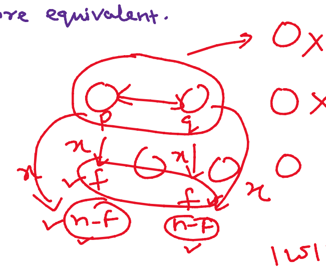
Minimization of DFA:

Sunday, April 18, 2021 12:01 PM

* It is a problem of minimization of states in a given DFA to construct the minimal DFA.
* Minimization of states can be done only with equivalent states or states which are equivalent.

Two states (P, q) are equivalent states

if $\delta(P, w) \in F$ OR if $\delta(P, w) \notin F \Rightarrow \delta(q, w) \notin F$
then $\delta(q, w) \in F$



* If length $|w| = 0$, then both states are called 0-equivalent
→ if $|w| = 1 \Rightarrow$ 1-equivalent
→ if $|w| = 2 \Rightarrow$ 2-equivalent
In general, if $|w| = n$, n-equivalent $\quad P \Leftrightarrow q$

$|w| = 0$

* Any two final states are 0-equivalent and any two non-final states are also 0-equivalent.
* Two states (P, q) are K-equivalent $(K \geq 0)$ if both $\delta(P, w)$ & $\delta(q, w)$ are final states or both $\delta(P, w) \notin F$ & $\delta(q, w) \notin F$, for all strings $|w| = K$.

P → f, q → f; P → nf, q → nf

n > K

Prop-1: Equivalence relations are reflexive, symmetric, and transitive

Prop-2: Partitioning of $Q <^{\Pi}_{\Pi_K}$ → K-equivalence classes

$Q = \{q_0, q_1, \ldots, q_K, \ldots q_n\}$

$q \Leftrightarrow P$

Prop-3: If P & q are K-equivalent, for all $K \geq 0$, then they are equivalent

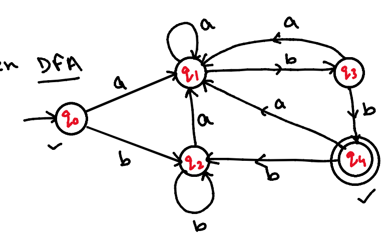
→ K - { q_0 q_{11} q_{13} }

Prop-4: If P & q are (K+1) equivalent, then they are K-equivalent

Prop-5: $\Pi_n = \Pi_{n+1}$ for some n. (Π_n denotes the set of equivalence classes under n-equivalence)

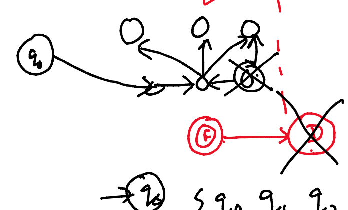
$\Pi_K = \Pi_{K+1}$, K+1, $= \Pi_{K+2}$

ex Given DFA



⇒

	a	b
→ q_0	q_1 nf	q_2 nf
q_1	q_1	q_3
q_2	q_1 nf	q_2 nf
q_3	q_1	q_4
(q_4)	q_1	q_2



$\{q_0\ q_1\ q_2\ q_3\}\{q_4\}$ F

* find the dead state that is unreachable from the final state, remove it.
* All the states should be reachable from the initial state and if particular state is unreachable from the initial state, remove it.
* Now find equivalent states & merge them.

$q_0 q_1$, $q_0 q_2$, $q_0 q_3$; ~~$q_1 q_0$~~, $q_1 q_2$, $q_1 q_3$; ~~$q_2 q_0$~~, ~~$q_2 q_1$~~, $q_2 q_3$; q_3 - - -

Π_0 0-equivalent sets ⇒ $[q_0\ q_1\ q_2\ q_3]$ $[q_4]$ * finding set of non-final states & final states

Π_1 1-equivalent sets ⇒ consider every pair of states in 0-equivalent set and check the equivalency based on final & non-final states

So $(q_0\ q_1)$ are equivalent, similarly $(q_0\ q_2)$ are equivalent as both are in same group of 0-equivalent class.

But $(q_0\ q_3)$ will not be in same group as q_3 goes to final state q_4 on accepting the input b.

⇒ $[q_0 q_1, q_2]$ $[q_3]$ $[q_4]$

$q_0 \neq q_3$ are not 1-equivalent

$q_0 q_1$, $q_0 q_2$; ~~$q_1 q_0$~~, $q_1 q_2$; ~~$q_2 q_0$~~, ~~$q_2 q_1$~~

$[q_0\ q_2]$ $[q_1]$ $\{q_3\}\{q_4\}$

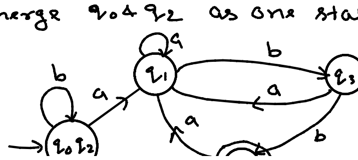
Π_2 2-equivalent sets ⇒ $[q_0, q_2]$ $[q_1]$ $[q_3]$ $[q_4]$

Π_3 3-equivalent " → $[q_0 q_2]$ $[q_1]$ $[q_3]$ $[q_4]$

Π_K K-equivalent " ⇒ $[q_0\ q_2]$ $[q_1]$ $[q_3]$ $[q_4]$

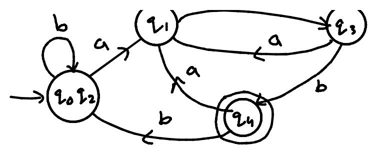
K → n

Now we can merge q_0 & q_2 as one state



⇒

	a	b
$q_0 q_2$	q_1	$q_0 q_2$

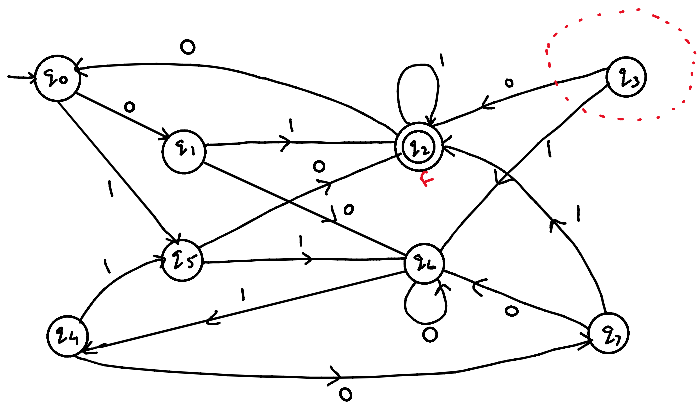


minimal DFA

$\Rightarrow$

	a	b
q_0q_2	q_1	q_0q_2
q_1	q_1	q_3
q_3	q_1	q_4
q_4	q_1	q_0q_2

ex



× remove

	0	1
$\rightarrow q_0$	q_1 nf	q_5 nf
q_1	q_6 nf	$^*q_2$ f
$^*q_2$	q_0	$^*q_2$
q_3	$^*q_2$	q_6
q_4	q_7 nf	q_5 nf
q_5	$^*q_2$ f	q_6
q_6	q_6 nf	q_4 f
q_7	q_6 nf	$^*q_2$ f

→ remove this entry because q_3 is unreachable from the initial state

$q_0\ q_5$

$[q_0\ q_4\ q_6]$ $[q_1 q_7]$ $[q_5]$

$q_0\ q_4$ ne

$q_4\ q_5$ ×

$q_6\ q_5$ ×

Now partitioning the states: $Q = \{q_0\ q_1\ q_2\ \cancel{q_3}\ q_4\ q_5\ q_6\ q_7\}$

$q_0\ q_1$
$q_0\ q_4$
$q_0\ q_5$

final → $Q_1^0 = \{q_2\} = F$

non final → $Q_2^0 = Q - Q_1^0$

$\Pi_0 = \{q_2\}, \{q_0\ q_1\ q_4\ q_5\ q_6\ q_7\}$

$\Pi_1 = \{q_2\}, \{q_0\ q_4\ q_6\}, \{q_1, q_7\}, \{q_5\}$ ✓

$\Pi_2 = \{q_2\}, \{q_0\ q_4\}, \{q_6\}, \{q_1 q_7\}, \{q_5\}$

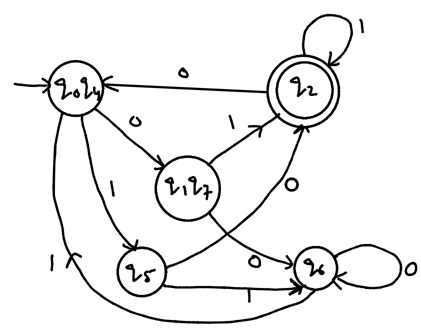
$\Pi_3 = \{q_2\}, \{q_0\ q_4\}, \{q_6\}, \{q_1 q_7\}, \{q_5\}$

$\vdots$

$\Pi_2 = \Pi_3 = \Pi_4 \cdots$ $\quad \Pi_K = \{q_2\}\ \{q_0\ q_4\}\ \{q_6\}\ \{q_1 q_7\}\ \{q_5\}$

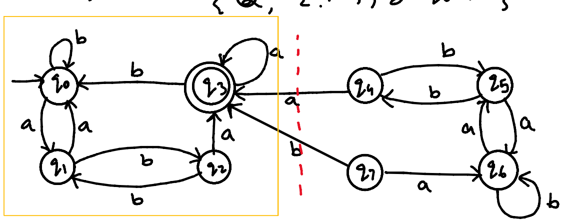
So Π_2 gives an equivalence class

& $M' = \{Q', \{0,1\}, \delta'\ q_0'\ F'\}$



Min DFA

ex



$\Pi_0 = \{q_0\ q_1\ q_2\}\ \{q_3\}$

$\Pi_1 = \{q_0\ q_1\}\ \{q_2\}\ \{q_3\}$

$\Pi_2 = \{q_0\}\ \{q_1\}\ \{q_2\}\ \{q_3\}$