APPROVAL SHEET

Title of Thesis: MY-	THESIS-TIT	ΓLE				
Name of Candidate:	DEGREE-					
Thesis and Abstract A	Approved:	MY-ADVISORS-NAME MY-ADVISORS-TITLE Department of Computer Science and Electrical Engineering				
Date Approved:						

Curriculum Vitae

Name: MY-FULL-NAME.

Permanent Address: MY-FULL-ADDRESS.

Degree and date to be conferred: DEGREE-NAME, GRADUATION-MONTH

GRADUATION-YEAR.

Date of Birth: MY-BIRTHDATE.

Place of Birth: MY-PLACE-OF-BIRTH.

Secondary Education: MY-HIGH-SCHOOL, MY-HIGH-SCHOOLS-CITY,

MY-HIGH-SCHOOLS-STATE.

Collegiate institutions attended:

University of Maryland Baltimore County, DEGREE-NAME MY-MAJOR, GRADUATION-YEAR.

MY-OTHER-DEGREES.

Major: MY-MAJOR.

Minor: MY-MINOR.

Professional publications:

FULL-CITATION-INFORMATION.

FULL-CITATION-INFORMATION.

Professional positions held:

EMPLOYMENT-INFO. (START-DATE - END-DATE).

EMPLOYMENT-INFO. (START-DATE - END-DATE).

ABSTRACT

Title of Thesis: Recoloring Web Pages For Color Vision Deficiency Users.

Vikas Bansal, Masters in Science, 2014

Thesis directed by: Dr. Lina Zhou, Associate Professor

Department of Information Systems

Dr. Tim Finin, Professor

Department of Computer Science and

Electrical Engineering

Colors are an important part of our life. They are commonly used to represent important information, specially, categories. Ability to differentiate between colors is important in performing routine tasks such as reading content, following traffic lights etc. This ability varies from person to person. Many people experience difficulty in reading content on web pages due to this variation. These difficulties result from the inability of individuals to sufficiently differentiate between colors. This condition of an individual is called Color Vision Deficiency (CVD). More than four percent of current population suffer from some kind of CVD, significantly affecting their web experience.

To improve the web experience of CVD users, we have presented an algorithm which can be used to recolor web pages such that the recolored web pages do not pose any difficulty to a CVD user. Replaced colors are chosen from a fixed set of color called Dichromacy Trichromacy Equivalency Plane (DTEP) set. While recoloring we also preserve the naturalness and contrast among foreground and background colors in different sections of the web page. A quantitative comparison with the existing tool SPRWeb[] shows that our algorithm performs better in preserving contrast among different sections of the web pages and doesnt differ much in preserving naturalness.

An additional step in to algorithm was added to induce the contrast in pairs according to the W3C guidelines. Quantitative experimentation of modified algorithm shows that

contrast ratio in each replacement pair is more than 4.5 as required for readability.

TITLE-OF-THESIS

by MY-FULL-NAME

Thesis submitted to the Faculty of the Graduate School of the University of Maryland in partial fulfillment of the requirements for the degree of DEGREE-NAME CURRENT-YEAR



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Chapter 1

INTRODUCTION

Website colors specially which are used in foreground text and background are responsible to provide legibility in the content. In addition to providing legibility, they also influence the subjective responses from the users. Depending on the color, a website may seem heavy or light, high or low in temperature and may tell about busyness.

While one solution to the problem of illegibility could be to educate web designers so that they only use a set of colors which are comfortable to both CVD and normal users. [we have developed one such technique - webpages colors downloaded]. Which is a kind of a prevention step. One such color combination could be to develop all the webpages using only black and white. But that itself defeats the purpose of existence of colors. Most of the web developers today, even being aware of the existence of CVD users, develop web pages considering only normal web users leading to confusion and frustration among CVD users.

Another solution is more of a rectifying step. In this, we recolor existing bad web pages to suit the need of both normal and CVD users.

- 1.1 SECTION-TITLE
- 1.2 SECTION-TITLE
 - 1.2.1 SECTION-TITLE
 - 1.2.2 SECTION-TITLE

Chapter 2

BACKGROUND AND RELATED WORK

In this section we will learn about CVD and the existing related work to deal with it. Which also formed the basis of this thesis.

2.1 Color Vision Deficiency (C.V.D.)

Color vision deficiency, is the inability or decreased ability to see color, or perceive color differences, under normal lighting conditions. Color blindness affects a significant percentage of the population. There is no actual blindness but there is a deficiency of color vision. The most usual cause is a fault in the development of one or more sets of retinal cones that perceive color in light and transmit that information to the optic nerve. This type of color blindness is usually a sex-linked condition. The genes that produce photopigments are carried on the X chromosome; if some of these genes are missing or damaged, color blindness will be expressed in males with a higher probability than in females because males only have one X chromosome (in females, a functional gene on only one of the two X chromosomes is sufficient to yield the needed photo-pigments).

Color blindness can also be produced by physical or chemical damage to the eye, the optic nerve, or parts of the brain. For example, people with achromatopsia suffer from a completely different disorder, but are nevertheless unable to see colors.

By cause CVD can be classified in to three types:

- Acquired
- Inherited: Inherited can further be classified in to three types:
 - Monochromacy: Also known as "total color blindness", is the lack of ability to distinguish colors (and thus the person views everything as if it were on a black and white television); caused by cone defect or absence. Monochromacy occurs when two or all three of the cone pigments are missing and color and lightness vision is reduced to one dimension. We are not dealing with this type in our current version of the algorithm.
 - Dichromacy: Dichromacy is a moderately severe color vision defect in which
 one of the three basic color mechanisms is absent or not functioning. Our algorithm can recolor web pages to suit all the Dichromats.
 - * Protanopia (1% of the males): Protanopia is a severe type of color vision deficiency caused by the complete absence of red retinal photoreceptors. It is a form of dichromatism in which the subject can only perceive light wavelengths from 400 to 650nm, instead of the usual 700nm. Pure reds cannot be seen, instead appearing black; purple colors cannot be distinguished from blues; more orange-tinted reds may appear as very dim yellows, and all orange-yellow-green shades of too long a wavelength to stimulate the blue receptors appear as a similar yellow hue. It is hereditary and sex-linked.
 - * Deuteranopia (1% of the males): Deuteranopia is a color vision deficiency in which the green retinal photoreceptors are absent, moderately affecting redgreen hue discrimination. It is a form of dichromatism in which

there are only two cone pigments present. It is likewise hereditary and sex-linked.

* Tritanopia (Less than 1% of males and females): Tritanopia is a very rare color vision disturbance in which there are only two cone pigments present and a total absence of blue retinal receptors. Blues appear greenish, yellows and oranges appear pinkish, and purple colors appear deep red. It is related to Chromosome "7".

Anomalous trichromacy

2.2 Diagnosis

The Ishihara color test, which consists of a series of pictures of colored spots, is the test most often used to diagnose redgreen color deficiencies. A figure (usually one or more Arabic digits) is embedded in the picture as a number of spots in a slightly different color, and can be seen with normal color vision, but not with a particular color defect. The full set of tests has a variety of figure/background color combinations, and enable diagnosis of which particular visual defect is present.

To test the type of color blindness users have, we did the Ishihara color test on each of them during our user study of the algorithm.

2.3 Related Work

Quite a work has been done to both simulate and correct CVD. Ichikawa et al. [16sprweb] were the first publishers of a website recolouring tool that improves accessibility for people with CVD. Iaccarino et al. [15sprweb] proposed the use of edge services to recolour website images in transit to the user. ColorBlindnessSimulateCorrect by Seewald Solutions is one of the examples of an application developed to both sim-



FIG. 2.1: Ishihara Test [Wikipedia]

ulate and correct CVD effect. It uses a linear transformation on rgb input. Daltonize developed by www.daltonize.org can be used to recolor web pages. They first convert RGB values of a color to LMS values and then compensate for color blindness by shifting wavelengths away from the portion of the spectrum invisible to the dichromat, towards the visible portion. Eyepilot [http://www.colorhelper.com/] enables computer users with redgreen color blindness to decipher color-coded maps and graphs by letting the user place a floating window on top of any picture to distinguish among the various color fields. The GNOME[http://en.wikipedia.org/wiki/GNOME] desktop environment provides users with the option to switch a color filter on and off choosing from a set of possible color transformations that displace the colors in order to disambiguate them. While all these tools have proved to be effective in inducing differentiability in the content and thus providing legibility, but none of them have been able to retain the naturalness1, differentiability2 of the original web page.

One such tool which preserves naturalness and improves color differentiability is kuhns tool. One improvement on naturalness preservation was done by David et al. in SPRWeb[]. Which is also the basis of this thesis. SPRWeb performs better in preserving naturalness and differentiability than Kuhns tool. But it fails to make sure than all the fore-

ground background color pairs have the minimum required contrast ratio threshold of 4.5 as per W3C guidelines.

2.4 Definitions

- O: Original set of colors as parsed from CSS files associated with the web page.
- **R**: Replaced set of colors as computed by the recoloring algorithm. And obviously, size of **R** is always equal to size of **O**.
- **P**: Number of color *pairs* in a web page. A *pair* of colors from a web page is of the form (fg,bg), where fg is a foreground text color and bg is a background color. There can be many such (fg,bg) color pairs.
- Perceptual naturalness[SPRweb]: It is a measure of how close is set R to set O. Ideally, if our algorithm is able to find a set such that R = O then the perceptual naturalness will be maximum and the cost of naturalness would be 0. Cost of naturalness quantifies naturalness factor. As defined in SPRWeb[], cost of naturalness can be quantified using the following expression:

$$pn = \frac{1}{N} * \sum_{i=1}^{N} \Delta_{L_{ab}}(O_i, R_i)$$

N Size of **O**.

 $\Delta_{L_{ab}}$ Euclidean distance in Lab color space.

Lab color space is explained in the next section. A *low* value of pn is highly desirable. A *low* value of pn as compared to a high value, essentially tells us that the

recolored web page having low value is closer to original web page in terms of colors. Optimizing the value of pn while computing the recolored set \mathbf{R} makes sure that the recolored version does not have abrupt colors as compared to the original version.

• **Perceptual differentiability[SPRweb]:** This factor maintains the differentiability among colors belonging to a pair in the original web page. Suppose the original web page has only one color pair, i.e size of **P** is 1. And the pair is (*Red*, *White*), then the recolored pair should have colors such that the difference in original pair should be maintained to its best possible extent. This factor can be quantified using the following equation:

$$pd = \frac{1}{size(\mathbf{P})} * \sum_{(X,Y) \in P_O, P_R} |\Delta_{L_{ab}}(X_f, X_g) - \Delta_{L_{ab}}(Y_f, Y_g)|$$

 P_O Set of pairs in original web page.

 P_R Set of pairs in recolored web page.

X A pair of color in original web page thus a member of P_O .

Y Replacement of color pair X thus a member of P_R .

 X_f and Y_f fg in pair X and Y respectively.

 X_q and Y_q bg in pair X and Y respectively.

• Subjective naturalness[SPRweb]: We need this factor in optimizing function to keep the subjective responses of users as is. For example, using this in optimization we can keep warm colors warm or heavy colors heavy. We are using subjective response model developed by [Ou et all] to compute the subjectivity factor. To preserve subjectivity of colors in original web page, we use subjective response space.

We discuss *subjective response space* in detail in future sections. Subjective naturalness cost can be quantified as follows:

$$srn = \frac{1}{N} * \sum_{i=1}^{N} \mathcal{O}_{u}(O_{i}, R_{i})$$

N Size of \mathbf{O} .

 \emptyset_u Euclidean distance in subjective response space.

• **Subjective differentiability[SPRweb]:** Similar to perceptual differentiability, subjective differentiability should be maintained among pairs of colors parsed from original web page. We again use *subjective response space* to quantify this factor as follows:

$$spd = \frac{1}{size(\mathbf{P})} * \sum_{(X,Y) \in P_O, P_R} |\mathcal{O}_u(X_f, X_g) - \mathcal{O}_u(Y_f, Y_g)|$$

 P_O Set of pairs in original web page.

 P_R Set of pairs in recolored web page.

X A pair of color in original web page thus a member of P_O .

Y Replacement of color pair X thus a member of P_R .

 X_f and Y_f fg in pair X and Y respectively.

 $X_g and Y_g$ bg in pair X and Y respectively.

• Cost function: = $W_{pn} * pn + W_{pd} * pd + W_{spn} * spn + W_{spd} * spd$

Where $W'_{ii}s$ are weights corresponding to the factors. We can choose any value for these factors depending upon the requirement. We keep a value 1 for all of them to treat all factors equally.

2.5 Color spaces

• Lab color space: A Lab color space is a color-opponent space with dimension L for lightness and a and b for the color-opponent dimensions, based on nonlinearly compressed CIE XYZ color space coordinates. The three coordinates of CIELAB represent the lightness of the color (L* = 0 yields black and L* = 100 indicates diffuse white; specular white may be higher), its position between red/magenta and green (a*, negative values indicate green while positive values indicate magenta) and its position between yellow and blue (b*, negative values indicate blue and positive values indicate yellow)[wikipedia]. The range of the three coordinates is as follows:

$$L^* \in [0,100]$$
 $a^* \in [-127,128]$
 $b^* \in [-127,128]$

Why did we use Lab color space?

- Lab color is designed to approximate human vision. It aspires to perceptual
 uniformity, and its L* component closely matches human perception of lightness.
- The L*a*b* color space includes all perceivable colors which means that its

- gamut exceeds those of the RGB and CMYK color models (for example, ProPhoto RGB includes about 90% all perceivable colors).
- One of the most important attributes of the L*a*b*-model is device independence. This means that the colors are defined independent of their nature of creation or the device they are displayed on. The L*a*b* color space is used e.g. in Adobe Photoshop when graphics for print have to be converted from RGB to CMYK, as the L*a*b* gamut includes both the RGB and CMYK gamut.
- **Subjective response space:** *Subjective response space* consists of three dimensions which can be formulated as follows using [Ou et all] model:

Activity(Active, Passive):

$$AP = -2.1 + 0.06\sqrt{(L^* - 50)^2 + (a^* - 3)^2 + (\frac{b^* - 17}{1.4})^2}$$

Temperature(Warm, Cool):

$$WC = -0.5 + 0.02(C^*)^{1.07}cos(H^* - 50^\circ)$$

Weight(Heavy, Light):

$$HL = -1.8 + 0.04(100 - L^*) + 0.45cos(H^* - 100^\circ)$$

Chroma:

$$C^* = \sqrt{a^{*2} + b^{*2}}$$

Hue:

$$\boldsymbol{H}^* = arctan(\boldsymbol{b}^*, \boldsymbol{a}^*)$$

 $L^*, a^* and \ b^*$ Dimensions of Lab color space

Chapter 3

SYSTEM ARCHITECTURE

In this chapter, we are going to present the approaches that we took to solve the problem in hand and then we will finally present our algorithm to recolor web pages.

3.1 Initial thoughts and approaches

We started of with a very intuitive idea. We first figured out that which colors are conflicting with rest of the color schema of the web page. After we figure that out, we replace those colors such that there are no more conflicts. In this idea, we need an initial knowledge of what two colors would be conflicting. And in addition to this, we also need to know what would be the safe colors which we can put in place of conflicting colors. To get an idea of this algorithm lets see the following example:

For simplicity, lets assume that we are recoloring web pages which are developed using only the following set of colors. Lets call this set as U(Universal Set):

- 1. Black(#000000)
- 2. Blue(#003366)
- 3. Orange(#FF9900)
- 4. Yellow(#FFCC00)

- 5. Red(#FF0000)
- 6. Green(#00FF00)

7. White(#FFFFF)

To recolor web pages for a particular type of CVD, lets say Protanopia, we need a kind of table like this:

	Black	Blue	Orange Yellow		Red	Green	White
Black	Х	✓	✓	✓	1	✓	✓
Blue	✓	Х	1	✓	1	1	1
Orange	✓	1	Х	✓	Х	Х	✓
Yellow	✓	1	1	Х	1	1	Х
Red	✓	1	Х	✓	Х	Х	✓
Green	✓	1	Х	✓	X	Х	1
White	✓	1	1	Х	1	1	Х

FIG. 3.1: Sample Look Up Table (LUT)

This look up table can help us determine the *conflict* of colors in a webpage. A *conflict* is defined as a situation when two or more colors are seen as similar color by a CVD person, leading to very low differentiability amongst the colors. And since the differentiability is low, one of them should be replaced with a color that can relatively increase the differentiability.

As we can see in Fig 3.2(a), two colors having hue properties as Orange and Green map to colors of similar hues, leading to conflict for a CVD person. LUT in Fig 3.1 lists all such conflicts for colors in **U** which can be experimentally determined.

Safe colors





- (a) As seen by a normal person
- (b) As seen by CVD person (Protanopia)

FIG. 3.2: Example of conflict

Safe colors are the colors which can replace the *conflicting* colors, removing the existing conflict and causing no more additional conflicts. For the given set **U**, some of the safe colors can be:

- 1. ColorA #CC00FF
- 2. ColorB #669999
- 3. ColorC #003300

So our table in Fig 2.1 would look something like this now:

	Black	Blue	Orange	Yellow	Red	Green	White	ColorA	ColorB	ColorC
Black	X	✓	1	✓	1	✓	✓	✓	✓	✓
Blue	✓	Х	1	✓	1	1	1	1	✓	✓
Orange	✓	✓	Х	✓	X	X	1	1	✓	✓
Yellow	✓	✓	1	X	1	1	Х	1	✓	✓
Red	✓	✓	Х	✓	Х	X	1	1	✓	✓
Green	✓	✓	Х	1	Х	Х	1	✓	✓	✓
White	✓	✓	1	Х	1	1	Х	1	1	1
ColorA	✓	✓	✓	✓	1	1	✓	X	✓	✓
ColorB	✓	✓	1	1	1	1	1	1	Х	1
ColorC	✓	✓	1	✓	1	1	1	1	✓	Х

FIG. 3.3: Sample Look Up Table (LUT)

Recoloring algorithm using LUT and safe colors

An algorithm such as Algorithm 1 can be implemented to recolor web pages.

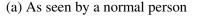
Another way of recoloring web pages could be to recolor them entirely using a predefined set of non-conflicting colors. Information on how to obtain these safe sets is provided in next section. We can perform Algorithm 2 utilizing these existing sets to recolor web pages:

3.1.1 How to obtain safe colors?

Most accurate way of obtaining these safe colors would be to show a set of colors to CVD users and then classify the colors as *safe* or *conflicting*. Although degree of CVD may slightly vary from user to user but we assume in our study that a safe set developed for some of the CVD users is applicable to most of them.

A similar study was done by [5]. They obtained a color palette as seen by a CVD users.







(b) As seen by CVD person (Protanopia)

FIG. 3.4: Color Palette

As we can observe, a CVD person can only see a subset of all possible colors. In other words, an entire set of colors(Fig 3.4(a)) is mapped to only a few different colors(Fig 3.4(b)). It is very probable that if two random colors are put into a single web page, they map to two colors that have very little differentiability between them. If they are used in

background and text formatting, a color blind person won't be able to read the text, which might be a crucial aspect from a website designers point of view. To resolve this, we need a particular set of colors, which when used together, map to colors having significant differentiability for a CVD user.

Lets say there are two colors in a web page C1 and C2 as seen by a normal person. They will be seen as C1' and C2' by a CVD user. C1' and C2' should be differentiable enough for a CVD person to read the content on the web page. Idea is to replace C1 and C2 with D1 and D2 such that D1' and D2', as seen by a CVD user, are differentiable enough.

Limitations of this approach

- We will have to manually generate a finite number of sets from the palette available at [5], so that we can choose a particular set while recoloring.
- Still, there can be a case in which none of the colors in *ColorHex* belongs to any of the sets, which restricts the scope of this approach.
- This approach leads to bad *perceptual naturalness*, as the complete set of original CSS colors will be replaced with a newer one.

3.1.2 SPRWeb

SPRweb[] by David et. al. is a tool that recolors websites to preserve subjective responses and improve color differentiability thus enabling users with CVD to have similar on-line experiences. To recolor web pages, it uses a mapping available in DTEP(Dichromacy Trichromacy Equivalence Plane) set.

To find the DTEP set previous researches, have used findings from unilateral dichromats (individuals who are dichromatic in one eye, but are trichromatic in the other) to identify

the set of colors that are perceived identically in dichromatic color vision and typical color vision.

DTEP set contains all the colors which are perceived same as by a normal person and a CVD person. It also lists all the colors which are perceived by a CVD person.

The key difference in our initial approach and that of SPRweb is the availability of a bigger set of colors and the inclusion of optimization of cost corresponding to *perceptual naturalness*, *perceptual differentiability*, *subjective naturalness* and *subjective differentiability*. We developed following algorithm implementing approach mentioned in SPRWeb to compare its performance with our final algorithm.

Performance Improvement in SPRWeb While implementing SPRWeb to do performance comparison, we came across some redundant calculations which were happening at every step. By storing the result of those calculations once and then reusing them gave us a significant performance improvement.

• Improvement Naturalness cost calculation: Procedure *PercepNaturalness* in Algorithm 6 computes a summation and has a time complexity of **O(n)** where n is the number of colors parsed from CSS file. And *PercepNaturalness* is called every time we make a change in our *ColorRep* set during optimization.

Instead of computing whole summation every time, we can compute the summation only once and then make changes in that for further replacement (because at each step only one color is being replaced and we can just change the sum according to that rather than re-computing whole summation), thus reducing computations.

We can rewrite our *PercepNaturalness* procedure as shown in Algorithm 8. As we can see that the for loop for summation will get executes only once. Thus the time complexity of one call to *PercepNaturalness* has an average cost of O(1) instead of

O(n).

Some other changes such as passing the value of the index where the colors is getting replaced and the value of color which is getting replaced will have to be passed as well to the *CalculateCost* procedure. But they all can be done without any added complexity.

• Improvement differentiability cost calculation: Procedure PercepDifferentiability in Algorithm 6 computes a double summation and has a time complexity of $O(n^2)$ where n is the number of colors parsed from CSS file. And PercepDifferentiability is called every time we make a change in our ColorRep set during optimization process.

Instead of computing the two summations every time, we can compute the inner summation only once and then make changes in that for further replacement (because at each step only one color is being replaced and we can just change the sum according to that change rather than re-computing the double summation), thus reducing computations.

We can rewrite our *PercepDifferentiability* procedure as shown in Algorithm 9. As we can see that the inner for loop for summation will get executes only once. Thus the time complexity of one call to *PercepNaturalness* has an average cost of O(n).

3.2 Our Approach

While browsing the content on-line, specially reading, color of the text and the background are of key significance. Contrast between those two colors is one of the several important factors among other factors as mentioned in Definitions sections. SPRWeb tries to optimize the cost corresponding to perceptual naturalness, perceptual differentiability, subjective naturalness and subjective differentiability for the entire parsed set of colors. But

at the very instant when user is reading a particular element of a web page, he/she may not be concerned about the other elements of the page. Therefore, we can optimize the cost corresponding to various factors for a particular element of the web page first and then repeat the same process for all the remaining elements.

The idea is that while optimizing the various factors for a particular element rather than the whole web page, we will have more color options to choose from and thus the possibility of doing quantitatively better.

We divide a web page in elements based on the foreground-background color combination. For example, if a web page has two division elements, each having one color for background and one for the foreground text color, then the web page has two elements to be processed. Each element having one foreground-background(fg-bg) color pair. We define, a color pair as a combination of foreground and background for a particular element on a page.

Initial step of our algorithm parses out such fg-bg color pairs. Such fg-bg color pairs can be extracted by analysis of HTML Document Object Model(DOM) structure and CSS files. One of the possible implementations is shown later in the section. Once we have all the the color pairs parsed out, we perform our algorithm on each of the color pair. Pseudo code of the algorithm is given in Algorithm 10. A textual description is as follows:

OurApproach defines our tool. The input is the CSS file corresponding to the web page and the Document Object Model(DOM) of the web page. Output is a CSS file which is recolored using our tool. **OurApproach** calls **NewCSSParser** whose input is also a CSS file and the DOM structure. The output is the pairs of foreground-background colors present in the CSS file. Procedure like **NewCSSParser** can be implemented by analyzing the DOM model and the CSS file both.

In the CSS file, we can read the contents as a string and use regular expressions to parse the IDs and classes which it has and their corresponding colors in block. A mapping can be maintained which has IDs and .class as keys and a list as value. List contains the color properties(foreground and background) in that ID or class.

To analyze the DOM structure of HTML web page, we can use tools such as BeautifulSoup. The Default pair is foreground and background colors of the body tag. We can take a default value if they are not defined. Using such modules, we can traverse the DOM tree and check the class or ID which they are using. If no class or IDs or font color attributes are present that means they simply inherit the color of the parent and no new color is added to the system. But if there is some ID or class or color or font color defined, we retrieve its color property from dictionary maintained and report a new pair. Pairs are stored in separate lists like foreground color in one list and background color in other list.

After we obtain a **Foreground** and **Background** color list, we pass one pair at once to procedure **FindRecolor**. **FindRecolor** returns the recolored pair corresponding to the input color. It optimizes the naturalness and differentiability among pairs. The three major steps which are present in **FindRecolor** can be explained as follows:

- **Preserving naturalness:** In the first step, we find a replacement fg' of a color fg from a pair fg-bg in DTEP space. The replacement is such that the distance between fg and fg' is as minimum as possible. This optimization maintains one of the perceptual naturalness as defined in SPRWeb[]. This ensures that the replacement color is as close to the original color as possible and there is no dramatic shift in replacement colors from original colors.
- Preserving differentiability: In the second step, we draw a shell around fg' with internal radius as d e and outer radius as d + e, to obtain a possible set of colors for choosing a replacement bg' for bg. d is the distance between original fg-bg color and e is the factor introduced to ensure that we reach a solution. This step optimizes the perceptual differentiability property. This perceptual differentiability is different

from what defined in SPRWeb. According to this property we try to maintain the contrast as it was present in original pair.

• **Preserving naturalness:** In the third step, we choose a replacement bg' for bg in possible set bg such that the distance between bg and bg' is minimum, again maintaining the perceptual naturalness property.

After we obtain the list of replacements for **Foreground** and **Background**, we need a list of replacement colors of size equal to original colors present CSS file. Such a list is needed to replace colors back in original CSS and also to make sure that if a color is present in both background and foreground of different elements, then no conflict should arise. To achieve this, we first search for a color, present in original color list, in Foreground and Background color list. If the same color is present both in Foreground and Background then that means we have two candidate replacements for the same color. We choose a replacement for both the colors such that we get minimum cost of naturalness and differentiability in the pairs that contain those two colors.

If a color is present only in Foreground and Background then we just put replacement of Foreground or Background in the replacement color list.

After obtaining the replacement color list, we rewrite the CSS files reflecting new colors.

3.3 Maintaining minimum contrast

Contrast is one of the major factors which is important in making a web page readable. In all the techniques discussed, measures are taken to ensure that the original colors and the contrast among them are preserved as much as possible. But there can be a case in which a color pair of replaced foreground and background exist which doesn't meeting the contrast criteria for reading.



FIG. 3.5: Recoloring leading to bad contrast

To resolve such conflicts, we can follow the W3C guidelines of contrast threshold as provided in WCAG 2.0.

3.3.1 W3C guidelines

- this technique is needed to make sure that users can read text that is presented over a background.
- If the background is a solid color (or all black or all white) then the relative luminance of the text can be maintained by making sure that each of the text letters have 4.5:1 contrast ratio (CR) with the background.
- If the background or the letters vary in relative luminance (or are patterned) then the background around the letters can be chosen or shaded so that the letters maintain a 4.5:1 contrast ratio with the background behind them even if they do not have that contrast ratio with the entire background.
- For example, if a letter is lighter at the top than it is a the bottom, it may be difficult to
 maintain the contrast ratio between the letter and the background over the full letter.
 In this case, the designer might darken the background behind the letter, or add a thin

black outline (at least one pixel wide) around the letter in order to keep the contrast ratio between the letter and the background above 4.5:1.

• The contrast ratio can sometimes be maintained by changing the relative luminance of the letters as the relative luminance of the background changes across the page.

The procedure to implement the method as suggested by W3C is follows:

• Measure the relative luminance: of foreground text using the formula:

$$L = 0.2126 * R + 0.7152 * G + 0.0722 * B$$

$$\begin{array}{ll} {\rm R} & {\rm if} \; R_{sRGB} <= 0.03928 \; {\rm then} \; R = R_{sRGB}/12.92 \; {\rm else} \; R = ((R_{sRGB}+0.055)/1.055)^{2.4} \\ {\rm G} & {\rm if} \; G_{sRGB} <= 0.03928 \; {\rm then} \; G = G_{sRGB}/12.92 \; {\rm else} \; G = ((G_{sRGB}+0.055)/1.055)^{2.4} \\ {\rm B} & {\rm if} \; B_{sRGB} <= 0.03928 \; {\rm then} \; B = B_{sRGB}/12.92 \; {\rm else} \; B = ((B_{sRGB}+0.055)/1.055)^{2.4} \\ R_{sRGB} & R_{8bit}/255 \\ G_{sRGB} & G_{8bit}/255 \\ \end{array}$$

$$\begin{array}{ll} B_{sRGB} & B_{8bit}/255 \\ B_{sRGB} & B_{8bit}/255 \end{array}$$

- Measure the relative luminance of the background pixels immediately next to the letter using same formula.
- Calculate the contrast ratio using the following formula.
- Check that the contrast ratio is equal to or greater than 4.5:1

$$contrast = (L1 + 0.05)/(L2 + 0.05)$$

- L1 Relative luminance of the lighter of the foreground or background colors
- L2 Relative luminance of the darker of the foreground or background colors.

3.3.2 Including W3C guidelines in our approach

To include W3C guidelines in our approach, we analyzed the luminance values of the colors present in DTEP space. Following is the plot which was obtained: X-axis is index of DTEP color. Y-axis is luminance value. The graph is oscillating in nature.

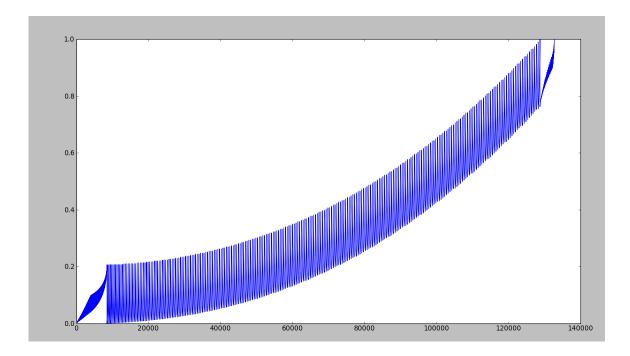


FIG. 3.6: Luminance plot for DTEP

If we see the plot of contrast ratio of a color with the rest of DTEP colors, it would be as follows: If we observe we can see that the graph is particularly divided in two regions, one

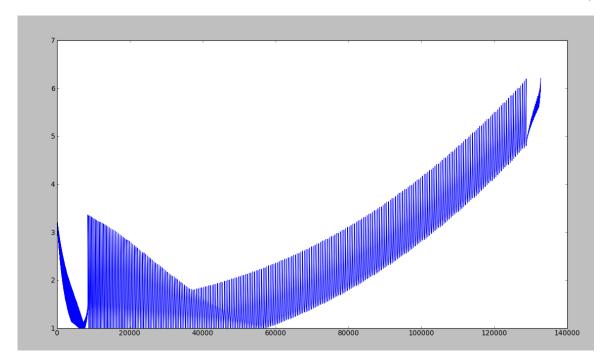


FIG. 3.7: contrast plot for DTEP

has a decreasing contrast ratio values followed by a unity point and then we have a region where contrast ratio is somewhat increasing.

If we arrange the DTEP set according to the luminance values, we can obtain a graph like in Fig 3.8:

And using this same sorted DTEP set, we can obtain the contrast ratio graph of one of the colors, lets call A, and the rest of the DTEP members as shown in Fig 3.9.

As we can observe in Fig 3.9, value of the contrast ratio starts from a value, lets call it a starting point and then plunges to 0. Lets call it a unity point. This unity point is essentially the index of occurrence of color A in DTEP set. After that, the contrast ratio increases and reaches its maximum value, lets call it a ending point.

Following observations can be made about starting point, unity point and ending point:

• Starting point can have a contrast value of [0,21]): As per the arrangement in

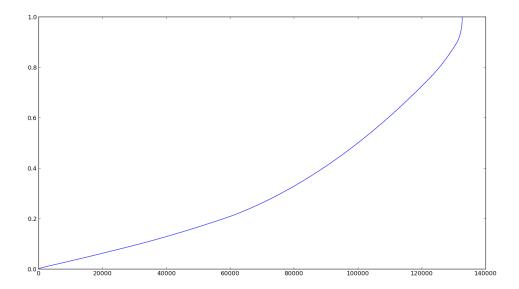


FIG. 3.8: SortedLuminancePlot

DTEP set (increasing order of luminance), it is evident that maximum value of contrast ratio can occur at starting point when it is paired up with the last value of DTEP.

- Contrast ratio strictly declines from starting point till unity point: At unity point, the color is paired up with itself thus giving a contrast ratio of 1. If we move slightly to the left of unity point, we will see colors which have less luminance value as compared to the color A under consideration. And since the colors to the left have a luminance value less than that of A, their luminance value will be in the denominator of the CR formula. And as we move backwards towards the starting point, the denominator will keep on decreasing (due to the sorted luminance order in DTEP set), increasing the net CR value.
- Contrast ratio strictly increases from unity point till ending point: If we move slightly rightwards towards the ending point, the value of luminance in colors would

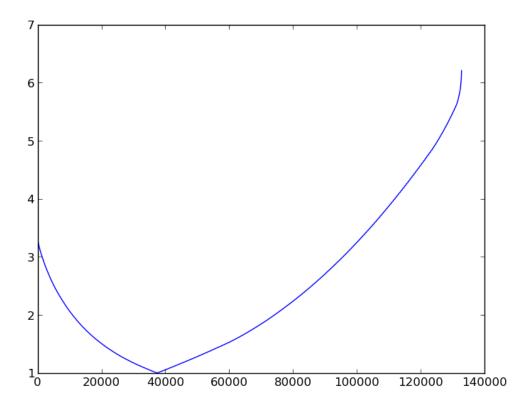


FIG. 3.9: Sorted contrast plot

increase. In the CR formula, luminance value of color on the right would be on top and that of color A, under consideration, would be in denominator. As we move right, numerator would increase, thus increasing net CR value.

To make sure that the colors we choose satisfy the minimum contrast threshold criteria, we draw a vertical line Y=4.5 in Fig 3.9. The possible plots that can be obtained after that are shown in Fig. 3.10, 3.11 and 3.12:

The gray area in Fig 3.10,3.11 and 3.12 show the region from where we can take the colors such that the W3C guideline of minimum threshold is met.

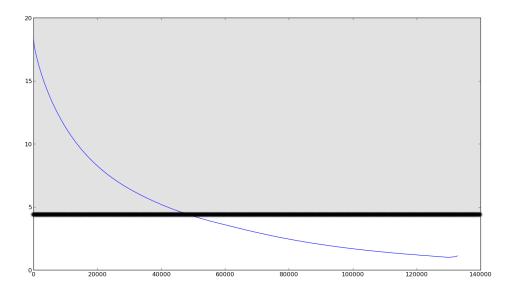


FIG. 3.10: Only starting point included

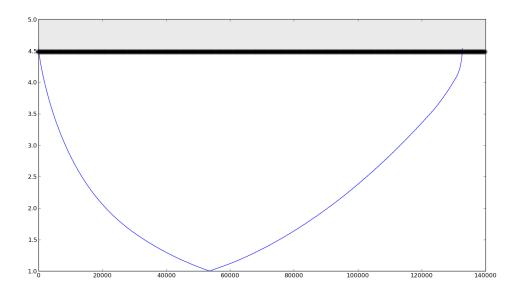


FIG. 3.11: Both starting and ending points included

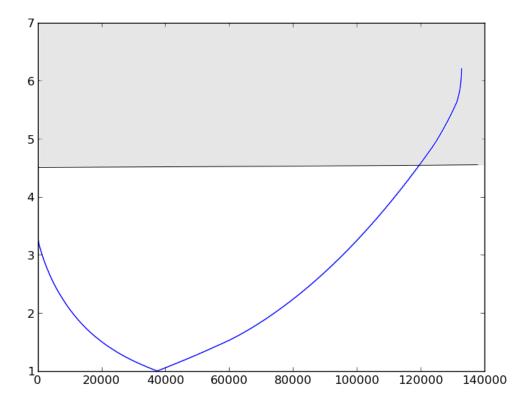


FIG. 3.12: Only ending point included

3.3.3 Implementation

Since we are aiming to make sure that the recolored web pages have color pairs such that the contrast ratio between them meets the 4.5:1 contrast threshold criteria, we need to restrict our possible color search space from DTEP set to a shorter set. Length and the properties of the possible set depends on the color being considered. As shown in Fig 3.10, 3.11, 3.12, the number of possibilities and their location depends on the color. A general approach to include this new criteria in recoloring of a pair, lets say fg-bg, would be to first find a replacement fg' for fg using the **FindRecolor** procedure in algorithm 11. And then obtain a curve like shown in Fig 3.10,11 and 12. Followed by finding the points which

lie in the gray region in DTEP set and putting them in new DTEPReduced set. And then finally searching a possible replacement bg' in the DTEPReduced search space using the same approach as mentioned in **FindRecolor** procedure in algorithm 11. But since there are many pairs possible in a web page, obtaining a curve at run time for each and every pair adds to computational complexity.

To resolve the problem of having to calculate the curves and find a DTEPReduced set corresponding to a color in color pair, we pre-compute a mapping of possible values. This mapping has each color as a key and DTEPReduced as value. A color, lets say x, has a DTEPReduced set such that all colors in DTEPReduced set when paired with x, produce a contrast ratio of more than 4.5. Thus, if in a pair fg-bg, where replacement for fg is fg', if we choose a replacement for bg in DTEPReduced set, then we would be sure that the recolored pair fg'-bg' has a contrast ratio of more than 4.5.

Algorithm 1 Recoloring 1.1

28:

```
1: procedure RECOLORWEB(CSSFILE)
       ColorsHex, ColorMap \leftarrow CSSParser(CSSFile)
2:
       for i \leftarrow 1 to ColorsHex.length do
3:
           for j \leftarrow 1 to ColorsHex.length do
4:
               if LUT[i][j] == 0 then
5:
6:
                  ColorsHex[i] \leftarrow S[S.length]
                  S.length \leftarrow S.length - 1
7:
       NewCSSFile \leftarrow ReplaceColors(CSSFile, ColorsHex, ColorMap)
8:
9:
       return NewCSSFile
10: procedure CSSPARSER(CSSFILE)
       a \leftarrow readLine(CSSFile)
11:
12:
       j \leftarrow 1
       while a \neq EOF do
13:
14:
           if re.find(\#xxxxxx||rgb(r,g,b)||hsl(h,s,l)) == true then
               Colors[j] \leftarrow a[StartOfFind : EndOfFind]
15:
               ColorMap[j++] \leftarrow (StartOfFind, EndOfFind)
16:
           a \leftarrow readLine(CSSFile)
17:
       for i \leftarrow 1 to Colors.length do
18:
           ColorsHex[i] \leftarrow ToHex(Colors[i])
19:
       return ColorsHex, ColorMap
20:
21: procedure REPLACECOLORS(CSSFILE, COLORSHEX, COLORMAP)
       a \leftarrow readLine(CSSFile)
22:
23:
       j \leftarrow 1
       while a \neq EOF do
24:
           if re.find(\#xxxxxx||rgb(r,g,b)||hsl(h,s,l)) == true then
25:
               a[ColorMap[j][0]:ColorMap[j][1]] \leftarrow ColorsHex[j++]
26:
           a \leftarrow readLine(CSSFile)
27:
       return NewCSSFile
```

Algorithm 2 Recoloring 1.2

```
1: procedure RECOLORWEB(CSSFILE)
2:
       ColorsHex, ColorMap \leftarrow \textit{CSSParser}(\textit{CSSFile})
       for i \leftarrow 1 to ColorsHex.length do
 3:
           for j \leftarrow 1 to SETS.length do

    ▷ SETS contains the predefined set of

4:
   non-conflicting colors
              if ColorsHex[i] \in SETS[j] then
5:
                   Replaced \leftarrow i
                                            ▶ Helps in keeping the original color and thus
 6:
   naturalness
7:
                   SetChosen \leftarrow j
                   break
8:
       for i \leftarrow 1 to ColorsHex.length do
9:
           if i == Replaced then
10:
               continue
11:
           ColorsHex[i] \leftarrow SETS[j][i]
12:
       NewCSSFile \leftarrow ReplaceColors(CSSFile, ColorsHex, ColorMap)
13:
       return NewCSSFile
14:
15: procedure CSSPARSER(CSSFILE)
16:
       Defined in Algorithm 1
17: procedure REPLACECOLORS(CSSFILE, COLORSHEX, COLORMAP)
       Defined in Algorithm 1
18:
```

Algorithm 3 SPRWeb

- 1: **procedure** SPRWEB(CSSFILE)
- 2: $ColorsHex, ColorMap \leftarrow CSSParser(CSSFile)$
- 3: $ColorsOrig \leftarrow HexToCIELab(ColorsHex)$
- 4: $DTEPSampled \leftarrow UniformSample(DTEP, 900) \triangleright gets 900 uniform samples$ from DTEP
- 5: $ColorsRep \leftarrow RAND(DTEPSampled, ColorsOrig.length)$ \triangleright Initializing Replaced color set with random colors from DTEPSampled
- 6: $FirstOuput \leftarrow FirstPass(ColorsOrig, ColorsRep, DTEPSampled)$
- 7: **for** $i \leftarrow 1$ to FirstOuput.length **do**
- 8: **for** $j \leftarrow 1$ to DTEP **do**
- 9: **if** distance(FirstOuput[i], DTEP[j]) < 5 **then** \triangleright Looks for all the points at 5 Lab units
- DTEPFiltered[i].append(DTEP[j])
- 11: $FinalOutput \leftarrow SecondPass(ColorsOrig, FirstOuput, DTEPFiltered)$
- 12: $ColorsHex \leftarrow CIELabToHex(FinalOutput)$
- $13: \qquad NewCSSFile \leftarrow ReplaceColors(CSSFile, ColorsHex, ColorMap)$
- 14: **return** NewCSSFile

Algorithm 4 SPRWeb:continue

```
15: procedure FirstPass(ColorsOrig,ColorsRep,DTEPSampled)
       cost \leftarrow CalculateCost(ColorsOrig, ColorsRep)
16:
17:
       ColorRep1 \leftarrow ColorsRep
                                         ▶ Initializing a variable to check optimization
       while true do
18:
          for i \leftarrow 1 to ColorsRep.length do
19:
              for j \leftarrow 1 to DTEPSampled do
20:
                 Value \leftarrow ColorsRep[i]
21:
                 ColorsRep[i] \leftarrow DTEPSampled[j]
22:

    b trying a new color from

   DTEPSampled
                 NewCost \leftarrow CalculateCost(ColorsOrig, ColorsRep) \triangleright Calculating
23:
   new cost after the change
                 if NewCost > cost then
24:
                     ColorsRep[i] \leftarrow Value
                                                  25:
          ColorsRep2 \leftarrow ColorsRep1
26:
          ColorsRep1 \leftarrow ColorRep
27:
          if ColorsRep1 == ColorRep2 then
28:
29:
              break
30:
       returnColorRep1
31: procedure CSSPARSER(CSSFILE)
       Defined in Algorithm 1
32:
33: procedure REPLACECOLORS(CSSFILE, COLORSHEX, COLORMAP)
       Defined in Algorithm 1
34:
```

Algorithm 5 SPRWeb:continue

```
35: procedure SecondPass(ColorsOrig,ColorsRep,DTEPFiltered)
36:
        cost \leftarrow CalculateCost(ColorsOrig, ColorsRep)
37:
        ColorRep1 \leftarrow ColorsRep
                                             ▶ Initializing a variable to check optimization
        while true do
38:
           for i \leftarrow 1 to ColorsRep.length do
39:
               for j \leftarrow 1 to DTEPFiltered[i] do
40:
                   Value \leftarrow ColorsRep[i]
41:
                   ColorsRep[i] \leftarrow DTEPFiltered[i][j]

    b trying a new color from

42:
    DTEPSampled
                   NewCost \leftarrow CalculateCost(ColorsOrig, ColorsRep) \triangleright Calculating
43:
    new cost after the change
                   if NewCost > cost then
44:
                       ColorsRep[i] \leftarrow Value > increased cost, Rejecting the color
45:
           ColorsRep2 \leftarrow ColorsRep1
46:
           ColorsRep1 \leftarrow ColorRep
47:
           if ColorsRep1 == ColorRep2 then
48:
               break
49:
        return Color Rep 1
50:
```

Algorithm 6 SPRWeb:continue

```
51: procedure CALCULATECOST(COLORSORIG,COLORSREP)
52:
       pn \leftarrow PercepNaturalness(ColorsOrig, ColorsRep)
       pd \leftarrow PercepDifferentiability(ColorsOrig, ColorsRep)
53:
       spn \leftarrow SubNaturalness(ColorsOrig, ColorsRep)
54:
       spd \leftarrow SubDifferentiability(ColorsOrig, ColorsRep)
55:
       cost \leftarrow pn + pd + spn + spd
56:
57:
       return cost
58: procedure PercepNaturalness(ColorsOrig,ColorsRep)
       pn \leftarrow 0
59:
       for i \leftarrow 1 to ColorsOrig.length do
60:
           pn \leftarrow pn + distance(ColorsOrig[i], ColorsRep[i])
61:
       pn \leftarrow pn/ColorsOrig.length
62:
       return pn
63:
64: procedure PercepDifferentiability(ColorsOrig,ColorsRep)
       pd \leftarrow 0
65:
       for i \leftarrow 1 to ColorsOrig.length do
66:
           for j \leftarrow i + 1 to ColorsRep.length do
67:
                      \leftarrow pd + abs(distance(ColorsOrig[i], ColorsOrig[j]) -
68:
   distance(ColorsRep[i], ColorsRep[j]))
       pd \leftarrow pd/((ColorsOrig.length) * (ColorsOrig.length - 1))
69:
       return pd
70:
```

Algorithm 7 SPRWeb:continue

```
71: procedure SUBNATURALNESS(COLORSORIG,COLORSREP)
```

72:
$$\mathbf{return} \ PercepNaturalness(Sub(ColorsOrig), Sub(ColorsRep))$$

74: **return**
$$PercepDifferentiability(Sub(ColorsOrig), Sub(ColorsRep))$$

76:
$$L \leftarrow Color[0]$$

77:
$$a \leftarrow Color[1]$$

78:
$$b \leftarrow Color[2]$$

79: **if**
$$a == 90$$
 then

80:
$$H \leftarrow 90$$

81: **else**

82:
$$H \leftarrow tan^{-1}b/a$$

83:
$$C \leftarrow sqrt(a^2 + b^2)$$

84:
$$activity \leftarrow -2.1 + 0.06 * sqrt((l-50)^2 + (a-3)^2 + ((b-17)/1.4)^2)$$

85:
$$temp \leftarrow -0.5 + 0.02 * C^{1.07} * cos(H - 50)$$

86:
$$weight \leftarrow -1.8 + 0.04 * (100 - l) + 0.45 * cos(H - 100)$$

87: **return** (activity,temp,weight)

Algorithm 8 Improvements in SPRWeb

```
1: procedure PERCEPNATURALNESS(COLORSORIG,COLORSREP,k,OLDCOLOR) ▷ k=index of replacement, OldColor = Color being replaced
```

2: **if** pn == InitialzedValue **then** \triangleright is true only once - during first computation of cost

```
3: for i \leftarrow 1 to ColorsOrig.length do
```

4:
$$pn \leftarrow pn + distance(ColorsOrig[i], ColorsRep[i])$$

5:
$$pn \leftarrow pn/ColorsOrig.length$$

6: **else**

7:
$$pn \leftarrow pn * ColorsOrig.length$$

8:
$$pn \leftarrow pn - distance(ColorsOrig[k], OldColor) + distance(ColorsOrig[k], ColorRep[k])$$

9: **return** *pn*

Algorithm 9 Improvements in SPRWeb

return pd

13:

```
1: procedure PercepDifferentiability(ColorsOrig,ColorsRep,k,OldColor)
   ⊳ k=index of replacement, OldColor = Color being replaced
       if pd == InitialzedValue then \triangleright is true only once - during first computation of
2:
   cost
           for i \leftarrow 1 to ColorsOrig.length do
 3:
              for j \leftarrow i + 1 to ColorsRep.length do
 4:
                      \leftarrow pd + abs(distance(ColorsOrig[i], ColorsOrig[j]) -
 5:
   distance(ColorsRep[i], ColorsRep[j]))
           pd \leftarrow pd/((ColorsOrig.length) * (ColorsOrig.length - 1))
6:
       else
7:
           pd \leftarrow pd * ((ColorsOrig.length) * (ColorsOrig.length - 1))
 8:
           for i \leftarrow 1 to ColorsOrig.length do
 9:
                      \leftarrow pd - abs(distance(ColorsOrig[i], ColorsOrig[k]) -
10:
   distance(ColorsRep[i], OldColor))
11:
                             pd + abs(distance(ColorsOrig[i], ColorsOrig[k])
              pd
   distance(ColorsRep[i],ColorRep[k])) \\
           pd \leftarrow pd/((ColorsOrig.length) * (ColorsOrig.length - 1))
12:
```

Algorithm 10 OurApproach

```
1: procedure OURAPPROACH(CSSFILE,DOM)
       Foreground, Background, ColorMap \leftarrow NewCSSParser(CSSFile,DOM)
2:
       ColorsOrig \leftarrow HexToCIELab(Foreground UBackground)
3:
       for j \leftarrow 1 to Foreground.length do
4:
          RecoloredForeground[i], RecoloredBackground[i]
5:
   FindRecolor(HexToCIELAB(Foreground[i]), HexToCIELAB(Background[i]))
6:
       for j \leftarrow 1 to ColorsOrig.length do
          if Foreground.count(ColorsOrig[i])! = 0 then
 7:
              FIndex \leftarrow Foreground.index(ColorsOrig[i])
8:
              if Background.count(ColorsOrig[i])! = 0 then
9:
                 BIndex \leftarrow Background.index(ColorsOrig[i])
10:
                 i \leftarrow xsuchmindist(RecoloredForeground[x], RecoloredBackground[x]) - dist(
11:
                 ColorRecolor[i] \leftarrow ColorsOrig[i]
12:
              else
13:
                 ColorRecolor[i] \leftarrow RecoloredForeground[FIndex]
14:
          ColorRecolor[i] \leftarrow RecoloredBackground[Background.index(ColorsOrig[j])]
15:
       ColorsHex \leftarrow CIELabToHex(ColorRecolor)
16:
       NewCSSFile \leftarrow ReplaceColors(CSSFile, ColorsHex, ColorMap)
17:
       return NewCSSFile
18:
```

Algorithm 11 OurApproach:continue

```
19: procedure FINDRECOLOR(FG,BG)
       minDist = MAXINF
20:
       for j \leftarrow 1 to DTEP do
21:
          if distance(bg, j) < minDist then
22:
              minDist = distance(bg, DTEP[j])
23:
24:
       for j \leftarrow 1 to DTEP do
           if distance(bq, j) < minDist + 0.5 then
25:
              minDistCircle.append(distance(bg, DTEP[j]))
26:
27:
       initial Differentiability \leftarrow distance(fg, bg)
28:
       a \leftarrow 1
       possibleSet \leftarrow 0
29:
       while possibleSet.length <= 0 do
30:
           for j \leftarrow 1 to DTEP do
31:
              if (distance(minDistCircle[0], DTEP[j]) > initialDiffrentiablity -
32:
   a*0.1 and distance(minDistCircle[0], DTEP[j]) < initialDifferentiablity + a*
   0.1): then
                  possibeSet.append(DTEP[j])
33:
              a = a * 2
34:
       minDist \leftarrow MAXINF
35:
       for j \leftarrow 1 to possibleSet do
36:
          if distance(fg, j) < minDist + 0.5 then
37:
              minDist = distance(fg, possibleSet[j])
38:
              replacementFg = possibleSet[j]
39:
       return replacement Fg, minDistCircle[0]
40:
```

Appendix A

APPENDIX-TITLE

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- A.1 SECTION-TITLE
- A.2 SECTION-TITLE
- A.2.1 SECTION-TITLE
- A.2.2 SECTION-TITLE