Problem_12_1

November 1, 2021

1 12_2 a

Creating 20×10 matrix A and $20 - vector\ b$

[ ]:
```
using LinearAlgebra
```

[ ]:
```
A = rand(20, 10)
b = rand(20)
```

[ ]:
```
20-element Vector{Float64}:
 0.2193376077737481
 0.828314922678014
 0.9704811257881327
 0.7055605363405624
 0.6422659841164733
 0.454724133046311
 0.5060667439016453
 0.7028353252352335
 0.5814800657829446
 0.17338869672303714
 0.5598290114342481
 0.5935715254953784
 0.7569578117533327
 0.6088065006769172
 0.5796830471346921
 0.004903974537650457
 0.6051702857417391
 0.012758850184721116
 0.7066561156609195
 0.52639779437309
```

1.1 a) Computing $\hat{x}$ using different methods

1.1.1 Using Backslash operator

[ ]:
```
x_hat = A\b
```

[ ]:
```
10-element Vector{Float64}:
 -0.22210324426742428
  0.24708705915627935
 -0.056812938783258035
  0.2246334978710109
 -0.054779798537748774
  0.1591558144547268
  0.3134779053748392
  0.12760724034917437
 -0.10379999413691628
  0.2576878577618959
```

1.1.2 Using $\hat{x} = (A^T A)^{-1} A^T b$

[ ]:
```
x_hat = inv(transpose(A)*A) * (transpose(A)*b)
```

[ ]:
```
10-element Vector{Float64}:
 -0.22210324426742512
  0.24708705915627957
 -0.05681293878325944
  0.22463349787101095
 -0.05477979853774961
  0.1591558144547276
  0.3134779053748393
  0.12760724034917503
 -0.10379999413691543
  0.25768785776189684
```

1.1.3 Using $\hat{x} = A \dagger b$

[ ]:
```
x_hat = pinv(A) * b
```

[ ]:
```
10-element Vector{Float64}:
 -0.22210324426742487
  0.2470870591562793
 -0.056812938783257216
  0.22463349787101106
 -0.05477979853774945
  0.1591558144547265
  0.31347790537483966
  0.12760724034917473
 -0.10379999413691525
  0.2576878577618955
```

1.2 b) To show that $||A(\hat{x}+\delta)-b||^2 > ||A\hat{x}-b||^2$

[ ]:
```
x_hat = A\b
```

[ ]:
```
10-element Vector{Float64}:
 -0.22210324426742428
  0.24708705915627935
 -0.056812938783258035
  0.2246334978710109
 -0.054779798537748774
  0.1591558144547268
  0.3134779053748392
  0.12760724034917437
 -0.10379999413691628
  0.2576878577618959
```

Creating δ as del

[ ]:
```
del = rand(10)
```

[ ]:
```
10-element Vector{Float64}:
 0.07330607570837477
 0.41324790815301404
 0.3048732597190005
 0.4657943479561444
 0.3420299758419425
 0.9048042129924343
 0.018976159790918157
 0.49575036974969544
 0.8588617836315418
 0.8805897813388359
```

Compute $||A(\hat{x}+\delta)-b||^2$

[ ]:
```
lhs = norm((A*(x_hat + del)) - b) ^ 2
```

[ ]:
```
127.19604170629499
```

Compute $||A\hat{x}-b||^2$

[ ]:
```
rhs = norm((A * x_hat) - b)^2
```

[ ]:
```
0.8629962845611574
```

Verifying $||A(\hat{x}+\delta)-b||^2 > ||A\hat{x}-b||^2$

[ ]:
```
if(lhs > rhs)
    print("Hence the statement is Verified")
end
```

```
Hence the statement is Verified
```

Computing for lower values of δ

[ ]:
```
for i in 1:10
    del = del/2
    lhs = norm((A*(x_hat + del)) - b) ^ 2
    rhs = norm((A * x_hat) - b)^2
    if(lhs > rhs)
        print("The statement is True\n")
    end
end
```

```
The statement is True
The statement is True
The statement is True
The statement is True
The statement is True
The statement is True
The statement is True
The statement is True
The statement is True
```