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Tutorial - 2

Ques 1. Lets take $n=4$,

① → $i=1$ $S=1$
$S<=n \Rightarrow 1<=4$
$i=2 \Rightarrow S=2+1=3$

② $S<=4 \Rightarrow 3<=4$
$i=3$, $S=3+3=6$

The S here increases with the Rate of i

$\therefore$ $1+2+3+\ldots\ldots+k$ [k = iteration for which $S<=n$]

$\Rightarrow \frac{k(k+1)}{2} < n$

$\Rightarrow \frac{k^2+k}{2} < n$

$\Rightarrow k^2 = O(\sqrt{n})$

Ques 2. for fibonacci series Recurrence Relation

$T(n) = T(n-1)+T(n-2)+1$, $n>1$

$T(2) = T(2-1)+T(2-2)+1$
$= T(1)+0+1 = 2$

$T(3) = T(3-1)+T(3-2)+1$
$= T(2)+T(1)+1 = 2+2 = 4$

$T.C = 2^0+2^1+2^2+\ldots\ldots+2^n$
$= \frac{2(2^n-1)}{1} = 2(2^n-1)$

$T.C = O(2^n)$

Since the call stack value never rises above n, so, $SC = O(n)$

Ques 3

```
for(int i=0; i<n; i++){
  for(int j=n; j>0; j=j/2){
      cout << "*" <<endl;
  }
}
```

T.C = O(nlogn)

```
for (int i=0; i<n; ++i)
  for (int j=0; j<n; ++j)
    for (int k=0; k<n; ++k)
      cout << "*" <<endl;
```

T.C = $O(n^3)$

```
for (int i=n; i>0; i=i/2)
  for (int j=n; j>0; j=j/2)
      cout << "*" <<endl;
```

T.C = O(log(logn))

Ques 5 Inner loop executes n/i time for each value of i. Its running time is O(nlogn).

Ques 6. In this case i takes value $2, 2^k, 2^{k^2}, 2^{k^3} \ldots\ldots 2^{k^{\log_k(\log(n))}}$

The last term must be less than or equal to n and we have

$$2^{k^{\log_k(\log(n))}} = 2^{\log(n)} = n$$

So there are in total $(\log_k(\log n))$ many iterations

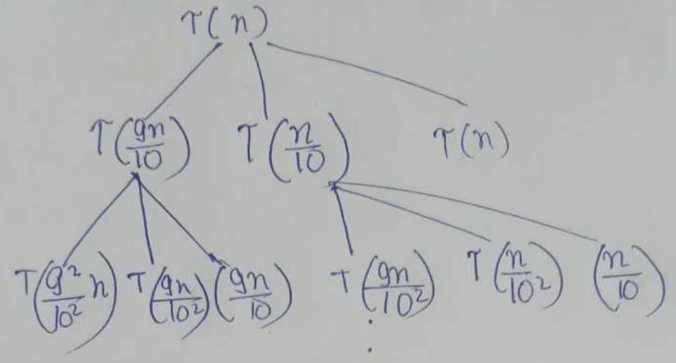
$\therefore$ T.C = O(log(log(n)))

Ques 7. If we split in this manner then the recurrence Relation will be

$$T(n) = T(9n/10) + T(n/10) + O(n)$$

where the first branch of the size $9n/10$ and second one is $\frac{n}{10}$

So - branches have nodes in 9:1 ratio



At 1st level the cost = n

At 2nd level the cost = $\frac{9n}{10} + \frac{n}{10} = n$

So time complexity = Summation of cost of all levels

$= O(n \log_{\frac{10}{9}} n)$ for longer branch

$= \Omega(n \log_{10} n)$ for shorter branch

The base of log does not matter as it is only a matter of constant.

Ques 8. a. $2^{2^n} > n! > 4^n > 2^n > 2^{\sqrt{n}} > n \log n > \log(n!) > n > \log^2 n >$

$\log n > \sqrt{n} > \log(\log n) > 1$

decreasing rate of ~~Cons~~ growth.

for which c(n)

$n! > 2(2^n) > n\log(n) > \log(n!) > 4n > 2n > 2\log(n) > \log 2n > \log n >$
$\sqrt{(\log n)} > \log(\log(n)) > 1$

$8^{2^n} > n! > 7n^3 > 8n^2 > n\log_6(n) > n\log_2(n) > \log(n!) > \log_8(n) >$
$\log_2(n) > 5n > 96$