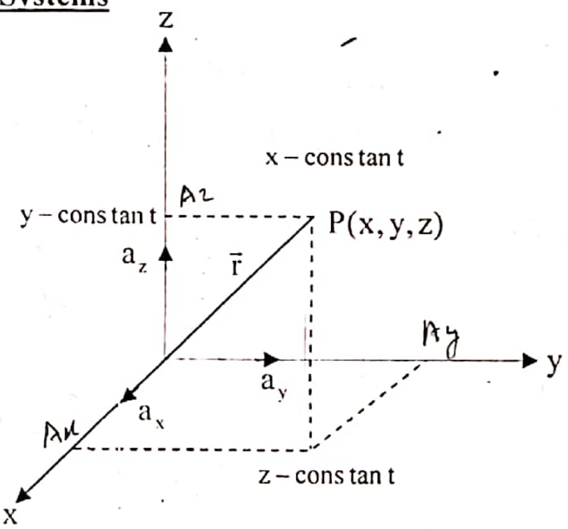
EMT NOTES (EE/EC)

Coordinate Systems

(1) Rectangular Coordinate System

$-\infty < x < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$

Three mutually perpendicular surfaces



$$\vec{r} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$$

$$\vec{A} = A_x a_x + A_y a_y + A_z a_z$$

$a_x . a_y = 0 \qquad a_x \times a_y = a_z$

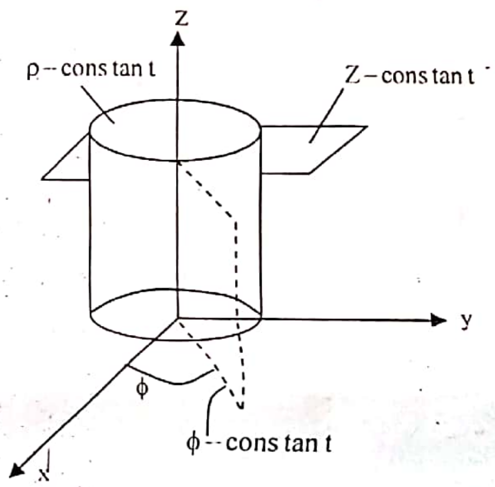
$a_y . a_z = 0 \qquad a_y \times a_z = a_x$

$a_z . a_x = 0 \qquad a_z \times a_x = a_y$

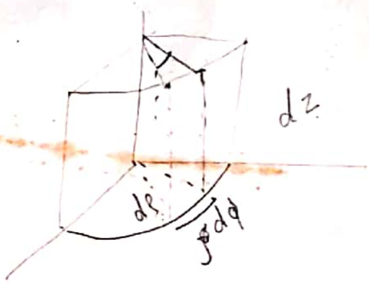
(2) Cylindrical Coordinate System

Three mutually perpendicular planes are –

(a) Cylinder of radius ρ (constant-ρ plane)
(b) Constant z-plane
(c) Constant ϕ- plane (Perpendicular to x-y plane and displaced by angle of ϕ from the x-axis.)



$0 \le \phi \le 2\pi$
$0 \le \rho \le \infty$
$-\infty \le Z \le \infty$

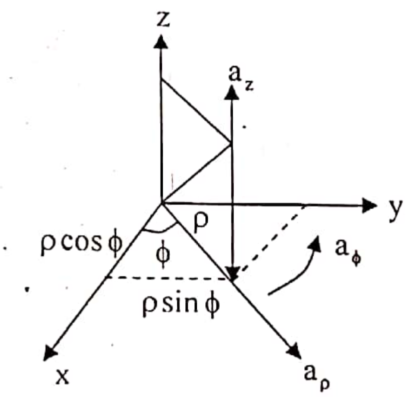


$x = \rho \cos \phi$
$y = \rho \sin \phi$
$z = Z.$

$\rho = \sqrt{x^2 + y^2}\,, \quad \tan \phi = y / x$

- **Conversion from Rectangular to cylindrical coordinate.**

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$



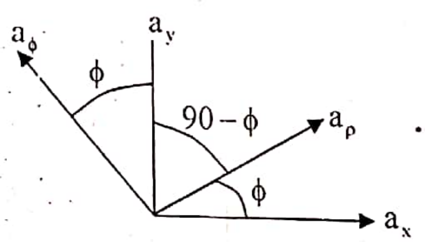
$$\left.\begin{aligned} \vec{A} &= A_x\vec{a}_x + A_y\,\vec{a}_y + A_z\vec{a}_z \\ \vec{A} &= A_\rho\vec{a}_\rho + A_\phi.\vec{a}_\phi + A_z a_z \end{aligned}\right\}$$

$a_x . a_\rho = \cos\phi$

$a_y . a_\rho = \sin\phi$

$a_x . a_\phi = \cos(90 + \phi) = -\sin\phi$

$a_y . a_\phi = \cos\phi$



(3) Spherical Coordinate System

Three mutually perpendicular planes are -

(1) A sphere of radius **r** centered at origin (constant r plane)

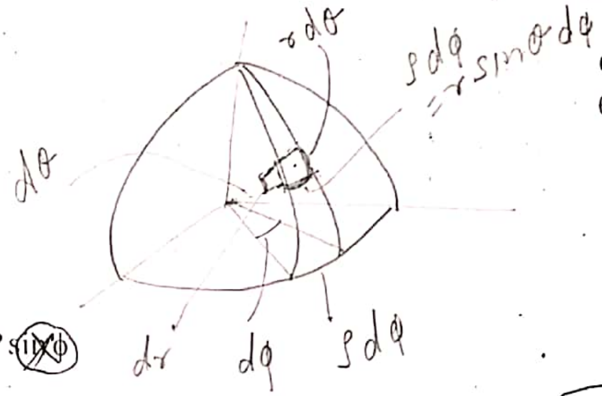
(2) A cone having a z axis edges its main axis and vertex at origin.

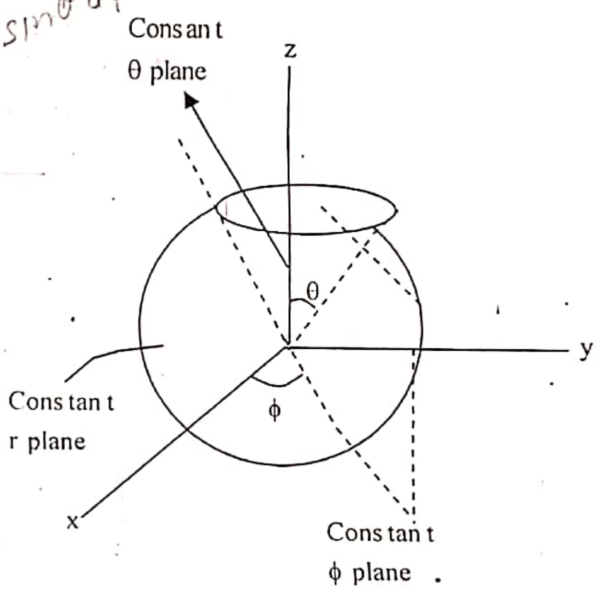
(3) A constant ϕ - plane which is perpendicular to x-y plane and displaced by an angle of ϕ from the x - axis

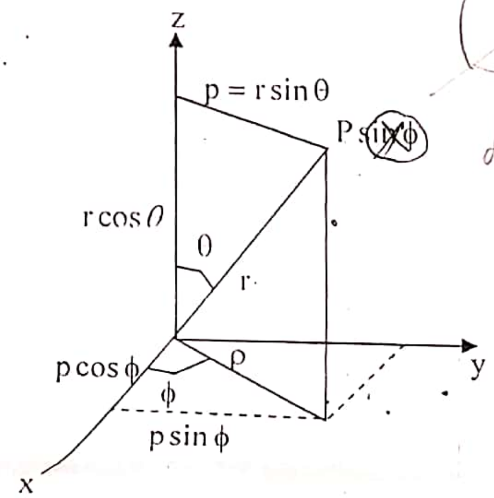
$0 \le r \le \infty$

$0 \le \Phi \le 2\pi$

$0 \le \theta \le \pi$







$x = \rho \cos \phi = r \sin\theta \cos\phi$

$y = \rho \sin \phi = r \sin\theta \sin\phi$

$z = r \cos \theta$

• Conversion from rectangular to

$A = A_x a_x + A_y a_y + A_z a_z$

$A = A_r a_r + A_\theta a_\theta + A_\phi a_\phi$

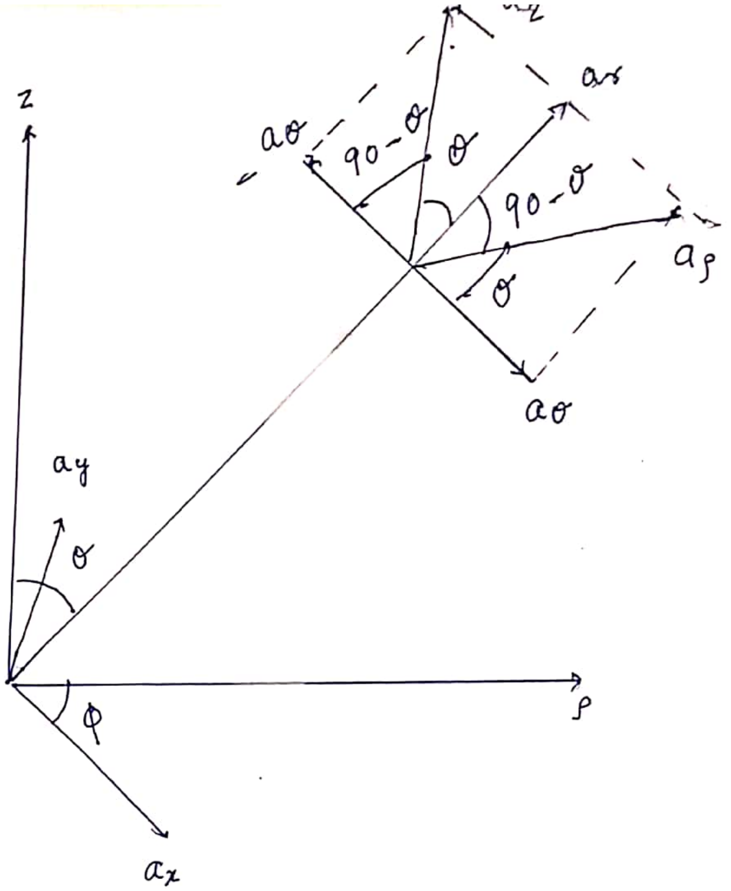
$A \cdot a_r = A_r$

$A_r = A_x \, a_x \cdot a_r + A_y \, a_y \cdot a_r$
$\quad + A_z \, a_z \cdot a_r$

$a_x \cdot a_r =$

$a_y \cdot a_r =$

$a_z \cdot a_r = \cos\theta$



Scanned with Cam

- **Conversion from rectangular to spherical coordinate**

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

➢ **General Co-ordinate System** (Orthogonal curvilinear Co-ordinate System)

	u	v	w	h_1	h_2	h_3 → scale factor
(1)	x	y	z	1	1	1
(2)	ρ	φ	Z	1	ρ	1
(3)	r	θ	φ	1	r	r sinθ

- **Differential length vector**

$$\vec{dl} = dl_1\,\vec{a}_u + dl_2\,\vec{a}_v + dl_3\,\vec{a}_w$$

where $dl_1 = h_1 du \quad dl_2 = h_2 dv, dl_3 = h_3 dw$

For rectangular coordinate

$\vec{dl} = dx\,a_x + dy\,a_y + dz\,a_z$

For cylindrical Coordinate

$\vec{dl} = d\rho\,a_\rho + \rho d\phi\,a_\phi + dz\,a_z$

For Spherical co-ordinate $\vec{dl} = dr\,a_r + r d\theta\,a_\theta + r\sin\theta d\phi\,a_\phi$

- **Differential surface area**

$\vec{ds} = \pm[dl_1 a_u \times dl_2 a_v]$

$= \pm[dl_2 a_v \times dl_3 a_w]$

$= \pm[dl_3 a_w \times dl_1 a_u]$

For Rectangular coordinate system

$\pm\, dxdy\, a_z$

$\pm\, dydz a_x$

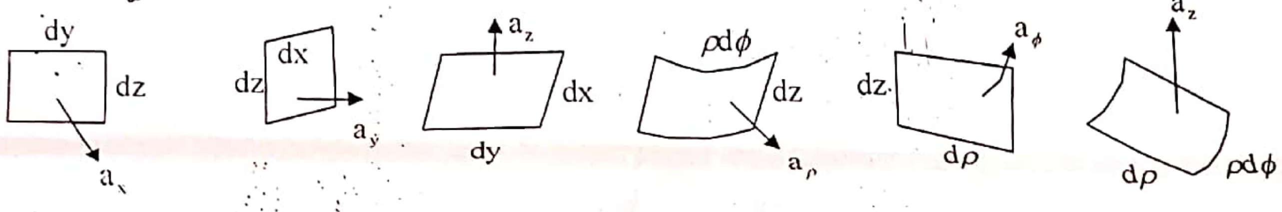
$\pm\, dxdz a_y$

For Cylindrical coordinate system

$\pm\, \rho d\rho d\phi a_z$

$\pm\, \rho d\phi dz a_\rho$

$\pm\, d\rho dz a_\phi$

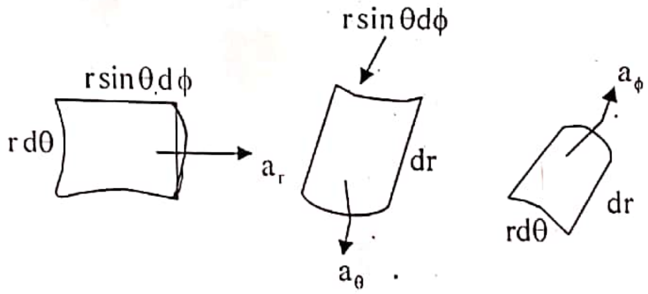


For spherical coordinate system

$= \pm\, r dr d\theta a_\phi$

$= \pm\, r^2 \sin\theta d\theta d\phi a_r$

$= \pm\, r\sin\theta dr d\phi a_\theta$



- **Differential volume** Rect Spherical For Spherical Co-ordinate

$dv = dl_1 dl_2 dl_3$ $dl = dx dy dz$ $dl = \rho d\rho d\phi dz$, $dv = r^2 \sin\theta dr d\theta d\phi$,

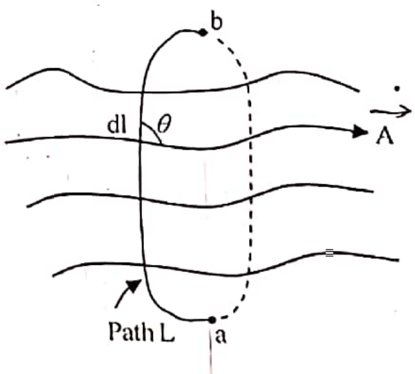
- **Line, Surface and Volume Integrals**

(1) Line Integral $\oint A\,\vec{dl} \rightarrow \oint \vec{A} \cdot \vec{dl}$

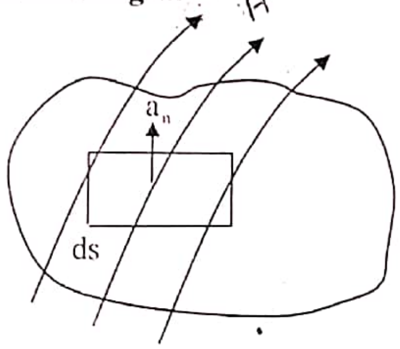
Circulation of vector $\vec{A}$ around the curve L (for closed path)

$\int A.\vec{dl} = \int_a^b |A| \cos\theta \vec{dl}$

Tangential component of $\vec{A}$ along the curve L



(2) **Surface Integral** $\vec{A}$



Vector field A is continuous in a region containing surface S. The surface integral or flux of $\overline{A}$ through surface S is given by

$$\psi = \int_S \overline{A}.a_n ds = \int_S \overline{A}.\overline{ds} = \iint \overline{A}.\overline{ds}$$

Where a_n is Unit normal vector at any point of surface S.
For closed surface

$$\psi = \oiint_S \overline{A}.\overline{ds} = \oint_S \vec{A}\cdot\vec{ds}$$

net outward flux of $\overline{A}$ from the surface S

(4) **Volume Integral**

For a closed surface

$$\iiint f\, dv$$

Scalar

➢ **Del Operator (∇)**

It is a 3–D differential vector operator and it is not a vector in itself but when it is operated on a sealer function it becomes a vector

$$\nabla = \frac{1}{h_1}\frac{\partial}{\partial u} a_u + \frac{1}{h_2}\frac{\partial}{\partial v} a_v + \frac{1}{h_3}\frac{\partial}{\partial w} a_w$$

The del operator useful in defining
(1) ∇f = gradient
(2) $\nabla \times \vec{F}$ = curl
(3) $\nabla . \vec{F}$ = divergence
(4) $\nabla^2 \vec{F}$ = Laplacian

(1) Gradient of a scalar Gradeent is a derivative of multi variable functn

The gradient of a scalar field f is a vector that represents both magnitude and direction

$$\nabla f = \frac{1}{h_1} a_u \frac{\partial f}{\partial u} + \frac{1}{h_2} a_v \frac{\partial f}{\partial v} + \frac{1}{h_3} a_w \frac{\partial f}{\partial w},$$

gradient is a direction to move

Temp of oven. – For making cookies

Note: **(1)** The magnitude of ∇f represents the rate of change of scalar quantity w.r.t given co-ordinate system.

(2) Direction of ∇f represents the direction in which the rate of change has its maximum value (Greatest rate of Increase of function) of a scalar functions

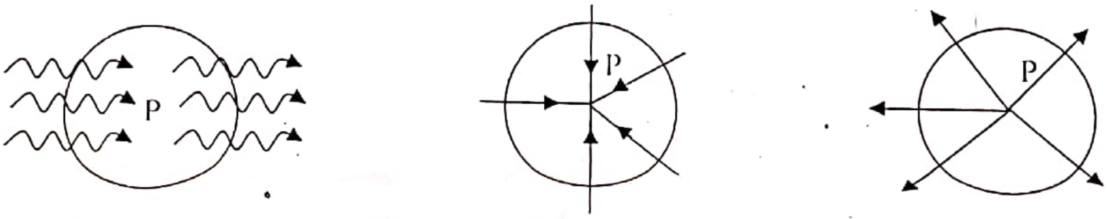
(2) Divergence of a Vector

Divergence of a vector is always a sealer quantity. Which can be defined as net outward flow of flux per unit volume over a closed incremental surface? As the volume shrinks about a point P.

$$\text{div}\,\vec{A} = \nabla.\vec{A} = \lim_{\Delta v \to 0} \frac{\iint \vec{A}.\vec{ds}}{\Delta v}$$

If vector represent velocity of fluid at each point in space, net flow of fluid out of a point

Physically div represent the rate of change of field strength in the direction of field (or any vector)



Zero divergence – divergence (sink) + divergence (source)

- If $\nabla.A = 0$ $\vec{A}$ is **solenoidal**, divergence free, incompressible Fiel
- divergence of vector $\vec{A}$. in the form of Tube

$$\nabla.\vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u}(A_u(h_2h_3)) + \frac{\partial}{\partial v}(A_v h_1 h_3) + \frac{\partial}{\partial w}(A_w h_1 h_2)\right]$$

Where $\vec{A} = \vec{A}_u \vec{a}_u + A_v \vec{a}_v + A_w \vec{a}_w$

moving in twisting on spinning manner

Curl measures the tendency of a vector Field to swirl around

(3) Curl of a Vector

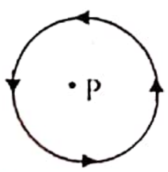
whose

Amount of pushing force along a path

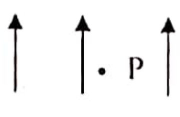
Curl of a vector is defined as axial or rotation vector whereas magnitude is the maximum circulation of vector $\vec{A}$ per unit area. As area $\to$ 0 and direction normal to the area

$$\text{Curl}\,\vec{A} = \nabla \times \vec{A} = \lim_{\Delta S \to 0} \frac{\int \vec{A}.\vec{dl}}{\Delta s}$$

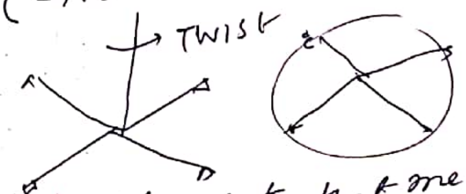
• Velocity vector in to water, and Padding of water movement by spin (stick to a Padde wheel).



Curl about P Exist



Curl about P is zero



if the Paddle rotate that mean Field is uneven on each side Curl will exist

- If $\nabla \times \vec{A} = 0$ field is **irrotational** , Field is conservative

Conservative Field

i A Field is said to be Conservative if its line Integral around a path is independent of Path

* $W = \oint F \cdot dl$ — Conservative independent of path. Energy is Conservative mean. Energy is Conserved.

* Non Conservative — Moving a object over a rough Surface

- $$\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 a_u & h_2 a_v & h_3 a_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

Where $\vec{A} = A_u \vec{a}_u + A_v \vec{a}_v + A_w \vec{a}_w$

(4) Laplacian of a scalar

$$\nabla^2 v = \text{div}(\text{grad } v)$$

$$\nabla^2 v = \nabla.(\nabla v)$$

- If $\nabla^2 v = 0$. The saslar field is always **harmonic**

Q. If $\vec{r} =$ $xa_x + ya_y + za_z$ and $r = |\vec{r}|$

(a) $\nabla r = \frac{\vec{r}}{r}$ (b) $\nabla.\vec{r} = 1$

(c) $\nabla(\vec{r}.\vec{r}) = 6$ (d) $\nabla \times \vec{r} = 0$

Which is the in correct statement.

➢ **Vector Identifies**

(i) $\nabla.(\nabla \times \vec{A}) = 0$

(ii) $\nabla \times (\nabla v) = 0$

(iii) $\nabla.(\nabla v) = \nabla^2 v$

(iv) $\nabla(fg) = f\nabla g + g\nabla f$

(v) $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla.\vec{A}) - \nabla^2 A$

(vi) $\nabla.(\vec{A} \times \vec{B}) = B.(\nabla \times \vec{A}) - \vec{A}.(\nabla \times \vec{B})$

Parallel vector $\vec{a} = k\vec{b}$, $\theta = 0°$ $\vec{a} \times \vec{b} = 0$

orthogonal vector $\vec{a} \cdot \vec{b} = 0$

orthonormal vector $\rightarrow \vec{a} \cdot \vec{b} = 0$ and $|a| = 1$, $|b| = 1$

$[1, 0, -1]$ $[1, \sqrt{2}, 1]$ $[1, -\sqrt{2}, 1]$

➢ **Stokes Theorem**

The line integral of tangential component of $\vec{A}$ around a close path L is equal to the surface integral of normal component of vector $\vec{A}$ over the surface closed by the path L.

$$\oint A.dL = \int_s (\nabla \times \vec{A}).\vec{ds}$$ — From defination of curl

➢ **Divergence Theorem**

Divergence theorem state that the normal component of a vector over a **closed surface** is equal to the volume integral of a divergence of the vector through out volume v and closed by the surface s.

$$\oint \vec{D}.\vec{ds} = \int_v (\nabla.\vec{D})\, dv$$ ⟶ From defination of Divergence

vector Normal to the surface

Q. Verify Divergence theorem for

$\overline{D} = \rho^2 \cos^2 \phi a_\rho + Z \sin \phi a_\phi$

$\rho = 4, \ 0 \le z \le 1$

$\int \vec{D} \cdot \vec{ds}$

$\vec{d_s} = \rho \, d\phi \, dz \, a_\rho$

$\vec{d_s} = d\rho \, dz \, a_\phi$

$\int_0^{2\pi} \int_0^1 3 \cos^2\phi \, d\phi \, dz \Big|_{\rho=4} = 64\pi$

$+ \int_0^4 \int_0^1 z \sin\phi \, d\rho \, dz$

Sol. $\nabla . D = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}\left(\rho^2 \cos^2 \phi\right)\rho + \frac{\partial}{\partial \phi} z \sin \phi\right]$

$= \frac{1}{\rho}\left[3\rho^2 \cos^2 \phi + z \cos \phi\right]$

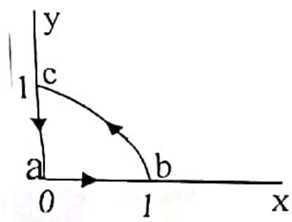
$= 3\rho \cos^2 \phi + \frac{z}{\rho} \cos \phi$

$\int_v \nabla . D \, dv = \int_0^4 \int_0^1 \int_0^{2\pi} \left(3\rho \cos^2 \phi + \frac{z}{\rho} \cos \phi\right) \rho d\rho d\phi dz$

$= 64\pi$

Q. For the given contour calculate the circulation of vector A around the given path.

$\overline{A} = 2\rho \cos \phi a_\rho + \rho a_\phi$



Sol. path ab $\rightarrow \phi = 0, z = 0, \ \rho = 0 \rightarrow 1$

path bc $\rightarrow z = 0, \cancel{\phi = \pi/2}$ $\cancel{z = 0}$ $\rho = 1$ $\phi, 0 - \pi/2$

path ca $\rightarrow \phi = \pi/2, z = 0, \rho = 0 \rightarrow 1$

$\oint \overline{A} \, \overline{dl} = \int \left(2\rho \cos \phi a_\rho + \rho a_\phi\right)\left(d\rho a_\rho + \rho d\phi a_\phi\right)$

$= \int \left(2\rho \cos \phi d\rho + \rho^2 d\phi\right)$

$= 2\frac{\rho^2}{2} \cos \phi + \rho^2 \phi.$

for path ab $= \rho^2 \cos \phi \Big|_{\rho=0, \phi=0}^{1} = 1$

for path bc $= \rho^2 \phi \Big|_0^{\pi/2} = \pi / 2$

for path ca $= \frac{2\rho^2}{2} \cos \phi \Big|_{1, \phi=90^\circ}^{0} = 0$

Circulation $\oint A \overline{dl} = \frac{\pi}{2} + 1$

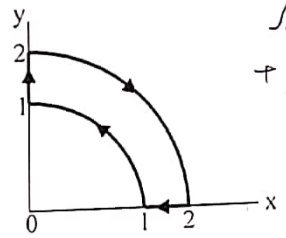
(c) $\oint_C \nabla \times V.dl = \iint_{S_C} \nabla \times V.ds$ (d) $\oint_C \nabla \times A.dl = \iint_{S_C} V.ds$

Q.3.2 If a given field is conservative, then the value of k is

$A = 3x^2yza_x + 2xy^2za_y + (x^3y - kxyz^2)a_z$

(a) – 5 (b) 5 (c) 4 (d) – 4

Q.3.3 $\int \vec{A} \cdot \vec{dl}$ for the contour shown in figure is, where $A = \rho \sin\phi\, a_\rho + \rho^2 a_\phi$



$d\vec{l} = d\rho a_\rho + \rho d\phi a_\phi + dz a_z$

$\int_0^1 \rho \sin\alpha\, d\rho |_{\phi=0} + \int_0^{\pi/2} \rho^3 d\phi |_{\rho=2}$

$+ \int_2^1 \rho \sin\phi\, d\rho |_{\phi=\pi/2} + \int_{\pi/2}^0 \rho^3 d\phi |_{\rho=1}$

$0 + 4\pi + \frac{1}{2} - \frac{1}{4} - \frac{\pi}{2}$

$= \frac{7\pi}{2} + \frac{1}{4} =$

(a) 9.5 (b) – 9.5 (c) 12.5 (d) – 12.5

Q.3.4 Which of the following option is 'TRUE'?

(a) div (grad V) = 0 ✓(b) div (curl $\bar{A}$) = 0

(c) curl (div $\bar{A}$) = 0 (d) grad (curl $\bar{A}$) = 0

Q.3.5 If $\bar{r} = xa_x + ya_y + za_z$, the position vector of point (x, y, z) and r = | r |. Which of the following is INCORRECT?

(a) $\nabla r = \frac{\bar{r}}{r}$ (b) $\nabla^2(\bar{r} \cdot \bar{r}) = 6$ (c) $\nabla \cdot \bar{r} = 1$ (d) $\nabla \times \bar{r} = 0$

Q.3.6 If Laplace's equation $\nabla^2 V = 0$, is satisfied by a scalar field V in a given region, than V is

(a) solenoidal (b) conservative ✓(c) harmonic (d) irrotational