Tutorial-4

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① $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

Using Master's Method comparing with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$a = 3, \; b = 2, \quad f(n) = n^2$

$k = \log_b a = \log_2 3 \simeq 1.6$

as $f(n) > n^k \Rightarrow n^2 > n^{1.6}$

so $T.C = O(n^2)$

② $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

comparing with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$k = \log_b a = \log_2 4 = 2$

As $f(n) = n^k \Rightarrow n^2 = n^2$

so $T.C = O(n^2 \log n)$

③ $T(n) = T\left(\frac{n}{2}\right) + 2^n$

comparing with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$k = \log_b a = \log_2 1 = 0$

As $f(n) > n^k \Rightarrow 2^n > n^0$

so $T.C = O(2^n)$

4. $T(n) = 2^n T(\frac{n}{2}) + n^n$

Compare it with $T(n) = aT(\frac{n}{b}) + f(n)$

$k = \log_2 2^n \Rightarrow n$

As $f(n) = n^k \Rightarrow n^n = n^n$

T.C = $O(n^n \log n)$

5. $T(n) = 16T(\frac{n}{4}) + n$

Compare it with $T(n) = 16T(\frac{n}{4}) + n$

$k = \log_4 16 = \log_4 4^2 = 2$

As, $f(n) < n^k \Rightarrow n < n^2$

T.C = $O(n^2)$

6. $T(n) = 2T(\frac{n}{2}) + n\log n$

Compare it with $T(n) = aT(\frac{n}{b}) + f(n)$

$k = \log_2 2 = 1$

As $f(n) > n^k \Rightarrow n\log n > n$

T.C = $O(n\log n)$

7. $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}$

Compare it with $T(n) = aT(\frac{n}{b}) + f(n)$

$k = \log_2 2 = 1$

As $f(n) < n^k \Rightarrow \frac{n}{\log n} < n$

$\therefore$ T.C = $O(n)$.

8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

Compare it with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$k = \log_b a = \log_4 2 = \frac{1}{2}$

As $f(n) > n^k \Rightarrow n^{0.51} > n^{0.5}$

$T.C = O(n^{0.51})$

9. $T.C = 0.5\,T\left(\frac{n}{2}\right) + \frac{1}{n}$

Compare it with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$k = \log_b a = \log_2 0.5 = \log_2 \frac{1}{2}$

$= \log_2 1 - \log_2 2 = 0 - 1 = -1$

As $f(n) = n^k \Rightarrow \frac{1}{n} = \frac{1}{n}$

$TC = O\left(\frac{\log n}{n}\right)$

10. $T(n) = 16T\left(\frac{n}{4}\right) + n!$

compare it with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$k = \log_4 16 = 2$

$f(n) > n^k \Rightarrow n! > n^2$

so $TC = O(n!)$

11. $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

Comparing it with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$k = \log_b a = \log_2 4 = 2$

As $f(n) < n^k \Rightarrow \log n < n^2$

so $TC = O(n^2)$

12. $T(n) = \sqrt{n}\, T\left(\frac{n}{2}\right) + \log n$

As comparing with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$a = \sqrt{n}$ so Master method not Applicable.

13. $T(n) = 3T\left(\frac{n}{2}\right) + n$

comparing it with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$