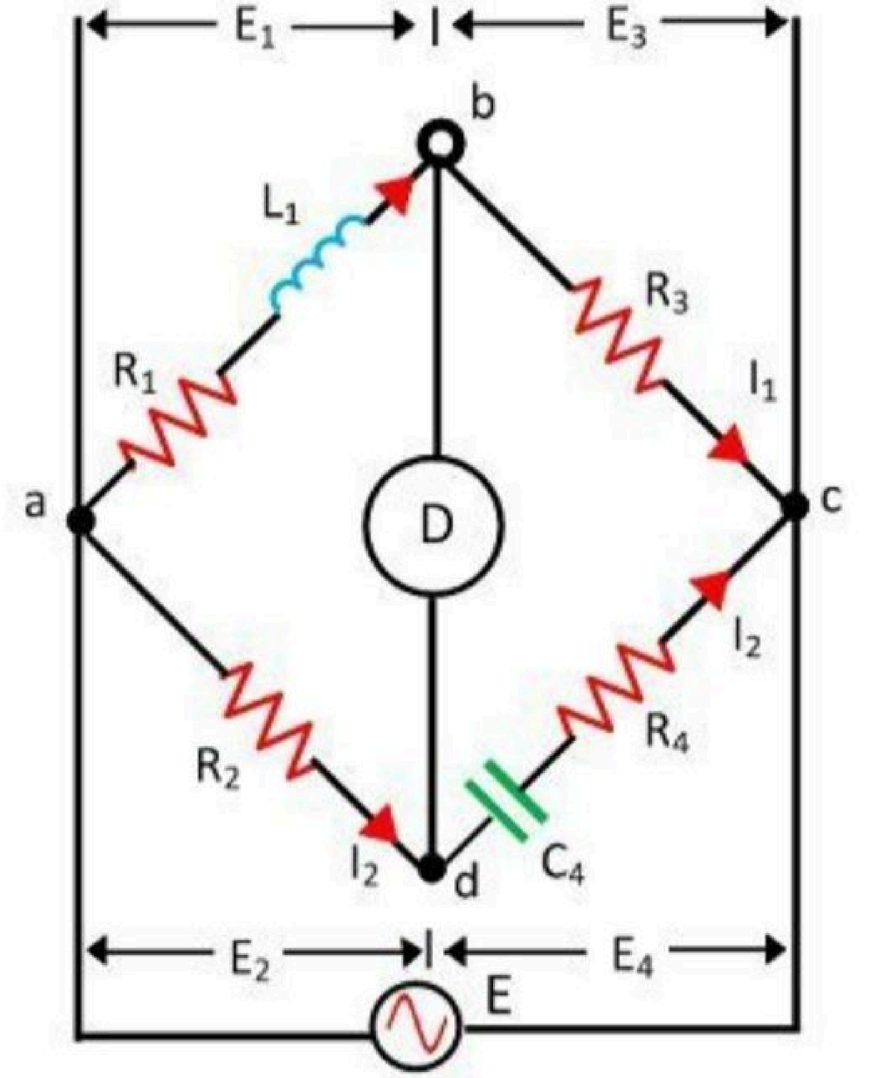
Hay’s Bridge

Definition: The **Hay’s bridge** is used for **determining** the **self-inductance** of the **circuit**. The bridge is the **advanced form** of **Maxwell’s bridge**. The Maxwell’s bridge is only appropriate for measuring the medium quality factor. Hence, for **measuring** the **high-quality factor** the **Hays bridge** is used in the circuit.

In **Hay's bridge**, the **capacitor** is **connected** in **series** with the **resistance**, the voltage drop across the capacitance and resistance are varied. And in Maxwell bridge, the **capacitance** is **connected** in **parallel** with the **resistance**. Thus, the magnitude of a voltage pass through the resistance and capacitor is equal.

Construction of Hay's Bridge

The unknown inductor L_1 is placed in the arm **ab** along with the resistance R_1. This unknown inductor is compared with the standard capacitor C_4 connected across the arm **cd**. The resistance R_4 is connected in series with the capacitor C_4. The other two non-inductive resistor R_2 and R_3 are connected in the arm **ad** and **bc** respectively.



Hay's Bridge

Circuit Globe

The C_4 and R_4 are adjusted for making the bridge in the balanced condition. When the bridge is in a balanced condition, no current flows through the detector which is connected to point **b** and **c** respectively. The potential drops across the arm **ad** and **cd** are equal and similarly, the potential across the arm **ab** and **bc** are equal.

Hay's Bridge Theory

Let,

L_1 – unknown inductance having a resistance R_1

R_2, R_3, R_4 – known non-inductive resistance.

C_4 – standard capacitor

At balance condition,

$$(R_1 + j\omega L_1)(R_4 - j/\omega C_4) = R_2 R_3$$

$$R_1 R_4 + \frac{L_1}{C_4} + j\omega L_1 R_4 - \frac{jR_1}{\omega C_4} = R_2 R_3$$

Separating the real and imaginary term, we obtain

$$R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3 \quad and\ L_1 = \frac{-\ R_1}{\omega^2 R_4 C_4}$$

Solving the above equation, we have

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2}$$

$$R_1 = \frac{\omega^2 C_4^2 R_2 R_3 R_4}{1 + \omega^2 R_4^2 C_4^2}$$

The quality factor of the coil is

$$Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega^2 C_4 R_4}$$

The equation of the unknown inductance and capacitance consists frequency term. Thus for finding the value of unknown inductance the frequency of the supply must be known.

For the high-quality factor, the frequency does not play an important role.

$$Q = \frac{1}{\omega^2 C_4 R_4}$$

Substituting the value of Q in the equation of unknown inductance, we get

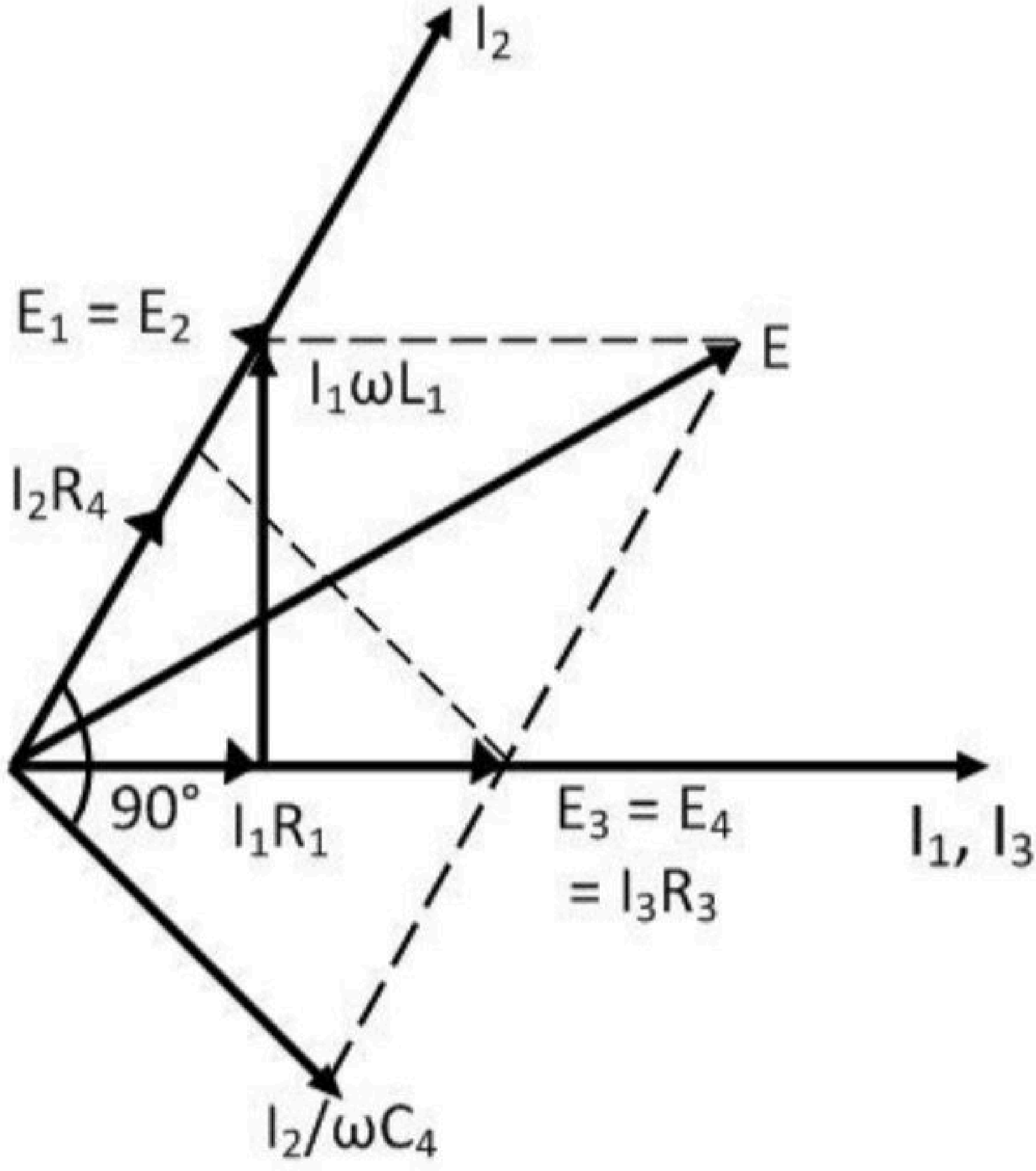
$$L_1 = \frac{R_2 R_3 C_4}{1 + (1/Q)^2}$$

For greater value of Q the 1/Q is neglected and hence the equation become

$$L_1 = R_2 R_3 C_4$$

Phasor Diagram of Hay's Bridge

The phasor diagram of the Hay's bridge is shown in the figure below. The magnitude and the phase of the **E_3** and **E_4** are equal and hence they are overlapping each other and draw on the horizontal axis. The current **I_1** flow through the purely resistive arm **bd.** The current **I_1** and the potential $E_3 = I_3 R_3$ are in the same phase and represented on the horizontal axis.



Phasor Diagram of Hay's Bridge

The current passes through the arm **ab** produces a potential drop $\mathbf{I_1R_1}$ which is also in the same phase of $\mathbf{I_1}$. The total voltage drop across the arm **ab** is determined by adding the voltage $\mathbf{I_1R_1}$ and $\boldsymbol{\omega I_1L_1}$.

The voltage drops across the arm **ab** and **ad** are equal. The voltage drop E_1 and E_2 are equal in magnitude and phase and hence overlap each other. The current I_2 and E_2 are in the same phase as shown in the figure above.

The current I_2 flows through the arms **cd** and produces the I_2R_4 voltage drops across the resistance and $I_2/\omega C_4$ voltage drops across the capacitor C_4. The capacitance C_4 lags by the currents 90°.

The voltage drops across the resistance C_4 and R_4 gives the total voltage drops across the arm **cd**. The sum of the voltage E_1 and E_3 or E_2 and E_4 gives the voltage drops **E**.

Advantages of Hay’s Bridge

The following are the advantages of Hay’s Bridge.

1. The Hays bridges give a simple expression for the unknown inductances and are suitable for the coil having the quality factor greater than the 10 ohms.
2. It gives a simple equation for quality factor.
3. The Hay’s bridge uses small value resistance for determining the Q factor.

Disadvantages of Hay's Bridge

The only disadvantage of this type of bridge is that it is not suitable for the measurement of the coil having the quality factor less than 10 ohms.

Note: The quality factor is a parameter which determines the relation between the stored energy and the energy dissipated in the circuit.