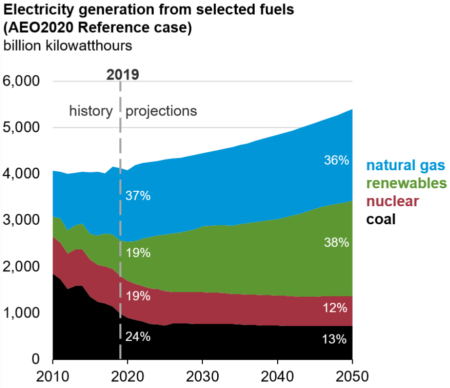
Bilevel and Multi-objective Optimization of Electricity Price Setting with Carbon Emission Consideration

Vikas Garg
Committee Chair: Dr Yongjia Song
Committee Members: Dr Scott J. Mason & Dr Michael Carbajales-Dale

Industrial Engineering, Clemson University

Why This Study?[1]

- Electricity ↑ from 4,000 to 5,500 billion kWh.
- Coal Contribution ↓ 24% to 13% & Gas ↓ from 37% to 36%.
- Coal Produces 0.74 lbs CO_2/kWh & Gas 0.40.
- Estimated Emissions in 2050 from Coal 530 billion lbs & from Gas 790 billion lbs.
- Total Emissions 1,320 billion lbs.



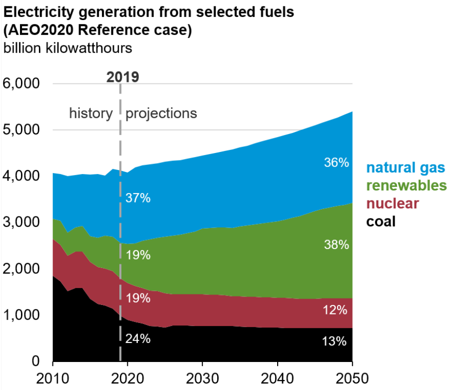
What if Coal Replaced by Gas:

- Total Emission Can be ↓ to 1,078, 18.33% reduction.
- How Performance of Electricity Sector is Affected?

[1] U.S. Energy Information Administration-AEO2020

Why This Study?[1]

- Electricity ↑ from 4,000 to 5,500 billion kWh.
- Coal Contribution ↓ 24% to 13% & Gas ↓ from 37% to 36%.
- Coal Produces 0.74 lbs CO_2/kWh & Gas 0.40.
- Estimated Emissions in 2050 from Coal 530 billion lbs & from Gas 790 billion lbs.
- Total Emissions 1,320 billion lbs.



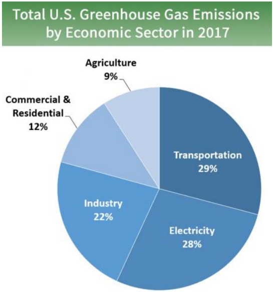
What if Coal Replaced by Gas:

- Total Emission Can be ↓ to 1,078, 18.33% reduction.
- How Performance of Electricity Sector is Affected?

[1] U.S. Energy Information Administration-AEO2020

Emissions in Electricity Sector

- Transportation (29%) and Electricity Production (28%) Major Source of Emissions[2].

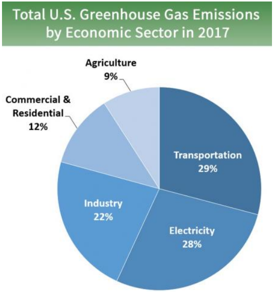


- Fossil Fuel Emission Contributed by Power Generation Sector are 42.50% (Li et al.,2018).
- Manufacturing Sector alone Contributes 38% Emissions due to Electricity Consumption (May et al.,2015).

[2]https://www.epa.gov/ghgemissions/sources-greenhouse-gas-emissions

Emissions in Electricity Sector

- Transportation (29%) and Electricity Production (28%) Major Source of Emissions[2].

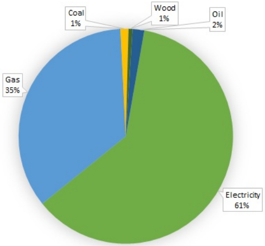


- Fossil Fuel Emission Contributed by Power Generation Sector are 42.50% (Li et al.,2018).
- Manufacturing Sector alone Contributes 38% Emissions due to Electricity Consumption (May et al.,2015).

[2] https://www.epa.gov/ghgemissions/sources-greenhouse-gas-emissions

Electricity Consumption & Prices

- Industrial Customers Consume 61% Energy from Electricity[3].
- Electricity Price Volatile, Increased by Approx. 20% since 2000[4].

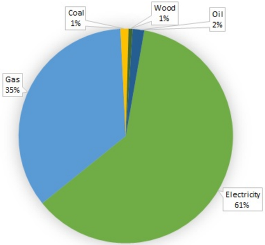


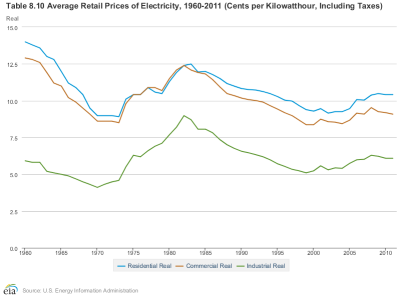
[3] https://iac.university/download

[4] https://www.eia.gov/totalenergy/data/annual/showtext.php?t=ptb0810

Electricity Consumption & Prices

- Industrial Customers Consume 61% Energy from Electricity[3].
- Electricity Price Volatile, Increased by Approx. 20% since 2000[4].





[3]https://iac.university/download
[4]https://www.eia.gov/totalenergy/data/annual/showtext.php?t=ptb0810

Demand Response Management

Figure: DR Tools

Demand Response Programs

- Incentive Based Programs (IBP)
  - Classical
    - Direct Control
    - Interruptible/Curtailable Programs
  - Market Based
    - Demand Bidding
    - Emergency DR
    - Capacity Market
    - Ancillary services market
- Price Based Programs (PBP)
  - Time of Use (TOU)
  - Critical Peak Pricing (CPP)
  - Extreme Day CPP (ED-CPP)
  - Extreme Day Pricing (EDP)
  - Real Time Pricing (RTP)

Demand Response Management

Figure: DR Tools

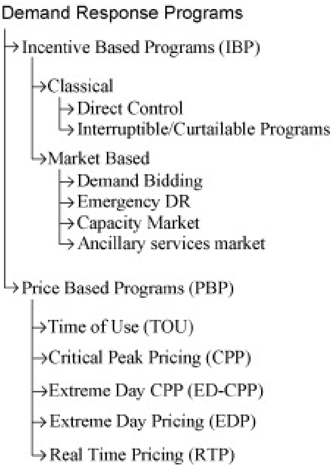


Figure: TOU vs TLOU

Research Questions?

1. Should the Retailer Consider Emissions in Electricity Price Setting?
2. How Weighted Profits are affected?
3. Emission Reduced with the Choice of Fuels & Price Change?
4. Emissions Impact Customers' Consumption Cost & Demand Shifting?

Research Questions?

1. Should the Retailer Consider Emissions in Electricity Price Setting?
2. How Weighted Profits are affected?
3. Emission Reduced with the Choice of Fuels & Price Change?
4. Emissions Impact Customers' Consumption Cost & Demand Shifting?

Research Questions?

1. Should the Retailer Consider Emissions in Electricity Price Setting?
2. How Weighted Profits are affected?
3. Emission Reduced with the Choice of Fuels & Price Change?
4. Emissions Impact Customers' Consumption Cost & Demand Shifting?

Research Questions?

1. Should the Retailer Consider Emissions in Electricity Price Setting?
2. How Weighted Profits are affected?
3. Emission Reduced with the Choice of Fuels & Price Change?
4. Emissions Impact Customers' Consumption Cost & Demand Shifting?

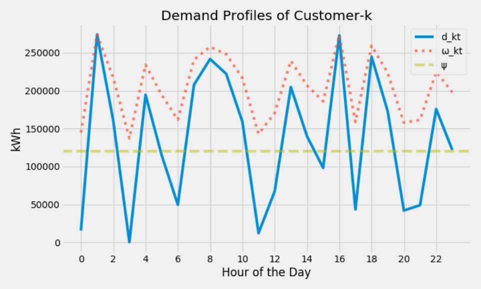


Figure: Demand Profiles & Shifting

Structure of the Problem

Leader's Problem:

maximize:- Profits of Retailer

minimize:- *Carbon emissions*

Subject to:

1. Electricity Demand Constraints
2. Number of price change allowed constraint
3. TLOU price setting constraints

Follower's Problem

minimize Electricicty Consumption Cost and Inconvenience Cost

Subject to:

1. Energy requirement constraint
2. Constraint for energy bought from manufacturer and competitor
3. TLOU electricity consumption capacity constraint

Sets & Parameters

- Sets

K	Set of different customers, indexed on k
T	Set of time periods (hr) of a day, indexed on t
S	Set of different types of fuel sources available, indexed on s

- Parameters

c_s	Unit cost of fuel s used
$\mathfrak{N}$	Maximum number of price changes allowed
ξ_{kt}	Inconvenience cost of the customers for shifting current demand
D_k	Daily current demand (kWh/day) of electricity from the customer k
d_{kt}	Current demand profile (kWh/hour) of customer k during time t
ω_{kt}	Maximum demand shifting capacity of customer k during time t
α_s	Unit electricity (kWh/unit fuel) generated per fuel source s
β_s	Unit CO_2 emissions (lbs/fuel source) produced by fuel source s
$\mathfrak{m}$	Maximum price difference allowed between two successive prices
γ_t	Unit prices offered by the competitor.

Sets & Parameters

- Sets

K	Set of different customers, indexed on k
T	Set of time periods (hr) of a day, indexed on t
S	Set of different types of fuel sources available, indexed on s

- Parameters

c_s	Unit cost of fuel s used
$\mathfrak{N}$	Maximum number of price changes allowed
ξ_{kt}	Inconvenience cost of the customers for shifting current demand
D_k	Daily current demand (kWh/day) of electricity from the customer k
d_{kt}	Current demand profile (kWh/hour) of customer k during time t
ω_{kt}	Maximum demand shifting capacity of customer k during time t
α_s	Unit electricity (kWh/unit fuel) generated per fuel source s
β_s	Unit CO_2 emissions (lbs/fuel source) produced by fuel source s
$\mathfrak{m}$	Maximum price difference allowed between two successive prices
γ_t	Unit prices offered by the competitor.

Decision Variables

- Leader's Problem Decision Variable

Π_t^ℓ	Lower unit price (¢/kWh) set in time t
Π_t^h	Higher unit price (¢/kWh) set in time t
X_t	Amount of electricity (kWh/hour) generated in time t
Y_{st}	Amount of fuel s used in time t
P_t	Binary variable, 1 if there is a price change between periods t and $t+1$ and 0 otherwise
Ψ	Limit set up by the retailer upto which lower prices will be applicable

- Follower's Problem Decision Variable

W_{kt}^ℓ	Energy consumption at lower price in time t (kWh) by customer k
W_{kt}^h	Energy consumption at higher price in time t (kWh) by customer k
V_{kt}^+	Demand shifted (kWh/hour) upwards from the current demand profile by customer k in time t
V_{kt}^-	Demand shifted (kWh/hour) downwards from the current demand profile by customer k in time t
U_{kt}	Electricity (kWh/hour) bought from retailer's competitor in time t by customer k

Decision Variables

- Leader's Problem Decision Variable

Π_t^ℓ	Lower unit price (¢/kWh) set in time t
Π_t^h	Higher unit price (¢/kWh) set in time t
X_t	Amount of electricity (kWh/hour) generated in time t
Y_{st}	Amount of fuel s used in time t
P_t	Binary variable, 1 if there is a price change between periods t and $t+1$ and 0 otherwise
Ψ	Limit set up by the retailer upto which lower prices will be applicable

- Follower's Problem Decision Variable

W_{kt}^ℓ	Energy consumption at lower price in time t (kWh) by customer k
W_{kt}^h	Energy consumption at higher price in time t (kWh) by customer k
V_{kt}^+	Demand shifted (kWh/hour) upwards from the current demand profile by customer k in time t
V_{kt}^-	Demand shifted (kWh/hour) downwards from the current demand profile by customer k in time t
U_{kt}	Electricity (kWh/hour) bought from retailer's competitor in time t by customer k

Leader's Problem

- Profits:

$$\text{Max } \sum_{k\in K}\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{*,\ell}(\Pi,\Psi)+\Pi_t^{h}\times W_{kt}^{*,h}(\Pi,\Psi)\right]$$
$$-\sum_{s\in S}\sum_{t\in T}c_s\times Y_{st}$$

- Emissions: $\text{Min } \sum_{t\in T}\sum_{s\in S}\beta_s\times Y_{st}$
- Subject To:
  Demand Satisfaction

$$\sum_{k\in K}\left[W_{kt}^{*,\ell}(\Pi,\Psi)+W_{kt}^{*,h}(\Pi,\Psi)\right]=X_t \qquad \forall t\in T$$

- Electricity Generation

$$\sum_{s\in S}\alpha_s\times Y_{st}=X_t \qquad \forall t\in T$$

- TLOU Price Setting

$$\left|\Pi_t^{\ell}-\Pi_{t+1}^{\ell}\right|\leq\mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\left|\Pi_t^{h}-\Pi_{t+1}^{h}\right|\leq\mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\Pi_t^{\ell}\leq\Pi_t^{h} \qquad \forall t\in T$$

- Maximum Price Changes

$$\sum_{t\in T}P_t\leq\mathfrak{M}$$

- Non-Negative and Binary Variable

$$\Pi_t^{\ell},\Pi_t^{h},X_t,Y_{st}\geq 0 \qquad \forall t,s$$

$$P_t\in\{0,1\} \qquad \forall t$$

Leader's Problem

- **Profits:**

$$\text{Max } \sum_{k\in K}\sum_{t\in T}\left[\Pi^\ell_t \times W^{*,\ell}_{kt}(\Pi,\Psi) + \Pi^h_t \times W^{*,h}_{kt}(\Pi,\Psi)\right] - \sum_{s\in S}\sum_{t\in T} c_s \times Y_{st}$$

- **Emissions:** $\text{Min } \sum_{t\in T}\sum_{s\in S} \beta_s \times Y_{st}$

- Subject To:
  Demand Satisfaction

$$\sum_{k\in K}\left[W^{*,\ell}_{kt}(\Pi,\Psi) + W^{*,h}_{kt}(\Pi,\Psi)\right] = X_t \qquad \forall t \in T$$

- Electricity Generation

$$\sum_{s\in S} \alpha_s \times Y_{st} = X_t \qquad \forall t \in T$$

- TLOU Price Setting

$$|\Pi^\ell_t - \Pi^\ell_{t+1}| \le \mathrm{m} \times P_t \qquad \forall t \in T \ni t < |T|$$

$$|\Pi^h_t - \Pi^h_{t+1}| \le \mathrm{m} \times P_t \qquad \forall t \in T \ni t < |T|$$

$$\Pi^\ell_t \le \Pi^h_t \qquad \forall t \in T$$

- Maximum Price Changes

$$\sum_{t\in T} P_t \le \mathfrak{N}$$

- Non-Negative and Binary Variable

$$\Pi^\ell_t, \Pi^h_t, X_t, Y_{st} \ge 0 \qquad \forall t, s$$

$$P_t \in \{0,1\} \qquad \forall t$$

Leader's Problem

- Profits:

$$\text{Max } \sum_{k\in K}\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{*,\ell}(\Pi,\Psi)+\Pi_t^{h}\times W_{kt}^{*,h}(\Pi,\Psi)\right]$$
$$-\sum_{s\in S}\sum_{t\in T} c_s\times Y_{st}$$

- Emissions: $\text{Min } \sum_{t\in T}\sum_{s\in S}\beta_s\times Y_{st}$
- Subject To:
  Demand Satisfaction

$$\sum_{k\in K}\left[W_{kt}^{*,\ell}(\Pi,\Psi)+W_{kt}^{*,h}(\Pi,\Psi)\right]=X_t \qquad \forall t\in T$$

- Electricity Generation

$$\sum_{s\in S}\alpha_s\times Y_{st}=X_t \qquad \forall t\in T$$

- TLOU Price Setting

$$\left|\Pi_t^{\ell}-\Pi_{t+1}^{\ell}\right|\leq \mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\left|\Pi_t^{h}-\Pi_{t+1}^{h}\right|\leq \mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\Pi_t^{\ell}\leq\Pi_t^{h} \qquad \forall t\in T$$

- Maximum Price Changes

$$\sum_{t\in T}P_t\leq\mathfrak{N}$$

- Non-Negative and Binary Variable

$$\Pi_t^{\ell},\Pi_t^{h},X_t,Y_{st}\geq 0 \qquad \forall t,s$$

$$P_t\in\{0,1\} \qquad \forall t$$

Leader's Problem

- **Profits:**

$$\text{Max } \sum_{k\in K}\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{*,\ell}(\Pi,\Psi)+\Pi_t^{h}\times W_{kt}^{*,h}(\Pi,\Psi)\right]$$
$$-\sum_{s\in S}\sum_{t\in T} c_s\times Y_{st}$$

- **Emissions:** $\text{Min } \sum_{t\in T}\sum_{s\in S}\beta_s\times Y_{st}$
- **Subject To:**
  Demand Satisfaction

$$\sum_{k\in K}\left[W_{kt}^{*,\ell}(\Pi,\Psi)+W_{kt}^{*,h}(\Pi,\Psi)\right]=X_t \qquad \forall t\in T$$

- **Electricity Generation**

$$\sum_{s\in S}\alpha_s\times Y_{st}=X_t \qquad \forall t\in T$$

- TLOU Price Setting

$$\left|\Pi_t^{\ell}-\Pi_{t+1}^{\ell}\right|\leq \mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\left|\Pi_t^{h}-\Pi_{t+1}^{h}\right|\leq \mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\Pi_t^{\ell}\leq \Pi_t^{h} \qquad \forall t\in T$$

- Maximum Price Changes

$$\sum_{t\in T}P_t\leq \mathfrak{N}$$

- Non-Negative and Binary Variable

$$\Pi_t^{\ell},\Pi_t^{h},X_t,Y_{st}\geq 0 \qquad \forall t,s$$

$$P_t\in\{0,1\} \qquad \forall t$$

Leader's Problem

- **Profits:**

$$\text{Max } \sum_{k\in K}\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{*,\ell}(\Pi,\Psi)+\Pi_t^{h}\times W_{kt}^{*,h}(\Pi,\Psi)\right]$$
$$-\sum_{s\in S}\sum_{t\in T} c_s\times Y_{st}$$

- **Emissions:** $\text{Min } \sum_{t\in T}\sum_{s\in S}\beta_s\times Y_{st}$
- **Subject To:**
  Demand Satisfaction

$$\sum_{k\in K}\left[W_{kt}^{*,\ell}(\Pi,\Psi)+W_{kt}^{*,h}(\Pi,\Psi)\right]=X_t \qquad \forall t\in T$$

- Electricity Generation

$$\sum_{s\in S}\alpha_s\times Y_{st}=X_t \qquad \forall t\in T$$

- **TLOU Price Setting**

$$\left|\Pi_t^{\ell}-\Pi_{t+1}^{\ell}\right|\leq \mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\left|\Pi_t^{h}-\Pi_{t+1}^{h}\right|\leq \mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\Pi_t^{\ell}\leq \Pi_t^{h} \qquad \forall t\in T$$

- Maximum Price Changes

$$\sum_{t\in T}P_t\leq \mathfrak{M}$$

- Non-Negative and Binary Variable

$$\Pi_t^{\ell},\Pi_t^{h},X_t,Y_{st}\geq 0 \qquad \forall t,s$$

$$P_t\in\{0,1\} \qquad \forall t$$

Leader's Problem

- Profits:

$$\text{Max} \ \sum_{k\in K}\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{*,\ell}(\Pi,\Psi)+\Pi_t^{h}\times W_{kt}^{*,h}(\Pi,\Psi)\right]$$
$$-\sum_{s\in S}\sum_{t\in T} c_s\times Y_{st}$$

- Emissions: $\text{Min} \ \sum_{t\in T}\sum_{s\in S}\beta_s\times Y_{st}$
- Subject To:
  Demand Satisfaction

$$\sum_{k\in K}\left[W_{kt}^{*,\ell}(\Pi,\Psi)+W_{kt}^{*,h}(\Pi,\Psi)\right]=X_t \qquad \forall t\in T$$

- Electricity Generation

$$\sum_{s\in S}\alpha_s\times Y_{st}=X_t \qquad \forall t\in T$$

- TLOU Price Setting

$$\left|\Pi_t^{\ell}-\Pi_{t+1}^{\ell}\right|\leq \mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\left|\Pi_t^{h}-\Pi_{t+1}^{h}\right|\leq \mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\Pi_t^{\ell}\leq\Pi_t^{h} \qquad \forall t\in T$$

- Maximum Price Changes

$$\sum_{t\in T}P_t\leq\mathfrak{N}$$

- Non-Negative and Binary Variable

$$\Pi_t^{\ell},\Pi_t^{h},X_t,Y_{st}\geq 0 \qquad \forall t,s$$

$$P_t\in\{0,1\} \qquad \forall t$$

Leader's Problem

- Profits:

$$\text{Max } \sum_{k\in K}\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{*,\ell}(\Pi,\Psi)+\Pi_t^{h}\times W_{kt}^{*,h}(\Pi,\Psi)\right]$$
$$-\sum_{s\in S}\sum_{t\in T}c_s\times Y_{st}$$

- Emissions: $\text{Min } \sum_{t\in T}\sum_{s\in S}\beta_s\times Y_{st}$
- Subject To:
  Demand Satisfaction

$$\sum_{k\in K}\left[W_{kt}^{*,\ell}(\Pi,\Psi)+W_{kt}^{*,h}(\Pi,\Psi)\right]=X_t \qquad \forall t\in T$$

- Electricity Generation

$$\sum_{s\in S}\alpha_s\times Y_{st}=X_t \qquad \forall t\in T$$

- TLOU Price Setting

$$\left|\Pi_t^{\ell}-\Pi_{t+1}^{\ell}\right|\le\mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\left|\Pi_t^{h}-\Pi_{t+1}^{h}\right|\le\mathfrak{m}\times P_t \qquad \forall t\in T\ni t<|T|$$

$$\Pi_t^{\ell}\le\Pi_t^{h} \qquad \forall t\in T$$

- Maximum Price Changes

$$\sum_{t\in T}P_t\le\mathfrak{N}$$

- Non-Negative and Binary Variable

$$\Pi_t^{\ell},\Pi_t^{h},X_t,Y_{st}\ge 0 \qquad \forall t,s$$

$$P_t\in\{0,1\} \qquad \forall t$$

Follower's Problem

- Objective Function:

$$\text{Min} \quad f(\Pi^*, \Psi^*) =$$

$$\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{\ell}+\Pi_t^{h}\times W_{kt}^{h}+\gamma_t\times U_{kt}\right]+\sum_{t\in T}\xi_{kt}\times V_{kt}^{+}$$

- Subject To:
  Hourly Demand

$$W_{kt}^{\ell}+W_{kt}^{h}+V_{kt}^{-}+U_{kt}=V_{kt}^{+}+d_{kt} \qquad \forall t\in T$$

- Daily Demand

$$\sum_{t\in T}\left[W_{kt}^{\ell}+W_{kt}^{h}+U_{kt}\right]=D_k$$

- Maximum Shifting Capability of Customer k:

$$V_{kt}^{+}\le\omega_{kt} \qquad \forall t\in T$$

- Maximum Consumption at Lower Price

$$W_{kt}^{\ell}\le\Psi \qquad \forall t\in T$$

- Non-Negativity Variables:

$$W_{kt}^{\ell}, W_{kt}^{h}, V_{kt}^{+}, V_{kt}^{-}, U_{kt}\ge 0 \qquad \forall t$$

Follower's Problem

- **Objective Function:**

$$\text{Min} \quad f(\Pi^*, \Psi^*) =$$

$$\sum_{t\in T}\left[\Pi_t^\ell \times W_{kt}^\ell + \Pi_t^h \times W_{kt}^h + \gamma_t \times U_{kt}\right] + \sum_{t\in T}\xi_{kt} \times V_{kt}^+$$

- **Subject To:**
  Hourly Demand

$$W_{kt}^\ell + W_{kt}^h + V_{kt}^- + U_{kt} = V_{kt}^+ + d_{kt} \qquad \forall t \in T$$

- Daily Demand

$$\sum_{t\in T}\left[W_{kt}^\ell + W_{kt}^h + U_{kt}\right] = D_k$$

- Maximum Shifting Capability of Customer k:

$$V_{kt}^+ \leq \omega_{kt} \qquad \forall t \in T$$

- Maximum Consumption at Lower Price

$$W_{kt}^\ell \leq \Psi \qquad \forall t \in T$$

- Non-Negativity Variables:

$$W_{kt}^\ell, W_{kt}^h, V_{kt}^+, V_{kt}^-, U_{kt} \geq 0 \qquad \forall t$$

Follower's Problem

- **Objective Function:**

$$\text{Min} \quad f(\Pi^*, \Psi^*) =$$

$$\sum_{t \in T} \left[\Pi_t^\ell \times W_{kt}^\ell + \Pi_t^h \times W_{kt}^h + \gamma_t \times U_{kt} \right] + \sum_{t \in T} \xi_{kt} \times V_{kt}^+$$

- **Subject To:**
  Hourly Demand

$$W_{kt}^\ell + W_{kt}^h + V_{kt}^- + U_{kt} = V_{kt}^+ + d_{kt} \qquad \forall t \in T$$

- **Daily Demand**

$$\sum_{t \in T} \left[W_{kt}^\ell + W_{kt}^h + U_{kt} \right] = D_k$$

- Maximum Shifting Capability of Customer k:

$$V_{kt}^+ \leq \omega_{kt} \qquad \forall t \in T$$

- Maximum Consumption at Lower Price

$$W_{kt}^\ell \leq \Psi \qquad \forall t \in T$$

- Non-Negativity Variables:

$$W_{kt}^\ell, W_{kt}^h, V_{kt}^+, V_{kt}^-, U_{kt} \geq 0 \qquad \forall t$$

Follower's Problem

- Objective Function:

Min $\quad f(\Pi^*, \Psi^*) =$

$$\sum_{t \in T} \left[\Pi_t^{\ell} \times W_{kt}^{\ell} + \Pi_t^{h} \times W_{kt}^{h} + \gamma_t \times U_{kt} \right] + \sum_{t \in T} \xi_{kt \times} V_{kt}^{+}$$

- Subject To:
  Hourly Demand

$$W_{kt}^{\ell} + W_{kt}^{h} + V_{kt}^{-} + U_{kt} = V_{kt}^{+} + d_{kt} \qquad \forall t \in T$$

- Daily Demand

$$\sum_{t \in T} \left[W_{kt}^{\ell} + W_{kt}^{h} + U_{kt} \right] = D_k$$

- Maximum Shifting Capability of Customer k:

$$V_{kt}^{+} \leq \omega_{kt} \qquad \forall t \in T$$

- Maximum Consumption at Lower Price

$$W_{kt}^{\ell} \leq \Psi \qquad \forall t \in T$$

- Non-Negativity Variables:

$$W_{kt}^{\ell}, W_{kt}^{h}, V_{kt}^{+}, V_{kt}^{-}, U_{kt} \geq 0 \qquad \forall t$$

Follower's Problem

- Objective Function:

$$\text{Min} \quad f(\Pi^*, \Psi^*) =$$

$$\sum_{t\in T} \left[\Pi_t^{\ell} \times W_{kt}^{\ell} + \Pi_t^{h} \times W_{kt}^{h} + \gamma_t \times U_{kt} \right] + \sum_{t\in T} \xi_{kt} \times V_{kt}^{+}$$

- Subject To:
  Hourly Demand

$$W_{kt}^{\ell} + W_{kt}^{h} + V_{kt}^{-} + U_{kt} = V_{kt}^{+} + d_{kt} \qquad \forall t \in T$$

- Daily Demand

$$\sum_{t\in T} \left[W_{kt}^{\ell} + W_{kt}^{h} + U_{kt} \right] = D_k$$

- Maximum Shifting Capability of Customer k:

$$V_{kt}^{+} \leq \omega_{kt} \qquad \forall t \in T$$

- Maximum Consumption at Lower Price

$$W_{kt}^{\ell} \leq \Psi \qquad \forall t \in T$$

- Non-Negativity Variables:

$$W_{kt}^{\ell}, W_{kt}^{h}, V_{kt}^{+}, V_{kt}^{-}, U_{kt} \geq 0 \qquad \forall t$$

Follower's Problem

- Objective Function:

$$\text{Min} \quad f(\Pi^*,\Psi^*) =$$

$$\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{\ell}+\Pi_t^{h}\times W_{kt}^{h}+\gamma_t\times U_{kt}\right]+\sum_{t\in T}\xi_{kt}\times V_{kt}^{+}$$

- Subject To:
  Hourly Demand

$$W_{kt}^{\ell}+W_{kt}^{h}+V_{kt}^{-}+U_{kt}=V_{kt}^{+}+d_{kt} \qquad \forall t\in T$$

- Daily Demand

$$\sum_{t\in T}\left[W_{kt}^{\ell}+W_{kt}^{h}+U_{kt}\right]=D_k$$

- Maximum Shifting Capability of Customer k:

$$V_{kt}^{+}\leq\omega_{kt} \qquad \forall t\in T$$

- Maximum Consumption at Lower Price

$$W_{kt}^{\ell}\leq\Psi \qquad \forall t\in T$$

- Non-Negativity Variables:

$$W_{kt}^{\ell}, W_{kt}^{h}, V_{kt}^{+}, V_{kt}^{-}, U_{kt}\geq 0 \qquad \forall t$$

KKT Conditions

- Follower's Dual

$$\mu^a_{kt} + \mu^b_k - \mu^c_{kt} \leq \Pi^\ell_t \qquad \forall t \in T \qquad [W^\ell_{kt}]$$

$$\mu^a_{kt} + \mu^b_k \leq \Pi^h_t \qquad \forall t \in T \qquad [W^h_{kt}]$$

$$-\mu^a_{kt} - \mu^d_{kt} \leq \xi_{kt} \qquad \forall t \in T \qquad [V^+_{kt}]$$

$$\mu^a_{kt} \leq 0 \qquad \forall t \in T \qquad [V^-_{kt}]$$

$$\mu^a_{kt} + \mu^b_k \leq \gamma_t \qquad \forall t \in T \qquad [U_{kt}]$$

- Complementary Slackness

$$(\Psi - W^\ell_{kt}) \times \mu^c_{kt} = 0 \qquad \forall k \in K, t \in T$$

$$(\omega_{kt} - V^+_{kt}) \times \mu^d_{kt} = 0 \qquad \forall k \in K, t \in T$$

$$(\Pi^\ell_t - \mu^a_{kt} - \mu^b_k + \mu^c_{kt}) \times W^\ell_{kt} = 0 \qquad \forall k \in K, t \in T$$

$$(\Pi^h_t - \mu^a_{kt} - \mu^b_k) \times W^h_{kt} = 0 \qquad \forall k \in K, t \in T$$

$$(\xi_{kt} + \mu^a_{kt} + \mu^d_{kt}) \times V^+_{kt} = 0 \qquad \forall k \in K, t \in T$$

KKT Conditions

- **Follower's Dual**

$$\mu^a_{kt} + \mu^b_k - \mu^c_{kt} \leq \Pi^\ell_t \qquad \forall t \in T \quad [W^\ell_{kt}]$$

$$\mu^a_{kt} + \mu^b_k \leq \Pi^h_t \qquad \forall t \in T \quad [W^h_{kt}]$$

$$-\mu^a_{kt} - \mu^d_{kt} \leq \xi_{kt} \qquad \forall t \in T \quad [V^+_{kt}]$$

$$\mu^a_{kt} \leq 0 \qquad \forall t \in T \quad [V^-_{kt}]$$

$$\mu^a_{kt} + \mu^b_k \leq \gamma_t \qquad \forall t \in T \quad [U_{kt}]$$

- **Complementary Slackness**

$$(\Psi - W^\ell_{kt}) \times \mu^c_{kt} = 0 \qquad \forall k \in K, t \in T$$

$$(\omega_{kt} - V^+_{kt}) \times \mu^d_{kt} = 0 \qquad \forall k \in K, t \in T$$

$$(\Pi^\ell_t - \mu^a_{kt} - \mu^b_k + \mu^c_{kt}) \times W^\ell_{kt} = 0 \qquad \forall k \in K, t \in T$$

$$(\Pi^h_t - \mu^a_{kt} - \mu^b_k) \times W^h_{kt} = 0 \qquad \forall k \in K, t \in T$$

$$(\xi_{kt} + \mu^a_{kt} + \mu^d_{kt}) \times V^+_{kt} = 0 \qquad \forall k \in K, t \in T$$

Linearize KKT Conditions

- Non-Linear Complementary Slackness Conditions.
- Linearized using BIG-M method.

$$M \times (1-\rho^e_{kt}) \geq \xi_{kt} + \mu^a_{kt} + \mu^d_{kt}, \quad V^+_{kt} \leq \omega_{kt} \times \rho^e_{kt} \quad \forall k \in K, t \in T$$

$$\Psi \times (1-\rho^a_{kt}) \geq \Psi - W^\ell_{kt}, \quad \mu^c_{kt} \leq M \times \rho^a_{kt}$$

$$M \times (1-\rho^f_{kt}) \geq 0 - \mu^a_{kt}, \quad V^-_{kt} \leq d_{kt} \times \rho^f_{kt} \quad \forall k \in K, t \in T$$

$$\omega_{kt} \times (1-\rho^b_{kt}) \geq \omega_{kt} - V^+_{kt}, \quad \mu^d_{kt} \leq M \times \rho^b_{kt}$$

$$M \times (1-\rho^g_{kt}) \geq \gamma_t - \mu^a_{kt} - \mu^b_k, \quad U_{kt} \leq (d_{kt} + \omega_{kt}) \times \rho^g_{kt} \quad \forall k \in K, t \in T$$

$$M \times (1-\rho^c_{kt}) \geq \Pi^\ell_t - \mu^a_{kt} - \mu^b_k + \mu^c_{kt}, \quad W^\ell_{kt} \leq \Psi \times \rho^c_{kt}$$

$$M \times (1-\rho^d_{kt}) \geq \Pi^h_t - \mu^a_{kt} - \mu^b_k, \quad W^h_{kt} \leq (d_{kt} + \omega_{kt} - \Psi) \times \rho^d_{kt}, \forall k \in K, t \in T$$

- Still Non-Linear due to Decision Variable Ψ.
- Used Bi-Sectional Search for Optimum Ψ.

Linearize KKT Conditions

- Non-Linear Complementary Slackness Conditions.
- Linearized using BIG-M method.

$$\Psi \times (1-\rho^a_{kt}) \geq \Psi - W^\ell_{kt}, \qquad \mu^c_{kt} \leq M \times \rho^a_{kt}$$

$$\omega_{kt} \times (1-\rho^b_{kt}) \geq \omega_{kt} - V^+_{kt}, \qquad \mu^d_{kt} \leq M \times \rho^b_{kt}$$

$$M \times (1-\rho^c_{kt}) \geq \Pi^\ell_t - \mu^a_{kt} - \mu^b_k + \mu^c_{kt}, \qquad W^\ell_{kt} \leq \Psi \times \rho^c_{kt}$$

$$M \times (1-\rho^e_{kt}) \geq \xi_{kt} + \mu^a_{kt} + \mu^d_{kt}, \qquad V^+_{kt} \leq \omega_{kt} \times \rho^e_{kt} \qquad \forall k \in K, t \in T$$

$$M \times (1-\rho^f_{kt}) \geq 0 - \mu^a_{kt}, \qquad V^-_{kt} \leq d_{kt} \times \rho^f_{kt} \qquad \forall k \in K, t \in T$$

$$M \times (1-\rho^g_{kt}) \geq \gamma_t - \mu^a_{kt} - \mu^b_k, \qquad U_{kt} \leq (d_{kt} + \omega_{kt}) \times \rho^g_{kt} \qquad \forall k \in K, t \in T$$

$$M \times (1-\rho^d_{kt}) \geq \Pi^h_t - \mu^a_{kt} - \mu^b_k, \qquad W^h_{kt} \leq (d_{kt} + \omega_{kt} - \Psi) \times \rho^d_{kt}, \forall k \in K, t \in T$$

- Still Non-Linear due to Decision Variable Ψ.
- Used Bi-Sectional Search for Optimum Ψ.

Linearize KKT Conditions

- Non-Linear Complementary Slackness Conditions.
- Linearized using BIG-M method.

$$M\times(1-\rho^e_{kt})\ge \xi_{kt}+\mu^a_{kt}+\mu^d_{kt}, \quad V^+_{kt}\le \omega_{kt}\times\rho^e_{kt} \quad \forall k\in K, t\in T$$

$$\Psi\times(1-\rho^a_{kt})\ge \Psi-W^\ell_{kt}, \quad \mu^c_{kt}\le M\times\rho^a_{kt}$$

$$M\times(1-\rho^f_{kt})\ge 0-\mu^a_{kt}, \quad V^-_{kt}\le d_{kt}\times\rho^f_{kt} \quad \forall k\in K, t\in T$$

$$\omega_{kt}\times(1-\rho^b_{kt})\ge \omega_{kt}-V^+_{kt}, \quad \mu^d_{kt}\le M\times\rho^b_{kt}$$

$$M\times(1-\rho^g_{kt})\ge \gamma_t-\mu^a_{kt}-\mu^b_{k}, \quad U_{kt}\le(d_{kt}+\omega_{kt})\times\rho^g_{kt} \quad \forall k\in K, t\in T$$

$$M\times(1-\rho^c_{kt})\ge \Pi^\ell_t-\mu^a_{kt}-\mu^b_{k}+\mu^c_{kt}, \quad W^\ell_{kt}\le\Psi\times\rho^c_{kt}$$

$$M\times(1-\rho^d_{kt})\ge \Pi^h_t-\mu^a_{kt}-\mu^b_{k}, \quad W^h_{kt}\le(d_{kt}+\omega_{kt}-\Psi)\times\rho^d_{kt}, \forall k\in K, t\in T$$

- Still Non-Linear due to Decision Variable Ψ.
- Used Bi-Sectional Search for Optimum Ψ.

Linearize KKT Conditions

- Non-Linear Complementary Slackness Conditions.
- Linearized using BIG-M method.

$$M \times (1-\rho^{e}_{kt}) \geq \xi_{kt} + \mu^{a}_{kt} + \mu^{d}_{kt}, \quad V^{+}_{kt} \leq \omega_{kt} \times \rho^{e}_{kt} \quad \forall k \in K, t \in T$$

$$\Psi \times (1-\rho^{a}_{kt}) \geq \Psi - W^{\ell}_{kt}, \quad \mu^{c}_{kt} \leq M \times \rho^{a}_{kt}$$

$$M \times (1-\rho^{f}_{kt}) \geq 0 - \mu^{a}_{kt}, \quad V^{-}_{kt} \leq d_{kt} \times \rho^{f}_{kt} \quad \forall k \in K, t \in T$$

$$\omega_{kt} \times (1-\rho^{b}_{kt}) \geq \omega_{kt} - V^{+}_{kt}, \quad \mu^{d}_{kt} \leq M \times \rho^{b}_{kt}$$

$$M \times (1-\rho^{g}_{kt}) \geq \gamma_t - \mu^{a}_{kt} - \mu^{b}_{k}, \quad U_{kt} \leq (d_{kt} + \omega_{kt}) \times \rho^{g}_{kt} \quad \forall k \in K, t \in T$$

$$M \times (1-\rho^{c}_{kt}) \geq \Pi^{\ell}_{t} - \mu^{a}_{kt} - \mu^{b}_{k} + \mu^{c}_{kt}, \quad W^{\ell}_{kt} \leq \Psi \times \rho^{c}_{kt}$$

$$M \times (1-\rho^{d}_{kt}) \geq \Pi^{h}_{t} - \mu^{a}_{kt} - \mu^{b}_{k}, \quad W^{h}_{kt} \leq (d_{kt} + \omega_{kt} - \Psi) \times \rho^{d}_{kt}, \forall k \in K, t \in T$$

- Still Non-Linear due to Decision Variable Ψ.
- Used Bi-Sectional Search for Optimum Ψ.

Linearize KKT Conditions

- Non-Linear Complementary Slackness Conditions.
- Linearized using BIG-M method.

$$M\times(1-\rho^e_{kt})\geq\xi_{kt}+\mu^a_{kt}+\mu^d_{kt}, \quad V^+_{kt}\leq\omega_{kt}\times\rho^e_{kt} \quad \forall k\in K,t\in T$$

$$\Psi\times(1-\rho^a_{kt})\geq\Psi-W^\ell_{kt}, \quad \mu^c_{kt}\leq M\times\rho^a_{kt}$$

$$M\times(1-\rho^f_{kt})\geq 0-\mu^a_{kt}, \quad V^-_{kt}\leq d_{kt}\times\rho^f_{kt} \quad \forall k\in K,t\in T$$

$$\omega_{kt}\times(1-\rho^b_{kt})\geq\omega_{kt}-V^+_{kt}, \quad \mu^d_{kt}\leq M\times\rho^b_{kt}$$

$$M\times(1-\rho^g_{kt})\geq\gamma_t-\mu^a_{kt}-\mu^b_k, \quad U_{kt}\leq(d_{kt}+\omega_{kt})\times\rho^g_{kt} \quad \forall k\in K,t\in T$$

$$M\times(1-\rho^c_{kt})\geq\Pi^\ell_t-\mu^a_{kt}-\mu^b_k+\mu^c_{kt}, \quad W^\ell_{kt}\leq\Psi\times\rho^c_{kt}$$

$$M\times(1-\rho^d_{kt})\geq\Pi^h_t-\mu^a_{kt}-\mu^b_k, \quad W^h_{kt}\leq(d_{kt}+\omega_{kt}-\Psi)\times\rho^d_{kt}, \forall k\in K,t\in T$$

- Still Non-Linear due to Decision Variable Ψ.
- Used Bi-Sectional Search for Optimum Ψ.

Experiments Setup

- Aim to Compare Models When Emissions Considered & When Not.
  - With CE: Model With Consideration of Emissions
  - Without CE: Model Without Consideration of Emissions

- With CE: Objective is the Weighted Sum of Profits & Emissions.

$$\boxed{\text{Max } \sum_{k\in K}\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{*,\ell}(\Pi,\Psi)+\Pi_t^{h}\times W_{kt}^{*,h}(\Pi,\Psi)\right]-\sum_{s\in S}\sum_{t\in T} c_s\times Y_{st}} - \boxed{\sum_{s\in S}\sum_{t\in T}\delta\times\beta_s\times Y_{st}}$$

- Without CE: Objective is the Profits only.

$$\boxed{\text{Max } \sum_{k\in K}\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{*,\ell}(\Pi,\Psi)+\Pi_t^{h}\times W_{kt}^{*,h}(\Pi,\Psi)\right]-\sum_{s\in S}\sum_{t\in T} c_s\times Y_{st}}$$

Experiments Setup

- Aim to Compare Models When Emissions Considered & When Not.
  - With CE: Model With Consideration of Emissions
  - Without CE: Model Without Consideration of Emissions

- With CE: Objective is the Weighted Sum of Profits & Emissions.

$$\boxed{\text{Max } \sum_{k\in K}\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{*,\ell}(\Pi,\Psi)+\Pi_t^{h}\times W_{kt}^{*,h}(\Pi,\Psi)\right]-\sum_{s\in S}\sum_{t\in T} c_s\times Y_{st}} - \boxed{\sum_{s\in S}\sum_{t\in T}\delta\times\beta_s\times Y_{st}}$$

- Without CE: Objective is the Profits only.

$$\boxed{\text{Max } \sum_{k\in K}\sum_{t\in T}\left[\Pi_t^{\ell}\times W_{kt}^{*,\ell}(\Pi,\Psi)+\Pi_t^{h}\times W_{kt}^{*,h}(\Pi,\Psi)\right]-\sum_{s\in S}\sum_{t\in T} c_s\times Y_{st}}$$

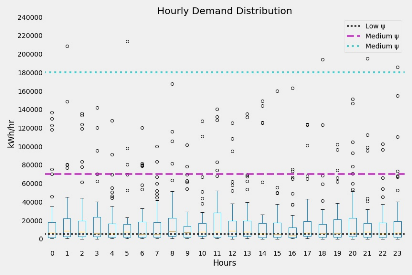
Optimal Ψ Search

Profits & Emission at Different Ψ

- A Low Ψ- 5% of Total Hourly Demand
- B Medium Ψ- 50% of Total Hourly Demand
- C High Ψ- 95% of Total Hourly Demand

Optimal Ψ Search

Profits & Emission at Different Ψ



Hourly Demand Distribution

- (A) Low Ψ- 5% of Total Hourly Demand
- (B) Medium Ψ- 50% of Total Hourly Demand
- (C) High Ψ- 95% of Total Hourly Demand

Data Input-Base Case

- Fuel Characteristics

Fuel	Unit	Price ¢	kWh	CO_2	kWh/CO2/¢	kWh/¢
Coal	Ton	3,925	5,543.44	4,086	0.0345%	141.23%
Gas	$10^3 * ft^3$	283	301.86	121	0.8815%	106.66%
Oil	Barrel	5,682	1,630.13	824	0.0348%	28.68%

- Unit Prices of the Competitor

12AM-3AM	4AM-7AM	8AM-12PM	1AM-6PM	7PM-9PM	10PM-11PM
10	15	20	10	15	10

- Customers' Inconvenience Cost $\xi_{kt} = \frac{d_{kt}}{\max_{t \in T} d_{kt}} \forall k, t$

Computational Stats

- Script Language Python 3.0 & Gurobi 9.0 used.
- Results obtained are close to Optimality, Gap $< 1\%$.
- Computational Time Varied between 99 sec-3,800 secs.
- Heuristics could be an Alternative for Fast Solutions [40].

	With CE		Without CE	
Ψ Value	Time (Sec)	Gap (%)	Time (Sec)	Gap (%)
Low	3,233	0.19%	3,800	0.07%
Medium	101	0.02%	99	0.01%
High	1,127	0.92%	2,133	0.45%

Profits & Emission Comparison[5]

	With CE			Without CE		
ΨValues	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	241.78	89.49	152.29	246.79	164.57	82.22
Medium	242.92	89.49	153.43	248.04	164.57	83.47
High	241.54	89.49	152.05	246.93	164.57	82.36

Profits = [Revenue - Cost of Fuel], Em-Cost = [Emissions Cost], Wt-Profits = [Profits - Em-Cost]

- Marginally higher Profits (0.50%) for Medium-Ψ
- 85% Lower Emission Costs for With CE.
- Profits higher by approx. 2% for Without CE.
- However, Weighted Profits are higher by 85% for With CE.

- Observations
  - Considering Emissions Improve Weighted Profits.
  - Emission Efficient Fuel $\propto$ Improved Weighted Profits

[5] Values in Million ¢

Profits & Emission Comparison[5]

	With CE			Without CE		
ΨValues	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	241.78	89.49	152.29	246.79	164.57	82.22
Medium	242.92	89.49	153.43	248.04	164.57	83.47
High	241.54	89.49	152.05	246.93	164.57	82.36

Profits = [Revenue - Cost of Fuel], Em-Cost = [Emissions Cost], Wt-Profits = [Profits - Em-Cost]

- Marginally higher Profits (0.50%) for Medium-Ψ
- 85% Lower Emission Costs for With CE.
- Profits higher by approx. 2% for Without CE.
- However, Weighted Profits are higher by 85% for With CE.

- Observations
  1. Considering Emissions Improve Weighted Profits.
  2. Emission Efficient Fuel $\propto$ Improved Weighted Profits

[5] Values in Million ¢

Profits & Emission Comparison[5]

	With CE			Without CE		
ΨValues	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	241.78	89.49	152.29	246.79	164.57	82.22
Medium	242.92	89.49	153.43	248.04	164.57	83.47
High	241.54	89.49	152.05	246.93	164.57	82.36

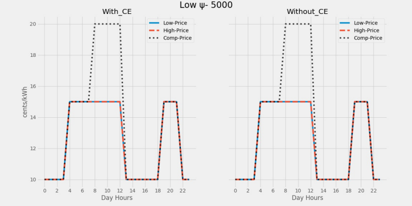
Profits = [Revenue - Cost of Fuel], Em-Cost = [Emissions Cost], Wt-Profits = [Profits - Em-Cost]

- Marginally higher Profits (0.50%) for Medium-Ψ
- 85% Lower Emission Costs for With CE.
- Profits higher by approx. 2% for Without CE.
- However, Weighted Profits are higher by 85% for With CE.

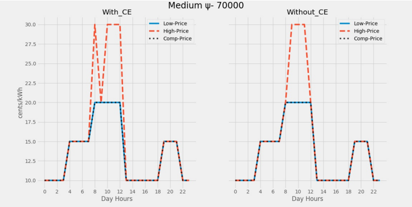
- Observations
  1. Considering Emissions Improve Weighted Profits.
  2. Emission Efficient Fuel $\propto$ Improved Weighted Profits

[5] Values in Million ¢

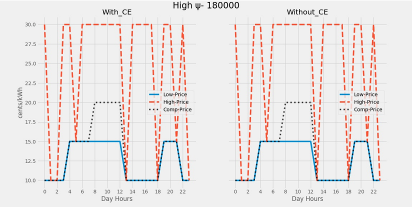
TLOU Prices



Low Ψ



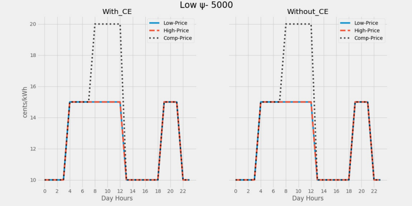
Medium Ψ



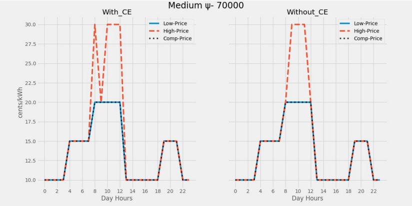
High Ψ

- A. Value of Ψ Affects Unit Rates.
- B. Retailer Prices Low during On-Peak hours.
- C. The Rates Should Impact Demand Shift.
- D. Maximum Demand Shift at Medium Ψ.
- E. Don't Seem to be Affected by Emissions.

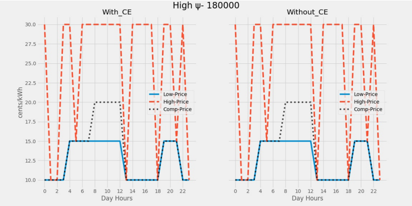
TLOU Prices



Low Ψ



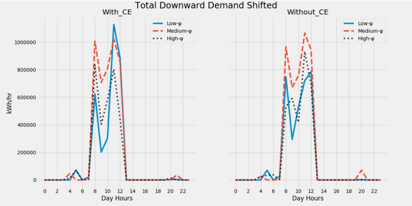
Medium Ψ



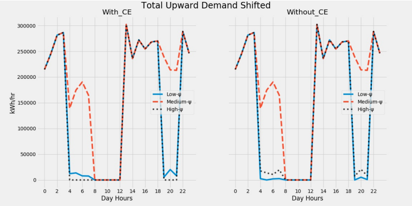
High Ψ

- A. Value of Ψ Affects Unit Rates.
- B. Retailer Prices Low during On-Peak hours.
- C. The Rates Should Impact Demand Shift.
- D. Maximum Demand Shift at Medium Ψ.
- E. Don't Seem to be Affected by Emissions.

Demand Shifts



Downward Shift

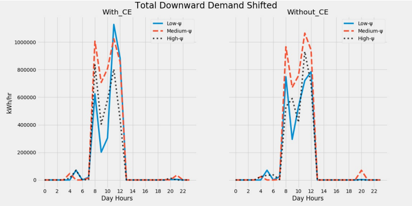


Upward Shift

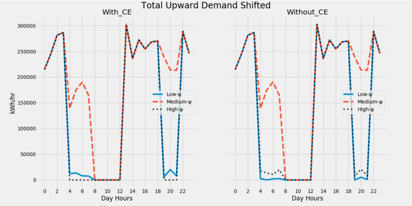
Models	Low Ψ	Medium Ψ	High Ψ
With CE	14.50%	20.15%	14.18%
Without CE	14.23%	20.15%	14.61%

- A Max. Demand Shift at Medium Ψ.
- B Demand Shift during On-Peak hours.
- C Demand Shift Seems Unaffected with Emission.

Demand Shifts



Downward Shift



Upward Shift

Models	Low Ψ	Medium Ψ	High Ψ
With CE	14.50%	20.15%	14.18%
Without CE	14.23%	20.15%	14.61%

- A Max. Demand Shift at Medium Ψ.
- B Demand Shift during On-Peak hours.
- C Demand Shift Seems Unaffected with Emission.

Customers' Costs

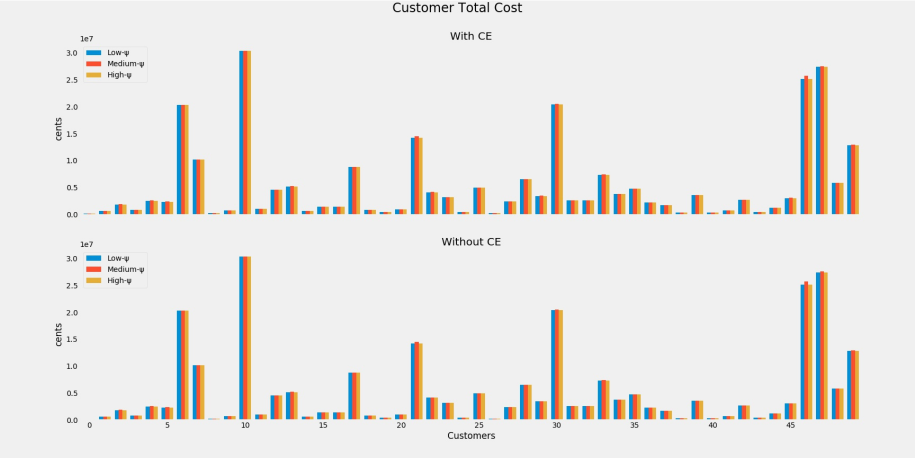


Figure: Customers' Cost Comparison at Different Ψ

Customers' Costs Savings

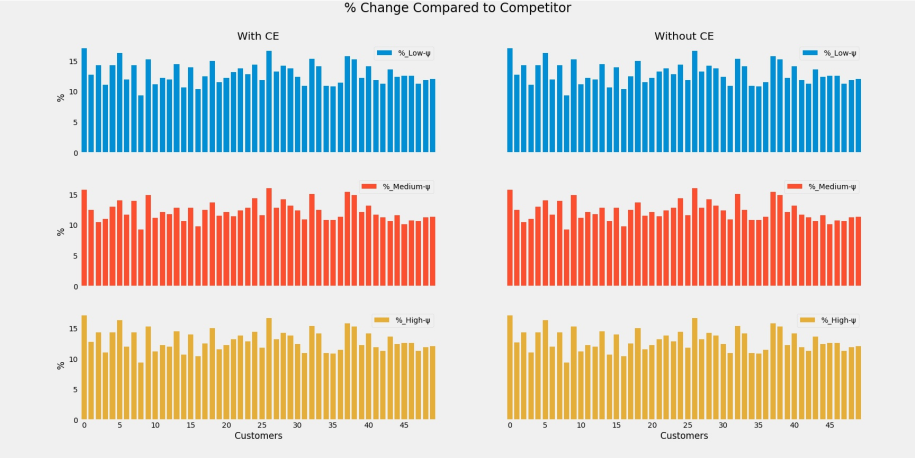


Figure: Customers' Cost Saving Compared to Competitor

Observations & Sensitivity

- Not Considering Emissions, Retailer's Profits are Marginally Higher.
- Emissions Improve the Weighted Profits of the Retailer.
- Unit Prices and Demand Shift of the Customers' Don't Seem to be Affected.
- Shifting to off-peak hours, Customers Save on Electricity Costs.

- Checking Sensitivity
  1. How the Fuel Supply Constraints Impact?
  2. Impacts of Emission Costs.
  3. Customers' Inconvenience Cost Impacts?

Observations & Sensitivity

- Not Considering Emissions, Retailer's Profits are Marginally Higher.
- Emissions Improve the Weighted Profits of the Retailer.
- Unit Prices and Demand Shift of the Customers' Don't Seem to be Affected.
- Shifting to off-peak hours, Customers Save on Electricity Costs.

- Checking Sensitivity
  1. How the Fuel Supply Constraints Impact?
  2. Impacts of Emission Costs.
  3. Customers' Inconvenience Cost Impacts?

Fuel Supply Constraint I

With CE						
	Base Case			Supply Constraint Included		
Ψ Values	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	241.78	89.49	152.29	244.97	119.52	125.44
Medium	242.92	89.49	153.43	244.97	119.52	125.44
High	241.54	89.49	152.05	243.75	119.52	124.22

Without CE						
	Base Case			Supply Constraint Included		
Ψ Values	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	246.79	164.57	82.22	244.19	127.04	117.15
Medium	248.04	164.57	83.47	245.48	127.04	118.44
High	246.93	164.57	82.36	245.48	127.04	118.44

- Profits 1.30% $\uparrow$ for all Ψ.
- With CE Emissions 25% $\uparrow$ & Weighted Profits 20% $\downarrow$
- Without CE Emissions 30% $\downarrow$ & Weighted Profits 30% $\uparrow$

Fuel Supply Constraint II

- Compared With CE & Without CE for Fuel Supply Constrained Scenario Only.

	With CE			Without CE		
ΨValues	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	244.97	119.52	125.44	244.19	127.04	117.15
Medium	244.97	119.52	125.44	245.48	127.04	118.44
High	243.75	119.52	124.22	245.48	127.04	118.44

- Emissions Still Low for With CE.
- With CE Weighted Profits 5%-7% Higher.
- 0%-2% Customers' Total Cost & % Cost Savings Variations.
- Demand Shifted Behavior Impacted.

Models	Low Ψ	Medium Ψ	High Ψ
With CE	20.15%	20.15%	14.30%
Without CE	14.20%	20.15%	20.15%

- Different With CE & Without CE TLOU Prices.

Fuel Supply Constraint II

- Compared With CE & Without CE for Fuel Supply Constrained Scenario Only.

	With CE			Without CE		
ΨValues	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	244.97	119.52	125.44	244.19	127.04	117.15
Medium	244.97	119.52	125.44	245.48	127.04	118.44
High	243.75	119.52	124.22	245.48	127.04	118.44

- Emissions Still Low for With CE.
- With CE Weighted Profits 5%-7% Higher.
- 0%-2% Customers' Total Cost & % Cost Savings Variations.
- Demand Shifted Behavior Impacted.

Models	Low Ψ	Medium Ψ	High Ψ
With CE	20.15%	20.15%	14.30%
Without CE	14.20%	20.15%	20.15%

- Different With CE & Without CE TLOU Prices.

Fuel Supply Constraint II

- Compared With CE & Without CE for Fuel Supply Constrained Scenario Only.

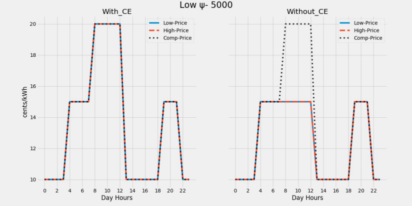
	With CE			Without CE		
ΨValues	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	244.97	119.52	125.44	244.19	127.04	117.15
Medium	244.97	119.52	125.44	245.48	127.04	118.44
High	243.75	119.52	124.22	245.48	127.04	118.44

- Emissions Still Low for With CE.
- With CE Weighted Profits 5%-7% Higher.
- 0%-2% Customers' Total Cost & % Cost Savings Variations.
- Demand Shifted Behavior Impacted.

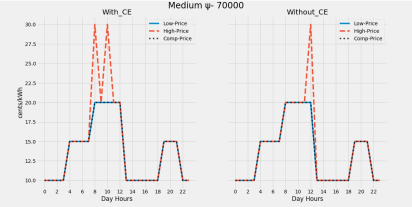
Models	Low Ψ	Medium Ψ	High Ψ
With CE	20.15%	20.15%	14.30%
Without CE	14.20%	20.15%	20.15%

- Different With CE & Without CE TLOU Prices.

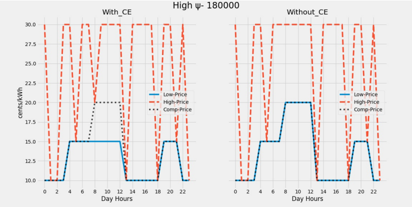
Fuel Supply Constraint III



Low Ψ

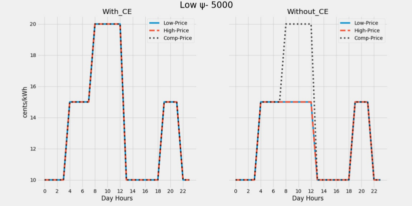


Medium Ψ

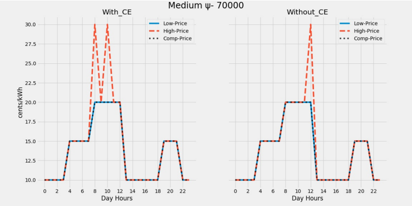


High Ψ

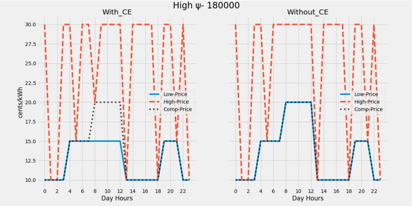
Fuel Supply Constraint III



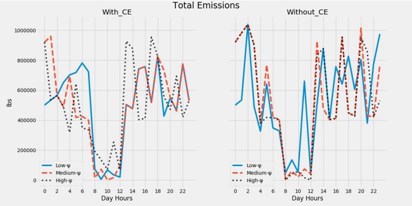
Low Ψ



Medium Ψ



High Ψ



Change in Hourly Emissions

Observations- Fuel Supply Constraint

1. Profits are Same.
2. Weighted Profits improved with Consideration of Emissions, 5%-7%.
3. Even with Mix of Fuels Low Emissions for With CE Model.
4. Hourly Emission Produced Vary.
5. Fuel Supply Restrictions Influence Demand Shifting Behaviors & TLOU Prices.
6. Customers Costs & % Savings Marginally Affected.

Low Emission Cost I

With CE						
	Base Case			Low Emission Cost		
Ψ Values	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	241.78	89.49	152.29	241.60	44.75	196.85
Medium	242.92	89.49	153.43	242.92	44.75	198.17
High	241.54	89.49	152.05	242.92	44.75	198.17

Without CE						
	Base Case			Low Emission Cost		
Ψ Values	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	246.79	164.57	82.22	247.10	82.28	164.82
Medium	248.04	164.57	83.47	248.04	82.28	165.76
High	246.93	164.57	82.36	248.04	82.28	165.76

- Profits Remain Almost Same for all Ψ.
- With CE Emissions 50% $\downarrow$ & Weighted Profits 30% $\uparrow$
- Without CE Emissions 50% $\downarrow$ & Weighted Profits 100% $\uparrow$

Low Emission Cost II

- Compared With CE & Without CE for Fuel Supply Constrained Scenario Only.

	With CE			Without CE		
ΨValues	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	241.60	44.75	196.85	247.10	82.28	164.82
Medium	242.92	44.75	198.17	248.04	82.28	165.76
High	242.92	44.75	198.17	248.04	82.28	165.76

- Emissions Still Low for With CE.
- With CE Weighted Profits 19.50% Higher.
- Customers Cost and % Cost Savings Remain Unchanged.
- Demand Shifted Behavior Impacted.

Models	Low Ψ	Medium Ψ	High Ψ
With CE	14.19%	20.15%	20.15%
Without CE	17.07%	20.15%	20.15%

- Different With CE & Without CE TLOU at Low Ψ.

Low Emission Cost II

- Compared With CE & Without CE for Fuel Supply Constrained Scenario Only.

	With CE			Without CE		
ΨValues	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	241.60	44.75	196.85	247.10	82.28	164.82
Medium	242.92	44.75	198.17	248.04	82.28	165.76
High	242.92	44.75	198.17	248.04	82.28	165.76

- Emissions Still Low for With CE.
- With CE Weighted Profits 19.50% Higher.
- Customers Cost and % Cost Savings Remain Unchanged.
- Demand Shifted Behavior Impacted.

Models	Low Ψ	Medium Ψ	High Ψ
With CE	14.19%	20.15%	20.15%
Without CE	17.07%	20.15%	20.15%

- Different With CE & Without CE TLOU at Low Ψ.

Low Emission Cost II

- Compared With CE & Without CE for Fuel Supply Constrained Scenario Only.

	With CE			Without CE		
ΨValues	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	241.60	44.75	196.85	247.10	82.28	164.82
Medium	242.92	44.75	198.17	248.04	82.28	165.76
High	242.92	44.75	198.17	248.04	82.28	165.76

- Emissions Still Low for With CE.
- With CE Weighted Profits 19.50% Higher.
- Customers Cost and % Cost Savings Remain Unchanged.
- Demand Shifted Behavior Impacted.

Models	Low Ψ	Medium Ψ	High Ψ
With CE	14.19%	20.15%	20.15%
Without CE	17.07%	20.15%	20.15%

- Different With CE & Without CE TLOU at Low Ψ.

High Emission Cost I

With CE						
	Base Case			High Emission Cost		
Ψ Values	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	241.78	89.49	152.29	241.92	134.24	107.68
Medium	242.92	89.49	153.43	242.92	134.24	108.67
High	241.54	89.49	152.05	242.92	134.24	108.67

Without CE						
	Base Case			High Emission Cost		
Ψ Values	Profits	Em-Cost	Wt-Profits	Profits	Em-Cost	Wt-Profits
Low	246.79	164.57	82.22	247.10	246.85	0.25
Medium	248.04	164.57	83.47	248.04	246.85	1.22
High	246.93	164.57	82.36	248.04	246.85	1.99

- With CE Emissions 50% ↑ & Weighted Profits 30% ↓
- Without CE Emissions 50% ↑ & Weighted Profits 100% ↓

Observations- Emission Cost Sensitivity

1. Profits are Higher When Emissions Not Considered.
2. Retailer Must Account Emission in Price Setting at Higher Emission Cost.
3. Retailer Generates Significantly Less Emissions.
4. With CE Weighted Profits Outweigh in both Scenarios.
5. Emission Costs Influence Demand Shifting Behaviors & TLOU Prices.
6. Customers Costs & % Savings Marginally Affected.

Observations- Emission Cost Sensitivity

1. Profits are Higher When Emissions Not Considered.
2. Retailer Must Account Emission in Price Setting at Higher Emission Cost.
3. Retailer Generates Significantly Less Emissions.
4. With CE Weighted Profits Outweigh in both Scenarios.
5. Emission Costs Influence Demand Shifting Behaviors & TLOU Prices.
6. Customers Costs & % Savings Marginally Affected.

Inconvenience Cost-Sensitivity I

- Inconvenience Cost Part of Customers' Objective.
- Emissions Produced Remain Same.
- Don't Impact the Retailer' Profits, Weighted Profits.
- Customers' % Cost Savings Impacted Significantly.

	Competitor	From retailer			
Inc-Cost	Total Cost	Elec Cost	Inc Cost	Total Cost	% Cost Savings
Base Case	28.81	25.49	0.21	25.70	10.80%
Low	28.81	25.49	0.04	25.52	11.42%
High	28.81	25.57	0.52	26.09	9.44%

- Demand Shift Behavior Changed. Low Inc 20.15% Demand Shifted.

Models	Low Ψ	Medium Ψ	High Ψ
With CE	14.54%	16.62%	14.54%
Without CE	14.29%	14.79%	14.51%

Inconvenience Cost-Sensitivity I

- Inconvenience Cost Part of Customers' Objective.
- Emissions Produced Remain Same.
- Don't Impact the Retailer' Profits, Weighted Profits.
- Customers' % Cost Savings Impacted Significantly.

	Competitor	From retailer			
Inc-Cost	Total Cost	Elec Cost	Inc Cost	Total Cost	% Cost Savings
Base Case	28.81	25.49	0.21	25.70	10.80%
Low	28.81	25.49	0.04	25.52	11.42%
High	28.81	25.57	0.52	26.09	9.44%

- Demand Shift Behavior Changed. Low Inc 20.15% Demand Shifted.

Models	Low Ψ	Medium Ψ	High Ψ
With CE	14.54%	16.62%	14.54%
Without CE	14.29%	14.79%	14.51%

Inconvenience Cost-Sensitivity I

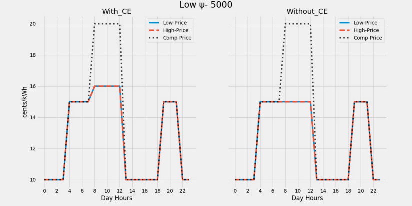
- Inconvenience Cost Part of Customers' Objective.
- Emissions Produced Remain Same.
- Don't Impact the Retailer' Profits, Weighted Profits.
- Customers' % Cost Savings Impacted Significantly.

	Competitor	From retailer			
Inc-Cost	Total Cost	Elec Cost	Inc Cost	Total Cost	% Cost Savings
Base Case	28.81	25.49	0.21	25.70	10.80%
Low	28.81	25.49	0.04	25.52	11.42%
High	28.81	25.57	0.52	26.09	9.44%

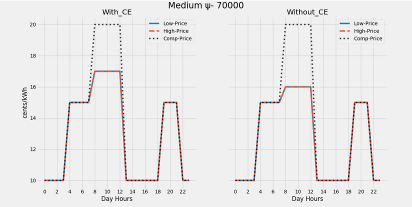
- Demand Shift Behavior Changed. Low Inc 20.15% Demand Shifted.

Models	Low Ψ	Medium Ψ	High Ψ
With CE	14.54%	16.62%	14.54%
Without CE	14.29%	14.79%	14.51%

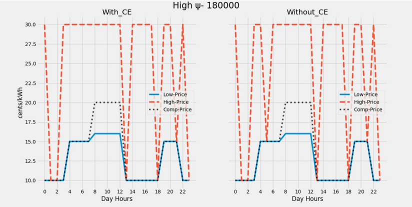
Inconvenience Cost-Sensitivity II



Low Ψ

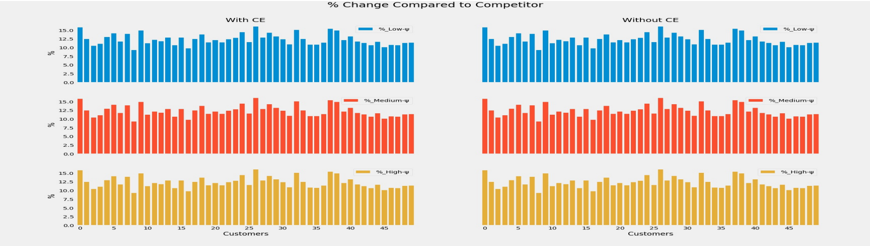


Medium Ψ



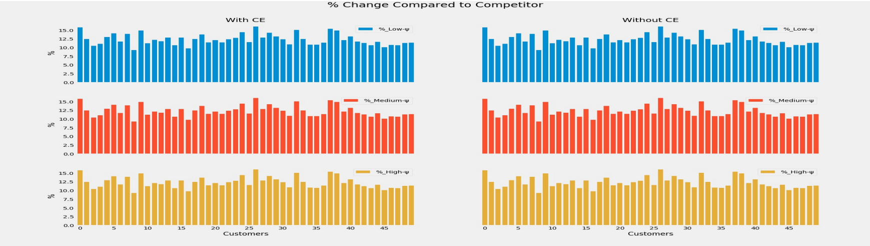
High Ψ

Inconvenience Cost-Sensitivity III

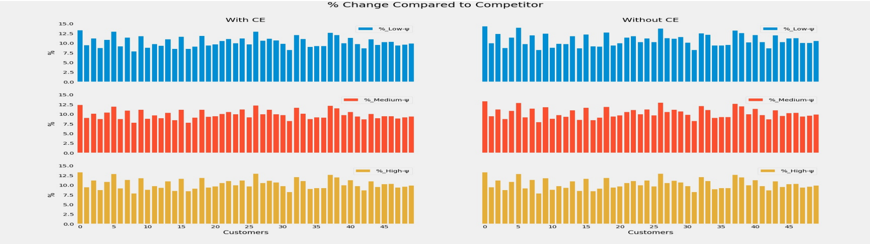


Low Inc Cost

Inconvenience Cost-Sensitivity III



Low Inc Cost



High Inc Cost

Observations- Inconvenience Cost Sensitivity

1. Emissions, Profits, Weighted Profits Remain Unchanged.
2. More Flexible the Customers More Demand Shift to Off-Peak Hours.
3. Inconvenience Cost Impact Retailer' TLOU Price Setting.
    - Uniform Prices When Customers are Flexible.
    - Less Flexible Customers $\propto$ With CE & Without CE Price Difference.
4. Flexible Customers Saves More on Total Costs.

Takeaway Messages

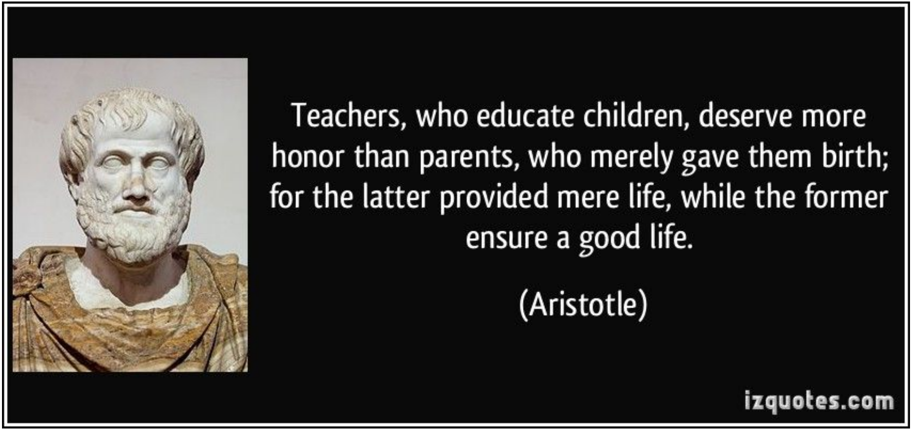
1. Even Profits are less by 2%, Emission Efficient Fuels Reduces Emission more than 80%.
2. O/w Include Emissions in Price Setting for Higher Weighted Profits.
3. Emissions Cost Don't Impact Customers Cost Savings. Low Cost Beneficial to Retailer.
4. Demand Shift and TLOU Prices impacted by Emission Cost & Inconvenience Cost.
5. Flexible Customer Help Flatten the Demand Peaks.

Future Research Ideas

1. Since contribution of distribution and transmission is 27.00% and 12.50% respectively [6] in electricity price, distribution and network can be included.
2. Customers can also generate electricity. Consideration of customer selling back excess electricity to utility can also be modeled.

[6] https://www.eia.gov/energyexplained/electricity/prices-and-factors-affecting-prices.php

Thank You


Teachers, who educate children, deserve more honor than parents, who merely gave them birth; for the latter provided mere life, while the former ensure a good life.
(Aristotle)
izquotes.com

```
Gurobi Optimizer version 9.0.1 build v9.0.1rc0 (win64)
Optimize a model with 26618 rows, 18218 columns and 71240 nonzeros
Model fingerprint: 0xca11b5fd
Variable types: 9794 continuous, 8424 integer (8424 binary)
Coefficient statistics:
  Matrix range     [1e+00, 2e+05]
  Objective range  [1e-02, 3e+06]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e-02, 3e+06]
Presolve removed 6168 rows and 2446 columns
Presolve time: 0.13s
Presolved: 20450 rows, 15772 columns, 60759 nonzeros
Variable types: 8357 continuous, 7415 integer (7415 binary)
Found heuristic solution: objective 2.048973e+08

Root relaxation: objective 2.470046e+08, 9329 iterations, 0.18 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 2.4700e+08    0 1054 2.0490e+08 2.4700e+08  20.6%     -    2s
     0     0 2.4614e+08    0  979 2.0490e+08 2.4614e+08  20.1%     -   13s
     0     0 2.4610e+08    0  945 2.0490e+08 2.4610e+08  20.1%     -   13s
     0     0 2.4610e+08    0  944 2.0490e+08 2.4610e+08  20.1%     -   13s
     0     0 2.4610e+08    0  958 2.0490e+08 2.4610e+08  20.1%     -   13s
     0     0 2.4610e+08    0  958 2.0490e+08 2.4610e+08  20.1%     -   13s
     0     0 2.4584e+08    0  649 2.0490e+08 2.4584e+08  20.0%     -   14s
     0     0 2.4582e+08    0  790 2.0490e+08 2.4582e+08  20.0%     -   14s
     0     0 2.4581e+08    0  763 2.0490e+08 2.4581e+08  20.0%     -   14s
     0     0 2.4581e+08    0  816 2.0490e+08 2.4581e+08  20.0%     -   14s
     0     0 2.4581e+08    0  815 2.0490e+08 2.4581e+08  20.0%     -   14s
     0     0 2.4581e+08    0  816 2.0490e+08 2.4581e+08  20.0%     -   14s
     0     0 2.4566e+08    0  394 2.0490e+08 2.4566e+08  19.9%     -   15s
     0     0 2.4564e+08    0  369 2.0490e+08 2.4564e+08  19.9%     -   15s
     0     0 2.4564e+08    0  395 2.0490e+08 2.4564e+08  19.9%     -   15s
     0     0 2.4564e+08    0  395 2.0490e+08 2.4564e+08  19.9%     -   16s
     0     0 2.4561e+08    0  311 2.0490e+08 2.4561e+08  19.9%     -   17s
     0     0 2.4561e+08    0  315 2.0490e+08 2.4561e+08  19.9%     -   17s
     0     0 2.4561e+08    0  319 2.0490e+08 2.4561e+08  19.9%     -   17s
     0     0 2.4561e+08    0  285 2.0490e+08 2.4561e+08  19.9%     -   18s
     0     0 2.4561e+08    0  284 2.0490e+08 2.4561e+08  19.9%     -   18s
     0     2 2.4561e+08    0  284 2.0490e+08 2.4561e+08  19.9%     -   19s
    19    24 2.3881e+08    5  436 2.0490e+08 2.4560e+08  19.9%   427   20s
    28    33 2.2759e+08    6 1022 2.0490e+08 2.4560e+08  19.9%   939   49s
    38    45 2.0674e+08    8  934 2.0490e+08 2.4560e+08  19.9%  1134   51s
    76    96 2.2694e+08   15  756 2.0490e+08 2.4560e+08  19.9%  1137   55s
   148   182 2.2263e+08   38  778 2.0490e+08 2.4560e+08  19.9%   753   94s
   183   216     cutoff   48      2.0490e+08 2.4560e+08  19.9%   624   96s
   713   751 2.1190e+08  145  833 2.0490e+08 2.4560e+08  19.9%   229  101s
```

Gurobi Log Output

[17]

```
  6635  4444 2.3865e+08   68  454 2.0501e+08 2.4550e+08  19.8%   224  925s
  6783  4564 2.1143e+08   84  954 2.0501e+08 2.4550e+08  19.8%   227  936s
  7014  4580 2.4549e+08   55  262 2.0501e+08 2.4550e+08  19.7%   238  954s
  7122  4682 2.4549e+08   64  117 2.0501e+08 2.4550e+08  19.7%   246  968s
  7304  4670 2.4544e+08   73  341 2.0501e+08 2.4550e+08  19.7%   263  982s

  7362  4699 2.0667e+08   77  924 2.0501e+08 2.4550e+08  19.7%   269 1101s
  7422  4863 2.4243e+08   85  682 2.0501e+08 2.4550e+08  19.7%   270 1115s
  7635  5012 2.4111e+08   99  774 2.0501e+08 2.4550e+08  19.7%   274 1129s
  7882  5067 2.0880e+08  114  752 2.0501e+08 2.4550e+08  19.7%   276 1319s
  8071  5163 2.1120e+08  143  640 2.0501e+08 2.4549e+08  19.7%   278 1335s
  8295  5269 2.4288e+08   79  604 2.0501e+08 2.4549e+08  19.7%   285 1356s
  8523  5374 2.0916e+08  127  907 2.0501e+08 2.4549e+08  19.7%   291 1377s
  8788  5785 2.4501e+08   75  530 2.0501e+08 2.4549e+08  19.7%   299 1399s
  9352  6717 2.1060e+08  249  737 2.0501e+08 2.4549e+08  19.7%   291 1429s
 10637  6933 2.1077e+08   67 1010 2.0501e+08 2.4549e+08  19.7%   275 1449s
 10927  7349 2.4202e+08   98  798 2.0501e+08 2.4549e+08  19.7%   280 1474s
 11454  8016 2.0876e+08   95  982 2.0501e+08 2.4549e+08  19.7%   280 1499s
 12246  8369 2.0640e+08  150  921 2.0501e+08 2.4549e+08  19.7%   275 1524s
 12701  8748 2.0889e+08  101  797 2.0501e+08 2.4549e+08  19.7%   279 1548s
 13171  8752 2.3306e+08  104  621 2.0501e+08 2.4549e+08  19.7%   280 1724s
 13175  8939 infeasible  105      2.0501e+08 2.4549e+08  19.7%   280 1756s
 13436  9072 2.4437e+08   78  358 2.0501e+08 2.4549e+08  19.7%   291 1789s
 13603  9141 2.4546e+08   94  293 2.0501e+08 2.4549e+08  19.7%   308 1826s
 13701  9357 2.4534e+08  110  597 2.0501e+08 2.4549e+08  19.7%   320 1859s
 13985  9711 2.4529e+08  139  620 2.0501e+08 2.4549e+08  19.7%   332 1891s
 14446  9891 2.4523e+08  157  850 2.0501e+08 2.4549e+08  19.7%   334 1926s
 14721 10149 2.4486e+08  180  367 2.0501e+08 2.4549e+08  19.7%   338 1958s
 15065 10669 2.4508e+08  204  478 2.0501e+08 2.4549e+08  19.7%   344 1991s
 15687 11020 2.2302e+08   58  641 2.0501e+08 2.4549e+08  19.7%   344 2025s
 16149 11414 2.4525e+08   74  269 2.0501e+08 2.4549e+08  19.7%   346 2073s
 16670 11795 2.4510e+08   86  261 2.0501e+08 2.4549e+08  19.7%   354 2108s
 17172 12022 2.3233e+08   72  975 2.0501e+08 2.4549e+08  19.7%   357 2301s
 17497 12267 2.1673e+08   73 1030 2.0501e+08 2.4549e+08  19.7%   355 2336s
 17898 12660 2.1592e+08   85  761 2.0501e+08 2.4549e+08  19.7%   356 2378s
 18426 12976 2.1437e+08   71 1005 2.0501e+08 2.4549e+08  19.7%   364 2416s
H18839 11791                    2.083593e+08 2.4549e+08  17.8%   373 2736s
H18840 11021                    2.101830e+08 2.4549e+08  16.8%   373 2736s
 18843 11206     cutoff   91      2.1018e+08 2.4549e+08  16.8%   373 2788s
H18848 11173                    2.102661e+08 2.4549e+08  16.8%   373 2788s
 19154 11863 2.4486e+08   76  292 2.1027e+08 2.4549e+08  16.8%   376 2827s
 20024 12195 2.4469e+08   55  328 2.1027e+08 2.4549e+08  16.8%   369 2868s
 20495 12586 2.3538e+08   80  662 2.1027e+08 2.4549e+08  16.8%   373 2907s
H21057   376                    2.454802e+08 2.4549e+08  0.00%   374 3017s
 21058   364 2.4548e+08   47  284 2.4548e+08 2.4549e+08  0.00%   374 3049s
```