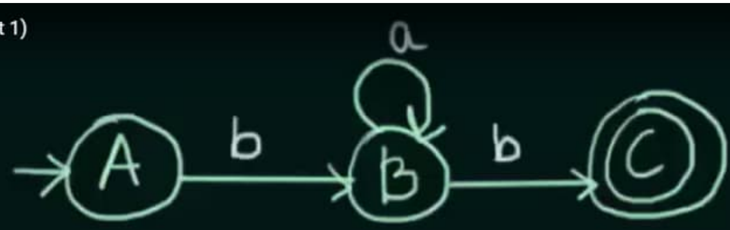
Convert the following Regular Expression to its equivalent Finite Automata:

$$10 + (0 + 11)\, 0^*1$$

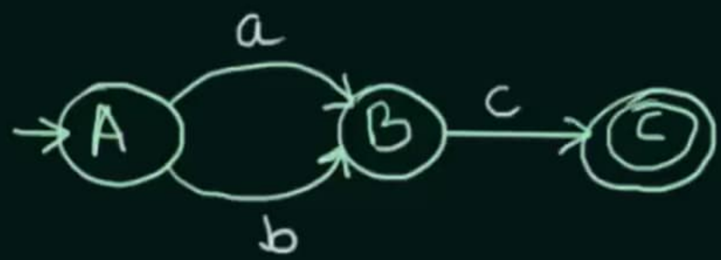




3) $a(bc)^*$



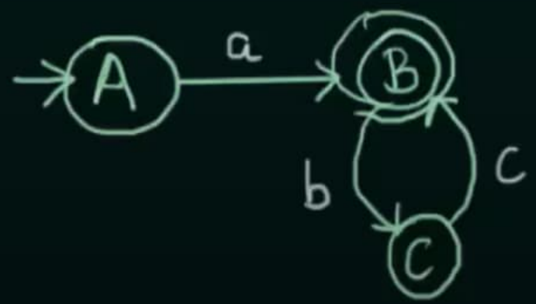
2) $(a+b)c$



ac ✓
bc ✓

3) $a(bc)^*$

a ✓ , abc ✓ , abcbc ✓ , abcbcbc ✓



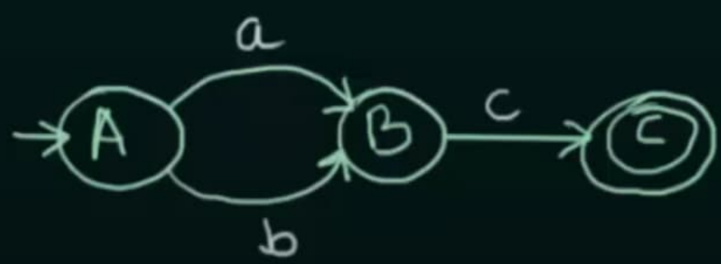
Conversion of Regular Expression to Finite Automata - Examples

Convert the following Regular Expressions to their equivalent Finite Automata:

1) $b\,a^*\,b$
2) $(a+b)\,c$
3) $a\,(bc)^*$

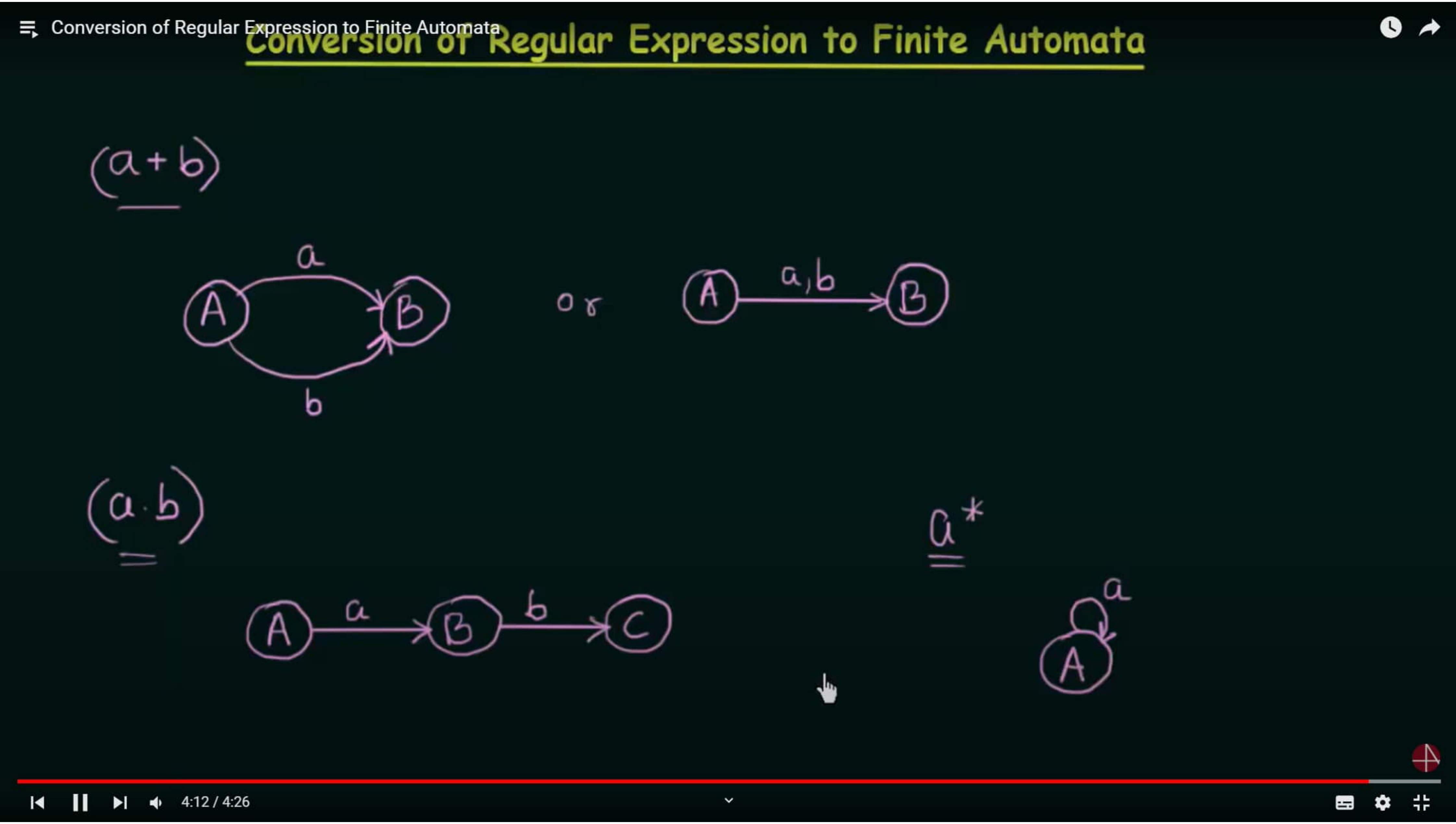
1) $b a^* b$ bb, bab, baab, ...

2) $(a+b)c$



ac ✓
bc ✓

3) $a(bc)^*$


Conversion of Regular Expression to Finite Automata
Conversion of Regular Expression to Finite Automata
(a+b)
a
A
B
b
or
A
a,b
B
(a.b)
A
a
B
b
C
a*
a
A
4:12 / 4:26

Final State (q2)

(11) $\Rightarrow$ $q_2 = q_1 1 + q_2 1$

$$\underbrace{q_2}_{R} = \underbrace{0^* 1}_{Q} + \underbrace{q_2}_{R} \underbrace{1}_{P}$$ Putting value of q_1 from (4)

$R = Q + RP$

$R = QP^*$

$q_2 = 0^* 1 (1)^*$

R = union of both Final states

$= 0^* + 0^* 1 1^*$

$= 0^* (\epsilon + 1 1^*)$

$\epsilon + RR^* = R^*$

$= 0^* 1^*$

$\Rightarrow$ Regular Expression

DFA to Regular Expression Conversion (when the DFA has Multiple Final States)

(I) $\Rightarrow$ $\underbrace{q_1}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_1}_{R}\underbrace{0}_{P}$

$R = Q + RP$

$R = QP^*$

Arden's theorem

$q_1 = \epsilon . 0^*$

$\epsilon . R = R$

$q_1 = 0^* \rightarrow$ (4)

Final state q_2

(II) $\Rightarrow$ $q_2 = q_1 1 + q_2 1$

$\underbrace{q_2}_{R} = \underbrace{0^* 1}_{Q} + \underbrace{q_2}_{R}\underbrace{1}_{P}$ Putting value of q_1 from (4)

$R = Q + RP$

$R = QP^*$

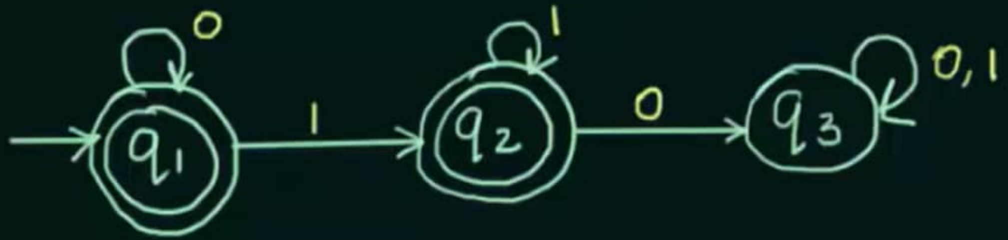
$q_2 = 0^* 1 (1)^*$

6:12 / 8:26

Designing Regular Expression - Examples (Part-4)

(When there are Multiple Final States)

Find the Regular Expression for the following DFA



$q_1 = \epsilon + q_1 0 \rightarrow$ (I)

$q_2 = q_1 1 + q_2 1 \rightarrow$ (II)

$q_3 = q_2 0 + q_3 0 + q_3 1 \rightarrow$ (III)

Final state (q_1)

(I) $\Rightarrow$ $\underbrace{q_1}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_1}_{R}\underbrace{0}_{P}$

$R = Q + RP$

$R = QP^*$ Arden's theorem

$q_1 = \epsilon . 0^*$

$\epsilon . R = R$

$q_1 = 0^* \rightarrow$ (4)

DFA to Regular Expression Conversion

$$q_4 = q_2 a + q_3 b + q_4 a + q_4 b \rightarrow \text{(IV)}$$

(I) $\Rightarrow$ $q_1 = \epsilon + q_2 b + q_3 a$

Putting values of q_2 and q_3 from (II) and (III)

$$q_1 = \epsilon + q_1 ab + q_1 ba$$

$$\underbrace{q_1}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_1}_{R}\underbrace{(ab+ba)}_{P}$$

$R = Q + RP$

$R = QP^*$ Arden's theorem

$$q_1 = \epsilon . (ab+ba)^*$$

$\epsilon . R = R$

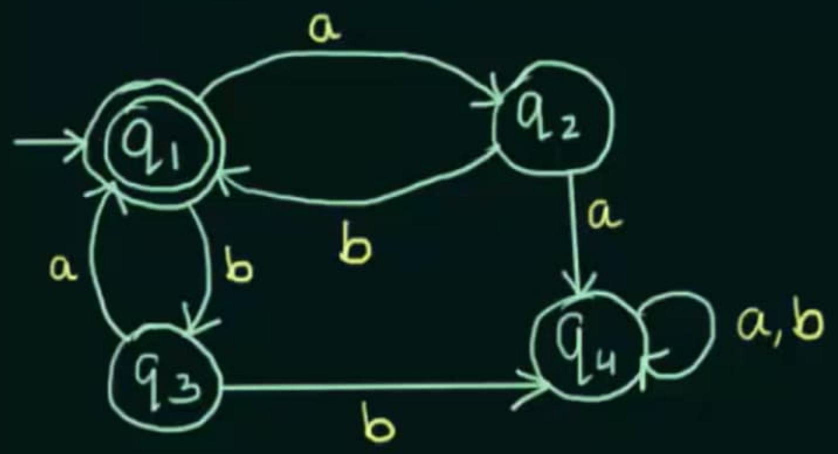
$$q_1 = (ab+ba)^*$$

$\hookrightarrow$ Regular Expression

6:01 / 6:18

Designing Regular Expression - Examples (Part-3)

Find the Regular Expression for the following DFA



$q_1 = \epsilon + q_2 b + q_3 a \rightarrow$ (I)

$q_2 = q_1 a \rightarrow$ (II)

$q_3 = q_1 b \rightarrow$ (III)

$q_4 = q_2 a + q_3 b + q_4 a + q_4 b \rightarrow$ (IV)

(I) $\Rightarrow$ $q_1 = \epsilon + q_2 b + q_3 a$

Putting values of q_2 and q_3 from (II) and (III)

$q_1 = \epsilon +$

NFA to Regular Expression Conversion

$$q_1 = \epsilon((a + a(b+ab)^*)b)^* \qquad \epsilon \cdot R = R$$

$$q_1 = (a + a(b+ab)^* b)^* \rightarrow (6)$$

Final state q_3

$$q_3 = q_2 a$$

$$= q_1 a (b+ab)^* a \quad \text{Putting value of } q_2 \text{ from } (5)$$

$$q_3 = (a + a(b+ab)^* b)^* \, a(b+ab)^* a \quad \text{Putting value of } q_1 \text{ from } (6)$$

= Required Regular Expression for the given NFA

12:31 / 13:36

NFA to Regular Expression Conversion

Putting value of q_2 from (5)

$$q_1 = \epsilon + q_1 a + ((q_1 a)(b+ab)^*)\, b$$

$$\underbrace{q_1}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_1}_{R} \underbrace{(a + a(b+ab)^*)b}_{P}$$

$$q_1 = \epsilon((a+a(b+ab)^*)b)^*$$

$$q_1 = (a+a(b+ab)^* b)^* \rightarrow (6)$$

$$R = Q + RP$$

$$R = QP^*$$

$$\epsilon \cdot R = R$$

Fin

10:44 / 13:36

$$\underbrace{q_2}_{R} = \underbrace{q_1 a}_{Q} + \underbrace{q_2}_{R}\underbrace{(b+ab)}_{P}$$

$$R = QP^*$$

$$q_2 = (q_1 a)(b+ab)^* \longrightarrow 5$$

3 $\rightarrow$ $q_1 = \epsilon + q_1 a + q_2 b$

Putting value of q_2 from 5

$$q_1 = \epsilon + q_1 a + ((q_1 a)(b+ab)^*)b$$

$$R = Q + RP$$

$$\underbrace{q_1}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_1}_{R}\underbrace{(a + a(b+ab)^*)b}_{P}$$

$$R = QP^*$$

$$q_1 = \epsilon((a+a(b+ab)^*)b)^*$$

NFA to Regular Expression Conversion

$$= (q_1 a + q_2 b + q_3 b)\, a$$

$$= q_1 aa + q_2 ba + q_3 ba \longrightarrow (4)$$

(2) $\Rightarrow$ $q_2 = q_1 a + q_2 b + q_3 b$ Putting value of q_3 from (1)

$$= q_1 a + q_2 b + (q_2 a)\, b$$

$$= q_1 a + q_2 b + q_2 ab$$

$$\underbrace{q_2}_{R} = \underbrace{q_1 a}_{Q} + \underbrace{q_2}_{R}\underbrace{(b + ab)}_{P}$$

$R = Q + RP$ Arden's Theorem

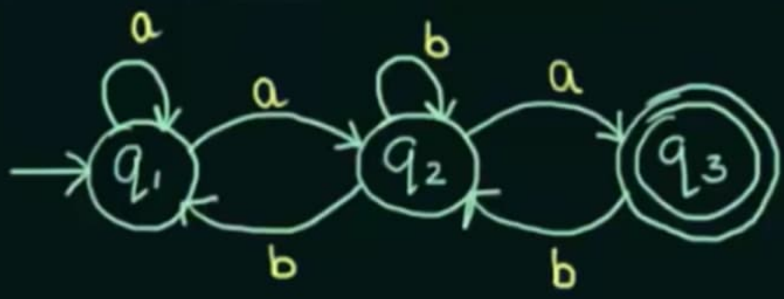
$R = QP^*$

$$q_2 = (q_1 a)(b + ab)^* \longrightarrow (5)$$

(3) $\Rightarrow$ $q_1 = \epsilon + q_1 a + q$

Designing Regular Expression - Examples (Part-2)

Find the Regular Expression for the following NFA



$$q_3 = q_2 a \rightarrow (1)$$

$$q_2 = q_1 a + q_2 b + q_3 b \rightarrow (2)$$

$$q_1 = \epsilon + q_1 a + q_2 b \rightarrow (3)$$

(1) $\Rightarrow$ $q_3 = q_2 a$

$$= (q_1 a + q_2 b + q_3 b) a$$

$$= q_1 aa + q_2 ba + q_3 ba \rightarrow (4)$$

(2) $\Rightarrow$ $q_2 = q_1 a + q_2 b + q_3 b$

Designing Regular Expressions

1) Language accepting strings of length exactly 2
2) Language accepting strings of length atleast 2
3) Language accepting strings of length atmost 2

Soln

1) $L_1 = \{aa, ab, ba, bb\}$

$R = aa + ab + ba + bb$

$= a(a+b) + b(a+b)$

$= (a+b)(a+b)$

2) $L_1 = \{aa, ab, ba, bb, aaa, \ldots\}$

$R = (a+b)(a+b)(a+b)^*$

3) $L_1 = \{\epsilon, a, b, aa, ab, ba, bb\}$

$R = \epsilon + a + b + aa + ab + ba + bb$

$= (\epsilon + a + b)(\epsilon + a + b)$

6:38 / 6:54

Designing Regular Expressions - Examples (Part-1)

Design Regular Expression for the following languages over {a,b}

1) Language accepting strings of length exactly 2
2) Language accepting strings of length atleast 2
3) Language accepting strings of length atmost 2

Soln

1) $L_1 = \{aa, ab, ba, bb\}$

$$R = aa + ab + ba + bb$$
$$= a(a+b) + b(a+b)$$
$$= (a+b)(a+b)$$

2) $L_1 = \{aa, ab, ba, bb, aaa, \ldots\}$

$$R = (a+b)(a+b)(a+b)^*$$

3) $L_1 = \{\epsilon, a, b, aa, ab, ba, bb\}$

$$R = \epsilon + a + b + aa + ab + ba + bb$$
$$= (\epsilon + a + b)(\epsilon + a + b$$

An Example Proof using Identities of Regular Expressions

Prove that (1+00*1) + (1+00*1) (0+10*1)* (0+10*1) is equal to 0*1 (0+10*1)*

$$LHS = (1+00^*1) + (1+00^*1)(0+10^*1)^*(0+10^*1)$$

$$= (1+00^*1)\left[\epsilon + (0+10^*1)^*(0+10^*1)\right]$$

$$\epsilon + R^*R = R^*$$

$$= (1+00^*1)(0+10^*1)^*$$

$$\epsilon . R = R$$

$$= (\epsilon . 1+00^*1)(0+10^*1)^*$$

$$= (\epsilon+00^*)1(0+10^*1)^*$$

$$= 0^*1(0+10^*1)^*$$

Arden's Theorem

$= Q + [Q + RP]P$

$= Q + QP + RP^2$

$= Q + QP + [Q + RP]P^2$

$= Q + QP + QP^2 + RP^3$

$\vdots$

$= Q + QP + QP^2 + \ldots\ldots QP^n + RP^{n+1}$

$$\boxed{R = QP^*}$$

$= Q + QP + QP^2 + \ldots\ldots QP^n + QP^* P^{n+1}$

$= Q[\epsilon + P + P^2 + \ldots\ldots P^n + P^* P^{n+1}]$

$R = \underline{\underline{QP^*}}$

Arden's Theorem

$$R = Q + RP$$

$$= Q + [Q + RP]P$$

$$= Q + QP + RP^2$$

$$= Q + QP + [Q + RP]P^2$$

$$= Q + QP + QP^2 + RP^3$$

$$\vdots$$

$$= Q + QP + QP^2 + \ldots\ldots QP^n + RP^{n+1}$$

5:30 / 7:34

ARDEN'S THEOREM

If P and Q are two Regular Expressions over Σ , and if P does not contain ϵ, then the following equation in R given by $R = Q + RP$ has a unique solution i.e. $R = QP^*$

$$R = Q + RP \longrightarrow (1)$$

$$= Q + QP^*P$$

$$= Q(\epsilon + P^*P)$$

$$= QP^*$$

Proved

$$R = QP^*$$

$$[\epsilon + R^*R = R^*]$$

$$R = Q + RP$$

$$= Q + Q + RP$$

Identities of Regular Expression

1) $\emptyset + R = R$
2) $\emptyset R + R\emptyset = \emptyset$
3) $\epsilon R = R\epsilon = R$
4) $\epsilon^* = \epsilon$ and $\emptyset^* = \epsilon$
5) $R + R = R$
6) $R^*R^* = R^*$
7) $RR^* = R^*R$
8) $(R^*)^* = R^*$
9) $\epsilon + RR^* = \epsilon + R^*R = R^*$
10) $(PQ)^*P = P(QP)^*$
11) $(P + Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$
12) $(P + Q)R = PR + QR$ and

    $R(P + Q) = RP + RQ$