Analysis of Koopman Operator using Deep Neural Networks

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Presentation Overview

Introduction

- To understand very complex function like brain,climate change or financial market , robotics,manufacturing system,autonomous vehicles and transportation networks etc.
- To understand very complex functions, data driven techniques are better than basic principle driven equations.
- Based on data driven, koopman operator is very efficient technique to study the non-linear dynamical systems in controls.

- Koopman operator is an efficient method to represent nonlinear dynamical system in terms of linear dynamical system.

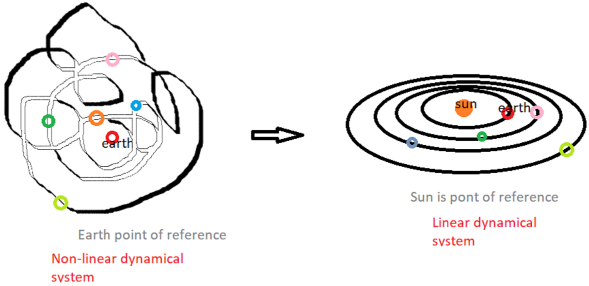

Figure: Change of reference

Frame title - Problems and Solutions

- Often equations are unknown or partially known.

  Model discovery with mechine learning.

- Nonlinear dynamics are still poorly understood.

  Coordinate transformations to linearize dynamics.

- High dimensionality often obscures dynamics.

  patterns exists facilitating reduction.

Frame title - Relevant Literature

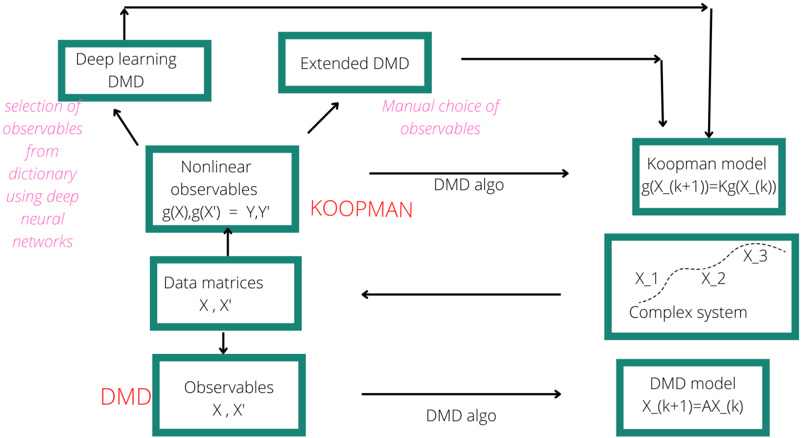

Figure: Comparison among DMD,EDMD,deepDMD

DMD algo

- Step 1: $X = U \sum V$
  Step 2: $A = X' X^{\dagger}$
  Step 3: $\tilde{A} W = W \wedge$
  Step 4: $\Phi = X' V \sum^{-1} W$
  Output: $x = \sum_{j=1}^{n} b_j \phi_j \exp(\lambda_j t)$
  Where,
  W=Eigenvectors
  $\wedge$=Eigenvalues
  $\tilde{A}$=Linear model of dynamical system
  ϕ=Eigen vectors of A
  x=modes
  if A=billion by billion matrix
  then $\tilde{A}$= few hundred by hundred order matrix from which we can easily get eigen values and eigen vectors

- As we know that in dynamic mode decomposition we do not have flexible choice of dictionary of observables, while in extended dynamic mode decomposition user can manually select observables from dictionary [2,6]. In deepDMD, selection of observables possible by deep neural network hence this method has advantage over EDMD.

Problem Statement

- The purpose of this project is to learn the deep neural network representaion of koopman operator so that linear estimation prediction,control and analysis of non-linear dynamical system using koopman operator can be done .

Frame Title - Proposed Solution

- Koopman operator theory tells that any non-linear dynamical system can be transformed into infinite dimensional linear operator.
- Because of infinite dimensional linear operator, it is not easy to find relevant eigen values and modes.Extended DMD is the method for finding such eigen values and modes.
- But in extended DMD there is need to define the observables by user, which is a big challenge .To address this challenge, deepDMD method uses auto-encoder network

Frame Title - Expected Outcomes

- In this project, i will simulate deep dynamic mode decomposition(deepDMD) for training koopman operators from data .
- I will try to tune deep neural networks so that it can be used to generate more efficient dictionaries, to obtain an accurate estimate of koopman operator.

Frame title - Deliverables

UP-Till	Tasks
1st evaluation (Sem-3, Aug 2022)	Literature survey
2nd evaluation (Sem-3, Nov 2022)	Literature survey, DMD algo simulation and work on data set.
1st evaluation (Sem-4, Feb 2023)	Simulation of deep dynamic mode decomposition for higher dimensional dynamic system .
2nd/Final evaluation (Sem-4, May 2023)	I will try to find more efficient dictionaries by tuning it's hyper parameters and Final report preparation .

* Tentative

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QUESTIONS

THANK YOU