Analysis of Koopman Operator using Deep Neural Networks

M.Tech Post Graduate Project Report

to be submitted by

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for the partial fulfillment of the degree

of

MASTER OF TECHNOLOGY IN

Communications and Signal Processing

under the supervision of

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August, 2022

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Timeline

UP-Till	Tasks
1^{st} evaluation (Sem-3, Aug 2022)	Literature survey
2^{nd} evaluation (Sem-3, Nov 2022)	Literature survey and work on data set .
1^{st} evaluation (Sem-4, Feb 2023)	I will implement deep dynamic mode decomposition for higher dimensional dynamic system .
2^{nd}/Final evaluation (Sem-4, May 2023)	I will try to find more efficient dictionaries by tuning it's hyper parameters and prepare final report .

Chapter 1

Introduction

In the past people have been doing dynamical system . The newton second law $F = ma$ is a dynamic system it can give you the trajectories of ballistics . You can use it to simulate things like the pendulum dynamics or planetary motion which are examples of dynamical system . But increasingly today the systems like we actually want to understand like brain,climate change or financial market, there are no first principles from which we can write down in as easy way to simulate and control framework . For example, there is no master equation for your brain, But you are increasingly getting more and more data .From this data your brain makes model and learn . We are going from first principles to data driven techniques . There are many challenges in very complex systems like-
(1). Non-linear dynamical system
(2). Unknown dynamics
(3). High dimensional
To solve these problems we start using DMD method then EDMD and then further on deepDMD .

1.0.1 Data driven dynamic systems

Let us consider a general dynamic equation [1,7],
$\dot{x} = fun(x, t, u, \beta)$
Where, x=state

t=time

u=actuation

β=parameter

The koopman operator is used to analyse the stability of non-linear system. To analysis the non-linear dynamic system, it tries to approximate with linear dynamic system . For very complex non-linear function, there will be required very large number of variables to convert into another linear coordinate system .

1.0.2 Change of coordinate to convert non-linear system into linear

Let us consider an example [13],

$\dot{y_1} = ay_1$

$\dot{y_2} = b(y_2 - y_1^2)$

Let us assume,

$z_1 = y_1$

$z_2 = y_2$

$z_3 = y_1^2$

So, $\dot{z_1} = \dot{y_1} = az_1$

$\dot{z_2} = \dot{y_2} = \lambda(z_2 - z_3)$

$\dot{z_3} = 2y_1\dot{y_1} = 2y_1ay_1 = 2ay_1^2 = 2az_3$

$\dot{\vec{z}} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & -b \\ 0 & 0 & 2a \end{bmatrix} \vec{y}$, On comparing

$\Rightarrow \frac{\partial z}{\partial t} = Kz$

Where,

$K = \begin{bmatrix} a & 0 & 0 \\ 0 & b & -b \\ 0 & 0 & 2a \end{bmatrix}$, K=koopman operator

Noted that,

Eigenfunction of koopman operator tells about mode of dynamics while eigen value tells about stability of mode.

Let us consider an another example,

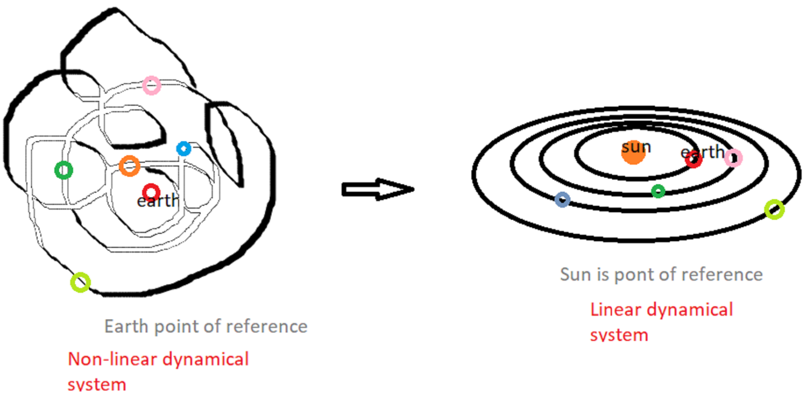

Fig. 1.1: Change of reference

Chapter 2

Literature Survey

2.0.1 Dynamic mode decomposition

It's purely data driven technique,it does not require any other knowledge of the system.DMD tries to find a best fit linear operator A_0 that advances X into X'.So the basic assumption here is that you can write down the dynamic system in terms of linear dynamic system $X' = A_0 X$.

Here,matrix generally are the order of millions by millions,so it has trillion elements.You don't want to actually compute matrix A.So DMD algorithm is trying to approximate the dominant eigen values and eigen vectors of matrix A_0 .Which can best fit in linear operator without computing A_0 [1,7].

DMD algo

Step 1: $X = U_r \sum V_r$

Step 2: $A_0 = X' X^{\dagger}$

Step 3: $\tilde{A}_0 W = W \wedge$

Step 4: $\Phi_0 = X' V_r \sum^{-1} W$

Output: $x = \sum_{i=1}^{n} b_i \phi_i \exp(\lambda_i t)$

Where,

W=Eigenvectors

$\wedge$=Eigenvalues

$\tilde{A}$=Linear model of dynamical system

ϕ_0=Eigen vectors of A_0

x=modes

if A_0=billion by billion matrix

then $\tilde{A}_0$= few hundred by hundred order matrix from which we can easily get eigen values and eigen vectors

2.0.2 Extended dynamic mode decomposition

As we know that in dynamic mode decomposition we do not have flexible choice of dictionary of observables, while in extended dynamic mode decomposition we have .Hence extended DMD has advantage over DMD [2,6].

Chapter 3

System Model

3.1 The problem and statement

The purpose of this project is to learn the deep neural network representation of koopman operator so that linear estimation prediction,control and analysis of non-linear dynamical system using koopman operator can be done .

3.2 Description of Scenario

Based on data driven, koopman operator is very efficient technique to study the non-linear dynamical systems in control[4]. It is used to predict the future state and try to find physical intuition of the system . In this paper i am going to highlight dynamic mode decomposition(DMD), extended dynamic mode decomposition(EDMD) and further more deep dynamic mode decomposition(deepDMD). I will implement novel deepDMD method, analyse the performance and try to improve the one of it's parameter or more . which automatically selects efficient deep dictionaries, while this novel method takes much low dimensional dictionaries . Extended DMD faces difficulty because this method manually select observables .

3.3 Deep dynamic mode decomposition

Koopman operator theory tells that any non-linear dynamical system can be transformed into infinite dimensional linear operator . Because of infinite dimensional linear operator, it is not easy to find relevant eigen values and modes.Extended DMD is the method for finding such eigen values and modes .But in extended DMD there is need to define the observables by user, which is a big challenge .To address this challenge, deepDMD method uses auto-encoder network [5] .

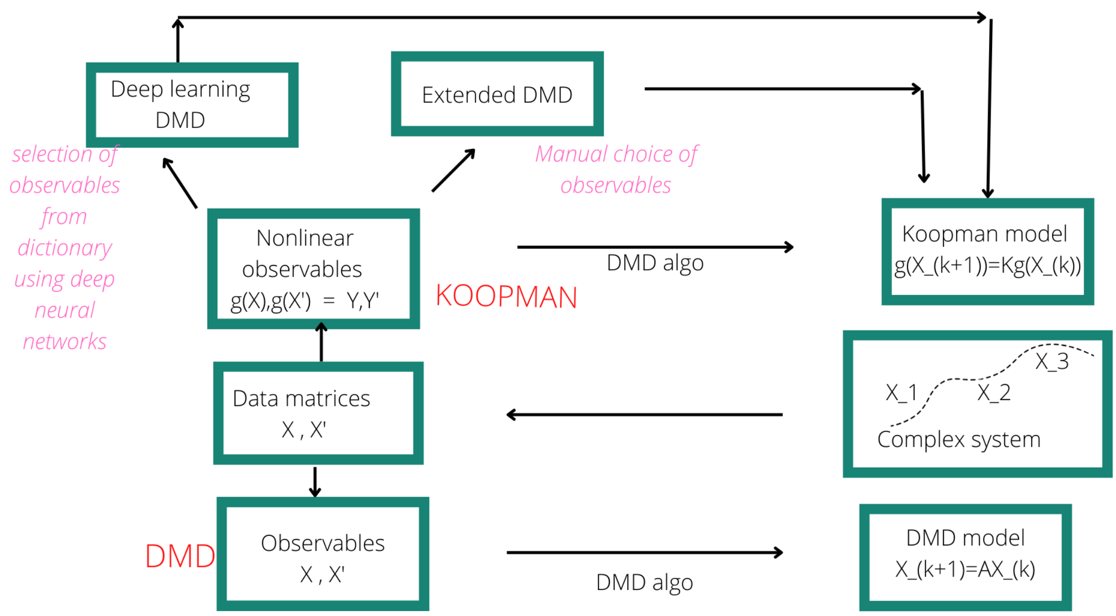

Fig. 3.1: Comparative analysis of DMD,EDMD and deepDMD

Chapter 4

Expected Outcomes

In this project i will implement deepDMD for training koopman operators from data. I will try to improve deep neural networks which can be used to generate more efficient dictionaries, so that accurate estimation of koopman operator can be perform.

Chapter 5

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