Adaptive Bidirectional Platoon Control Using a Coupled Sliding Mode Control Method

Ji-Wook Kwon and Dongkyoung Chwa

Abstract—**This paper proposes an adaptive bidirectional platoon-control method for an interconnected vehicular system using a coupled sliding mode control (CSMC) to improve the performance and stability of the bidirectional platoon control and to guarantee string stability. The previous work in the field of platoon control is based on two strategies, i.e., the leader–predecessor and bidirectional strategies. In the case of the leader–predecessor strategy, all vehicles should use the information of all the leading and preceding vehicles. On the other hand, the bidirectional strategy uses the information of its neighboring preceding and following vehicles. Due to the drawbacks of the bidirectional strategy, most previous work has preferred to employ the leader–predecessor strategy, which can guarantee stability and improved performance. The bidirectional strategy is, however, advantageous in that its implementation of the actual system becomes much more feasible than that of the leader–predecessor strategy. Thus, to employ the platoon-control law to an actual system, we propose the platoon-control law using a CSMC method for an interconnected vehicular system based on the bidirectional strategy such that the problems arising from communication devices in the previous work can be overcome. In particular, unlike the previous work using the bidirectional strategy, the proposed adaptive platoon-control law can lead to improved control performance of the whole system and can guarantee string stability. The stability analysis and simulation results of the proposed method in the presence of uncertainties and disturbances are included to demonstrate the practical application of the proposed algorithm.**

Index Terms—**Adaptive bidirectional platoon control, coupled sliding mode control (CSMC), interconnected vehicular system, string stability, uncertainties and disturbances.**

I. Introduction

PLATOON control is an interconnected vehicle control maintaining the desired space between adjacent vehicles. This interconnected vehicular system with a platoon control has become a popular research area since there has been much interest in the intelligent transportation system (ITS), the adaptive cruise control (ACC), and the intelligent highway system due to increased traffic congestion and collisions [1]–[17]. In these applications, each vehicle in the interconnected system should be controlled to maintain the desired space between the vehicles and to guarantee the stability of the whole system, which is called *string stability* [3]–[5]. Due to the slinky-type effect [6], the string of the vehicles can be destabilized [6], [7]. Accordingly, the string stability of the interconnected system cannot be guaranteed even when the desired space can be maintained between the neighboring vehicles (i.e., even when the platoon is stable, it may not be string stable) since the stability of the whole system cannot be considered the individual platoon-control algorithm, which does not consider string stability [1], [2].

To guarantee string stability and maintain the desired space, much research has been proposed in [3]–[17]. As stated in [6], there have been two control strategies, i.e., the leader–predecessor and bidirectional strategies. First, the leader–predecessor strategy is a platoon-control mechanism by which the information of the leader and that of the preceding vehicle should be employed. Moreover, these platoon-control algorithms using the leader information have handled the issues of the robustness against the model uncertainties and the disturbances of the interconnected system [3]–[12]. Although the leader information can be useful in achieving string stability, it can be difficult to transfer the leader information to the other vehicles that are far from the leader in real time when the interconnected system consists of many vehicles. Second, the bidirectional strategy has been studied to improve the feasibility of the implementation of the platoon-control laws to the actual interconnected system. Since the information of the adjacent vehicles can be acquired by onboard sensors, the bidirectional strategy can be employed to the actual interconnected system with more ease and with a lower cost than the leader–predecessor strategy. Still, the bidirectional structure suffers from high sensitivity to the length of the platoons and lower performance compared with the leader–predecessor strategy [6], [16], [17]. Due to these weaknesses of the bidirectional structure, whereas most of the platoon-control research work has been based on the leader–predecessor strategy, there has been relatively little research work on the platoon control based on the bidirectional strategy.

In spite of its theoretical drawbacks, the bidirectional strategy can be a good solution in developing practical applications (e.g., the ITS and the ACC) in the sense that the performance can be improved and the sensitivity to the length of the platoon

Manuscript received July 2, 2013; revised September 9, 2013, December 6, 2013, and February 14, 2014; accepted February 23, 2014. Date of publication April 2, 2014; date of current version September 26, 2014. The work of J.-W. Kwon was supported by the Korean Ministry of Science, ICT and Future Planning through the "Information Technology Consilience Creative Program," supervised by the National Information Technology Industry Promotion Agency (NIPA), under Grant NIPA-2014-H0201-14-1001. The work of D. Chwa was supported by the Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology under Grant 2012006233. The Associate Editor for this paper was Y. Wang.

J.-W. Kwon is with Yonsei Institute of Convergence Technology, Yonsei University, Incheon 406-840, Korea.

D. Chwa is with the Department of Electrical and Computer Engineering, College of Information Technology, Ajou University, Suwon 443-749, Korea (e-mail: dkchwa@ajou.ac.kr).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TITS.2014.2308535

can be decreased. In addition, the limitation of the bidirectional strategy arises from the fact that most of the work on platoon control has been based on linear control laws [6]. These linear control methods require the additional procedure of the Jacobian linearization of the nonlinear platoon system. Moreover, the analysis and synthesis of the nonlinear model describing the interconnected system should be more clearly studied since the linear model can only describe the exact information of the system around an equilibrium point. If each vehicle can achieve a robust and uniform tracking performance considering the adjacent vehicles in the interconnected system, then each desired distance between the neighboring vehicles can be maintained and the string stability of the overall system can be guaranteed more easily. In spite of the advantage of the nonlinear control laws, it is reported in [6] that most platoon-control algorithms have been limited to the linear control laws.

With these points in mind, this paper proposes an adaptive bidirectional platoon-control law for an interconnected vehicular system to guarantee string stability using the coupled sliding mode control (CSMC) method developed in [18] and [19]. We first need to choose a coupled sliding surface (CSS) to employ the CSMC method. As in [18] and [19], the sliding surfaces of the vehicles in the interconnected system are coupled in the form of their linear combination. Therefore, the CSS of the ith vehicle should be generated with the distance error of the ith vehicle and those of the following vehicle [i.e., the $(i+1)$th vehicle] in such a way that the magnitude of the error propagation transfer function remains smaller than or equal to one. The finite-time convergence of the CSS to zero can guarantee that the distance error between the vehicles converges to zero and that string stability can be achieved. Moreover, to improve the performance and stability of the platoon system using the proposed bidirectional platoon-control law, the additional control schemes are included in the following way. First, we employ the model-based adaptive sliding mode control mechanism to compensate for the parameter uncertainties and the disturbances in the vehicle dynamics. Second, the leader can be controlled using the same controller as that of the following vehicles, unlike in the previous work in which the leader moves independently with the whole system. This homogeneity of the proposed control law can allow that the same form of the control law can be employed to all the vehicles in the platoon-control system without the additional consideration of the role of the vehicles (e.g., the leader and the follower). This independence of the control law on the role of the vehicles can lead to the easy implementation of the platoon-control law to actual systems.

The contribution of the proposed adaptive bidirectional platoon-control law based on the CSMC can be summarized as follows. First, the implementation of the proposed bidirectional platoon-control law to the actual system can be done in a simpler way than the previous platoon-control laws based on the leader–predecessor strategy. The acquisition of the information using the onboard sensors can solve the problems of the communication devices (e.g., time delay, data drop, and limitation of the bandwidth) that are mainly used in the leader–predecessor strategy. In addition, since the leading vehicle can be controlled by the same control law as the other vehicles, the proposed bidirectional control law can be designed without considering the role of the vehicles. Second, the performance and the stability of the bidirectional strategy can be improved in the following sense. The CSMC can guarantee string stability and the convergence of the tracking error to zero via the convergence of the CSS to zero. The robust nonlinear control scheme can recover the degraded performance of the bidirectional strategy mentioned in the previous work. Moreover, the leading vehicle using the proposed CSMC-based control law can decrease the magnitudes of the errors. Third, the error transfer function representing the error propagation characteristics can be adjusted in terms of the weighting factor in the CSS. Unlike the previous work, in which the error transfer function is dependent on the frequency, the proposed platoon-control law is independent of the frequency in the form of the constant error propagation function. Finally, we can compensate for the model uncertainties, such as the parameter uncertainties and the disturbances for the bidirectional strategy, which can improve the performance of the bidirectional strategy for the actual interconnected system where model uncertainties, estimation errors, disturbances, etc., are present.

This paper is organized as follows. We describe the interconnected vehicular system and the problems considered in this paper in Section II. In Section III, we propose the adaptive bidirectional platoon-control law using the CSMC strategy, which is a homogeneous control scheme for all the vehicles. In order to demonstrate the usefulness of the proposed control algorithm, the simulation results are presented in Section IV. Finally, the conclusions of this study are given in Section V.

II. Model Description and Problem Formulation

In this paper, the vehicles in the interconnected system can be assumed to be described by the nonlinear dynamic model in [4], [9], and [10]. Consider the interconnected vehicular system described in Fig. 1. Here, x_i is the position of the ith vehicle, x_i^d is the desired distance between the adjacent vehicles, the distance (or tracking) error can be chosen as $e_i = (x_{i-1} - x_i) - x_i^d$ for $i = 1, \ldots, n$, and the index of a leading vehicle is set to be 0. Here, it can be noted that x_0^d for the leading vehicle is zero since the leading vehicle should track the reference trajectory. Desired distance x_i^d can be chosen without loss of generality as a fixed constant under a constant spacing policy [5], [7], [8]. This policy can provide advantages such as high vehicle density and low energy consumption. To achieve this policy, we should design the platoon-control law to guarantee more stringent string stability [7], [8]. The following model describes the dynamics of the vehicles in the platoon to be used for the proposed control law:

$$\ddot{x}_i = \frac{u_i - c_i \dot{x}_i^2 - f_i}{M_i} + \delta_i \tag{1}$$

where u_i, c_i, f_i, M_i, and δ_i are the control input, the effective aerodynamic drag coefficient, the rolling resistance friction, the effective inertia, and the system uncertainties/disturbances of the vehicle, respectively. In particular, the uncertainties and disturbances in the dynamics in (1) that can arise from various

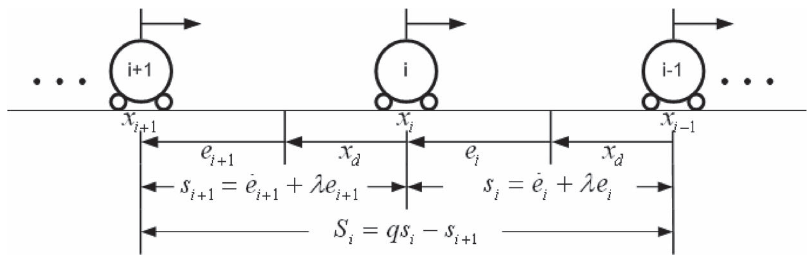


Fig. 1. Interconnected vehicular system.

factors such as the mass of passengers, the engine time constant, the estimated velocities, accelerations, and wind gust [7], [8] are considered in the form of an unknown time-varying function as δ_i. Of course, there have been more complicated dynamic models described by the nonlinear differential equations with the bounded functions with respect to the state variables. However, since the complex dynamic model considers the bounded functions and their unknown bounds can be estimated by using the proposed disturbance estimation method presented later, the dynamic model in (1) can be employed in a reliable way. In this paper, it is assumed that parameters c_i, f_i, and M_i are unknown constants and that the disturbances are bounded as $|M_i\delta_i| \le D_i$ for an unknown positive constant D_i.

From the dynamics of the member vehicles, we can design the bidirectional platoon-control law to achieve the following objectives: 1) each vehicle maintains the desired distance with respect to the preceding vehicle (i.e., stability) and 2) string stability is guaranteed.

To these ends, we will show that all distance errors e_i converge to zero for $i = 1, 2, \ldots, n$ with time such that all vehicles follow the previous vehicle while maintaining desired distance x_i^d and that the string stability defined as in [3] and [4] can be guaranteed.

Definition 1 (String Stability): Origin $x_i = 0$, with $i \in N$ in Fig. 1 and the dynamics in (1), is string stable if, given any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\|e_i(0)\|_\infty < \delta \Rightarrow \sup_i \|e_i(\cdot)\|_\infty < \varepsilon.$$

Definition 2 [Asymptotic (Exponential) String Stability]: Origin $e_i = 0$, with $i \in N$ in Fig. 1 and the dynamics in (1), is asymptotically (exponentially) string stable if it is string stable and $e_i \to 0$ asymptotically (exponentially) for all $i \in N$.

Definition 3 (Strong String Stability): Origin $e_i = 0$, with $i \in N$ in Fig. 1 and the dynamics in (1), is string stable in the strong sense if error propagation transfer function $H_i(s) := E_{i+1}(s)/E_i(s)$ satisfies $|H_i(s)| \le 1$ for all $i \in N$.

Then, it can be shown that the tracking error of the platoon system is uniformly bounded from the fact that, given any $\gamma > 0$, there exists $\delta > 0$ such that

$$\sup_i \max\{|e_i(0)|, |\dot{e}_i(0)|\} < \delta$$
$$\Rightarrow \sup_i \sup_{t \ge 0} \max\{|e_i(t)|, |\dot{e}_i(t)|\} < \gamma.$$

Accordingly, we need to show $|H_i(s)| \le 1$ to prove the string stability of the platoon system in the strong sense using the proposed control laws, which will be done in the following section.

III. Adaptive Bidirectional Platoon-Control Law Using a CSMC Method

In this section, the adaptive bidirectional platoon-control law using the CSS is proposed. Consider the interconnected vehicular system described in Fig. 1 and the dynamics of the vehicles with distance error e_i for $i = 1, \ldots, n$. As mentioned before, the control objective in this section is to make e_i converge to zero as time goes on and to guarantee string stability. We can choose each sliding surface as $s_i = \dot{e}_i + \lambda e_i$, where λ is a positive constant. The time derivative of s_i becomes

$$\dot{s}_i = \ddot{e}_i + \lambda\dot{e}_i = (\ddot{x}_{i-1} - \ddot{x}_i) + \lambda\dot{e}_i. \tag{2}$$

Since the information of the preceding and following vehicles can be filtered to obtain its time derivatives, as in Section IV, there is no loss of generality in assuming that the time derivatives of the position and velocity are available. Moreover, since the convergence of sliding surface s_i to zero cannot guarantee string stability, we choose the CSS of the ith vehicle for the control of the total platoon system as

$$S_i = qs_i - s_{i+1} \tag{3}$$

where $q > 0$ is a weighting factor. Since s_{n+1} does not exist in the case of the last vehicle (i.e., $i = n$), we set $s_{n+1} = 0$. It should be noted here that S_i is chosen in such a way that, when S_i becomes zero for all $i = 1, \ldots, n$, at the same time, s_i also becomes zero for all $i = 1, \ldots, n$, as shown in the following lemma.

Lemma 1 (Equivalence of the Convergence of the CSS and Each Sliding Surface Toward Zero): S_i becomes zero for all $i = 1, \ldots, n$ if and only if s_i becomes zero for all $i = 1, \ldots, n$ at the same time.

Proof: To show that sliding surface s_i can be zero if and only if S_i becomes zero for $i = 1, \ldots, n$, we can describe the relationship between S_i and s_i in (3) as $\mathbf{S} = \mathbf{Q}\mathbf{s}$, where

$$\mathbf{s} = [\, s_1 \quad s_2 \quad \cdots \quad s_n \,]^T$$
$$\mathbf{S} = [\, S_1 \quad S_2 \quad \cdots \quad S_n \,]^T$$
$$\mathbf{Q} = \begin{bmatrix} q & -1 & \cdots & 0 & 0 \\ 0 & q & -1 & \cdots & 0 \\ & & \vdots & & \\ 0 & 0 & \cdots & q & -1 \\ 0 & 0 & \cdots & 0 & q \end{bmatrix}.$$

Since $q > 0$ is constant, $\mathbf{Q}$ is invertible. Accordingly, it follows the equivalence of $\mathbf{S} = 0$ and $\mathbf{s} = 0$. (Q.E.D.)

As shown in Lemma 1, we should design the control law to make the CSS converge to zero. First, the time derivative of S_i in (3) can be described as

$$\begin{aligned}\dot{S}_i &= q\dot{s}_i - \dot{s}_{i+1}\\ &= q\left\{\ddot{x}_{i-1} - \ddot{x}_i + \lambda\dot{e}_i^n\right\} - \left\{\ddot{x}_i - \ddot{x}_{i+1} + \lambda\dot{e}_{i+1}^n\right\}\\ &= \frac{(q+1)}{M_i}\left(u_i - c_i\dot{x}_i^2 - f_i + \delta_i'\right) + A_i \end{aligned} \tag{4}$$

where $\delta_i' = M_i\delta_i$, and $A_i = q\ddot{x}_{i-1} + \ddot{x}_{i+1} + \lambda(q\dot{e}_i^n - \dot{e}_{i+1}^n)$. In Fig. 1 and (1)–(4), we can design the novel adaptive platoon control law that guarantees the stability of each vehicle and the string stability of the whole interconnected system as

$$\begin{aligned}u_i &= \hat{c}_i\dot{x}_i^2 + \hat{f}_i + \hat{D}_i\mathrm{sgn}(S_i)\\ &\quad + \frac{\hat{M}_i}{q+1}A_i + \frac{k}{q+1}S_i + \frac{\bar{k}}{q+1}\mathrm{sgn}(S_i)\end{aligned} \tag{5}$$

where $\bar{k}$ and k are positive constants, and $\hat{c}_i$, $\hat{f}_i$, $\hat{D}_i$, and $\hat{M}_i$ are the estimates of c_i, f_i, D_i, and M_i, respectively. The parameter adaptation laws for the unknown parameters and disturbance can be determined as

$$\dot{\hat{c}}_i = \gamma_i^c(q+1)S_i\dot{x}_i^2 \tag{6a}$$
$$\dot{\hat{f}}_i = \gamma_i^f(q+1)S_i \tag{6b}$$
$$\dot{\hat{D}}_i = \gamma_i^D(q+1)|S_i| \tag{6c}$$
$$\dot{\hat{M}}_i = \gamma_i^M A_i S_i \tag{6d}$$

where design parameters γ_i^c, γ_i^f, γ_i^D, and γ_i^M are positive constants for $i = 1, \ldots, n$. In the case of the last vehicle that does not have a following vehicle, the control input and adaptive laws of the estimates of the unknown parameters can be expressed as

$$\begin{aligned}u_n &= \hat{c}_n\dot{x}_n^2 + \hat{f}_n + \hat{D}_n\mathrm{sgn}(S_n) + \frac{\hat{M}_n}{q}A_n\\ &\quad + \frac{1}{q}\left\{kS_n + \bar{k}_n\mathrm{sgn}\left(S_n\right)\right\}\end{aligned} \tag{7}$$

where $S_n = qs_n$, $A_n = q\lambda\dot{e}_n$, and

$$\dot{\hat{c}}_n = \gamma_n^c qS_n\dot{x}_n^2 \tag{8a}$$
$$\dot{\hat{f}}_n = \gamma_n^f qS_n \tag{8b}$$
$$\dot{\hat{D}}_n = \gamma_n^D q|S_n| \tag{8c}$$
$$\dot{\hat{M}}_n = \gamma_n^M A_n S_n. \tag{8d}$$

By using the proposed adaptive platoon-control law in (5), the CSS can converge to zero in finite time, as in the following theorem.

Theorem 1 (Tracking Performance of the Adaptive Bidirectional Platoon-Control Law Using CSMC): When the control law in (5) and the adaptive rules in (6) are employed to the interconnected vehicular system in Fig. 1, the stability of the vehicles can be guaranteed in the sense that sliding surfaces S_i and s_i, and the distance error variables e_i for $i = 1, \ldots, n$ converge to zero asymptotically.

Proof: To show the stability of the vehicular system, we choose the Lyapunov function candidate as

$$V = \sum_{i=1}^{n} V_i \tag{9}$$

where $V_i = (M_i/2)S_i^2 + (\tilde{c}_i^2/2\gamma_i^c) + (\tilde{f}_i^2/2\gamma_i^f) + (\tilde{D}_i^2/2\gamma_i^D) + (\tilde{M}_i^2/2\gamma_i^M)$. Estimation errors $\tilde{c}_i$, $\tilde{f}_i$, $\tilde{D}_i$, and $\tilde{M}_i$ are defined as $\tilde{c}_i := c_i - \hat{c}_i$, $\tilde{f}_i := f_i - \hat{f}_i$, $\tilde{D}_i := D_i - \hat{D}_i$, and $\tilde{M}_i := M_i - \hat{M}_i$, respectively. Using (4), the time derivative of the Lyapunov function candidate for each vehicle can be obtained as

$$\begin{aligned}\dot{V}_i &= M_iS_i\dot{S}_i + \frac{\tilde{c}_i\dot{\tilde{c}}_i}{\gamma_i^c} + \frac{\tilde{f}_i\dot{\tilde{f}}_i}{\gamma_i^f} + \frac{\tilde{D}_i\dot{\tilde{D}}_i}{\gamma_i^D} + \frac{\tilde{M}_i\dot{\tilde{M}}_i}{\gamma_i^M}\\ &= M_iS_i\left\{-\frac{(q+1)}{M_i}\left(u_i - c_i\dot{x}_i^2 - f_i + \delta_i'\right) + A_i\right\}\\ &\quad + \frac{\tilde{c}_i\dot{\tilde{c}}_i}{\gamma_i^c} + \frac{\tilde{f}_i\dot{\tilde{f}}_i}{\gamma_i^f} + \frac{\tilde{D}_i\dot{\tilde{D}}_i}{\gamma_i^D} + \frac{\tilde{M}_i\dot{\tilde{M}}_i}{\gamma_i^M}.\end{aligned} \tag{10}$$

Substituting (5) and (6) into (10) gives

$$\begin{aligned}\dot{V}_i &= M_iS_i\left[\frac{(q+1)\tilde{c}_i\dot{x}_i^2}{M_i} + \frac{(q+1)\tilde{f}_i}{M_i}\right.\\ &\qquad - \frac{(q+1)\delta_i'}{M_i} - \frac{(q+1)\hat{D}_i\mathrm{sgn}(S_i)}{M_i}\\ &\qquad \left. + A_i - \frac{\hat{M}_i}{M_i}A_i - \frac{k}{M_i}S_i - \frac{\bar{k}}{M_i}\mathrm{sgn}(S_i)\right]\\ &\quad + \frac{\tilde{c}_i\dot{\tilde{c}}_i}{\gamma_i^c} + \frac{\tilde{f}_i\dot{\tilde{f}}_i}{\gamma_i^f} + \frac{\tilde{D}_i\dot{\tilde{D}}_i}{\gamma_i^D} + \frac{\tilde{M}_i\dot{\tilde{M}}_i}{\gamma_i^M}\\ &\le (q+1)\tilde{c}_i\dot{x}_i^2S_i + (q+1)\tilde{f}_iS_i - (q+1)\tilde{D}_i|S_i|\\ &\quad + \tilde{M}_iA_iS_i - kS_i^2 - \bar{k}|S_i| - \frac{\tilde{c}_i\dot{\hat{c}}_i}{\gamma_i^c} - \frac{\tilde{f}_i\dot{\hat{f}}_i}{\gamma_i^f}\\ &\quad - \frac{\tilde{D}_i\dot{\hat{D}}_i}{\gamma_i^D} - \frac{\tilde{M}_i\dot{\hat{M}}_i}{\gamma_i^M}\\ &= -kS_i^2 - \bar{k}|S_i|.\end{aligned} \tag{11}$$

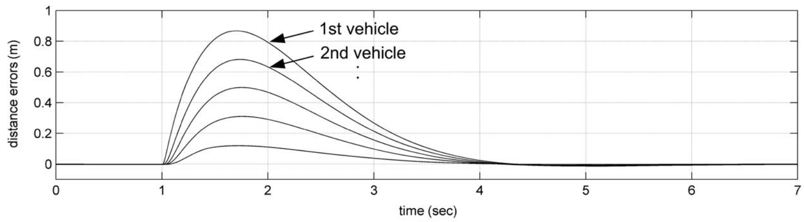
Therefore, it follows that $\dot{V} = \sum_{i=1}^{n}\dot{V}_i \le \sum_{i=1}^{n}\{-kS_i^2 - \bar{k}|S_i|\}$. Thus, $V(t)$ is bounded for all times, and accordingly, S_i, $\tilde{c}_i$, $\tilde{f}_i$, $\tilde{D}_i$, $\tilde{M}_i \in L_\infty$. This yields $\hat{c}_i$, $\hat{f}_i$, $\hat{D}_i$, $\hat{M}_i \in L_\infty$. Since $V_i(0)$ is bounded and $V_i(t)$ is nonincreasing and bounded, it can be shown by Barbalat's Lemma [20]–[22] that S_i converges to zero.

Then, we choose the following Lyapunov function:

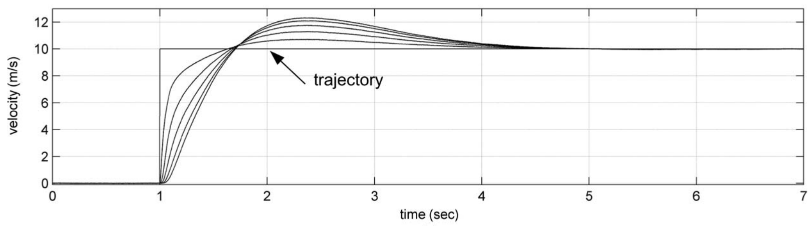
$$\begin{aligned}\bar{V}_i &= V_i - \frac{\tilde{c}_i^2}{2\gamma_i^c} - \frac{\tilde{f}_i^2}{2\gamma_i^f} - \frac{\tilde{D}_i^2}{2\gamma_i^D} - \frac{\tilde{M}_i^2}{2\gamma_i^M}\\ &= \frac{M_i}{2}S_i^2\end{aligned}$$

whose time derivative becomes

$$\begin{aligned}\dot{\bar{V}}_i = &-kS_i^2 - \bar{k}|S_i| + \tilde{c}_i(q+1)S_i\dot{x}_i^2\\ &+ \tilde{f}_i(q+1)S_i + \tilde{D}_i(q+1)|S_i| + \tilde{M}_iA_iS_i.\end{aligned} \tag{12}$$



(a)



(b)

Fig. 2. Performance of the proposed bidirectional control law based on CSMC when the leader control is achieved by considering the following vehicle. (a) Distance errors. (b) Velocities.

From the fact that $\tilde{c}_i$, $\tilde{f}_i$, $\tilde{D}_i$, $\tilde{M}_i \in L_\infty$, it can be concluded that $|\tilde{c}_i| \leq \bar{C}_i$, $|\tilde{f}_i| \leq \bar{F}_i$, $|\tilde{D}_i| \leq \bar{D}_i$, and $|\tilde{M}_i| \leq \bar{M}_i$ for positive constants $\bar{C}_i$, $\bar{F}_i$, $\bar{D}_i$, and $\bar{M}_i$, respectively. When $\bar{k}$ satisfies

$$\bar{k} \geq (q+1)\bar{C}_i \left|\dot{x}_i^2\right| + (q+1)\bar{F}_i + (q+1)\bar{D}_i + \bar{M}_i|A_i| + \bar{\xi}_i \quad (13)$$

for a constant $\bar{\xi}_i > 0$, (12) becomes

$$\begin{aligned}\dot{\bar{V}}_i =& -kS_i^2 - \bar{k}|S_i| + \tilde{c}_i(q+1)S_i\dot{x}_i^2 + \tilde{f}_i(q+1)S_i \\ &+ \tilde{D}_i(q+1)|S_i| + \tilde{M}_iA_iS_i \\ \leq& -kS_i^2 - \bar{\xi}_i|S_i| \leq -\bar{\xi}_i\sqrt{2\bar{V}_i}. \end{aligned} \quad (14)$$

The Lyapunov function of the whole system can be arranged as

$$\begin{aligned}\dot{\bar{V}} \leq& \sum_{i=1}^{n}\left(-\bar{\xi}_i\sqrt{2\bar{V}_i}\right) \leq -\sqrt{2}\underline{\xi_i}\sum_{i=1}^{n}\left(\sqrt{\bar{V}_i}\right) \\ \leq& -\sqrt{2}\underline{\xi_i}\sqrt{\sum_{i=1}^{n}\bar{V}_i} = -\sqrt{2}\underline{\xi_i}\sqrt{\bar{V}} \end{aligned} \quad (15)$$

where $\underline{\xi}_i$ satisfies $\underline{\xi}_i := \min_i \bar{\xi}_i$ such that $0 < \underline{\xi}_i \leq \bar{\xi}_i$ for all i. Thus, we can see that S_i can converge to zero in finite time. In (8)–(15), $V(t)$ is bounded for all times, and the finite-time convergence of $V(t)$ (i.e., S_i) toward zero can be shown. This, in turn, implies that S_i and e_i in Lemma 1 asymptotically converge to zero for all i [21], [22]. (Q.E.D.)

In Theorem 1, the asymptotic stability of the distance error of each vehicle is guaranteed by the proposed control law. In addition, the string stability of the proposed control law can be shown, as in the following theorem, based on the previous definition of the string stability.

Remark 1: When saturation function $\mathrm{sat}(a)$ defined as $\mathrm{sat}(a) = \mathrm{sgn}(a)$ for $|a| \geq 1$ and $\mathrm{sat}(a) = a$ for $|a| < 1$ is used in the control input [see (5)] instead of the signum function, the ultimate boundedness of the position errors of the vehicles can be easily guaranteed instead of their asymptotic convergence. Furthermore, the ultimate bound can be easily adjusted by the width of the boundary layer such that satisfactory performance can be maintained as well [21], [22].

Remark 2: The finite-time stability is not dependent on the number of vehicles, which can be seen from the fact that $\underline{\xi}_i := \min_i \bar{\xi}_i$ determines the finite-time stability, as in (15), and $\bar{\xi}_i$ can be chosen as an appropriate positive number irrespective of the number of vehicles (i.e., $\underline{\xi}_i$ does not decrease toward zero, even when the number of vehicles becomes quite large).

Theorem 2 (String Stability of the Adaptive Bidirectional Platoon-Control System Using CSMC): The proposed bidirectional platoon-control law using CSMC in (5) for the interconnected system in Theorem 1 can guarantee the strong string stability for all $0 < q \leq 1$ after some finite time.

Proof: To analyze the string stability of the platoon system using the proposed adaptive bidirectional control law based on the CSMC method in Theorem 2, we will show that $H_i(s)$ satisfies $|H_i(s)| \leq 1$ for all $i \in N$ (refer to Definition 3) after some finite time. Since $S_i = qs_i - s_{i+1} = 0$ can be reached in finite time, we can get the relationship described as $q(\dot{e}_i + \lambda e_i) = (\dot{e}_{i+1} + \lambda e_{i+1})$, which, in turn, can be changed via the Laplace transform as $(s+\lambda)E_{i+1}(s) = q(s+\lambda)E_i(s)$, i.e., $E_{i+1}(s) = qE_i(s)$. Thus, the string stability is always satisfied for $0 < q \leq 1$. In particular, the nonlinear system reduces to a linear time-invariant system in finite time, in which case it can be analyzed that the magnitude of the frequency response of the ratio between the previous and following distance errors is not dependent on the frequency at all. In addition, we can adjust the error propagation rate by choosing weighting factor q appropriately. (Q.E.D.)

It has been shown that the proposed platoon-control law can improve the performance and stability of the bidirectional strategy. We can achieve the convergence of the distance errors and the reduction of the magnitudes of the errors. Yet, these results cannot imply that the proposed control law can solve the problem that the error of the first vehicle can grow by the increment of the number of the vehicles. Nevertheless, we have also shown that the control input that increases with the increasing number of vehicles can be reduced by the bidirectional strategy when the leader control is achieved by considering

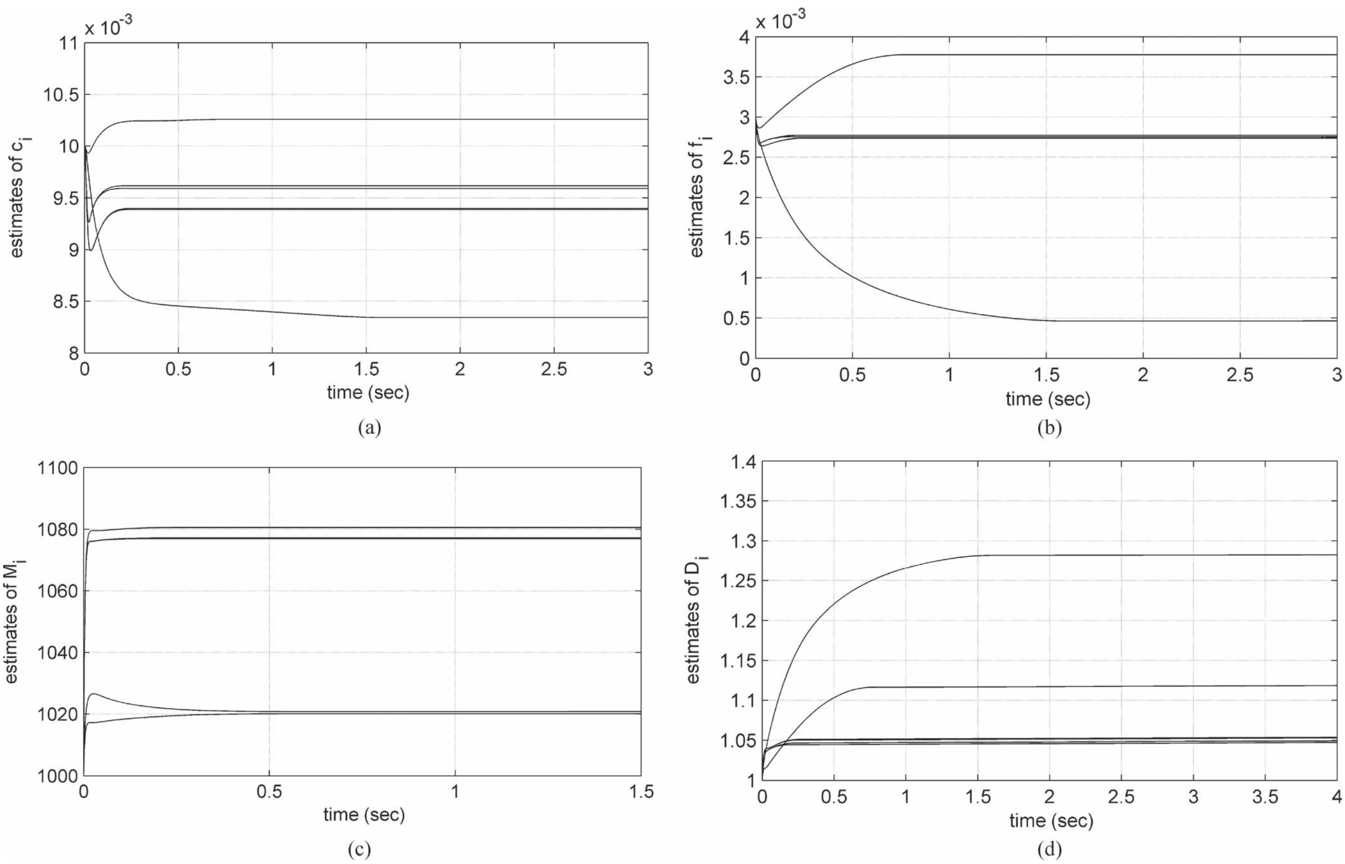


Fig. 3. Estimates of the parameter uncertainties and disturbances. (a) Estimate of c_i. (b) Estimate of f_i. (c) Estimate of M_i. (d) Estimate of D_i.

the following vehicle. As mentioned in Section I, the proposed bidirectional platoon-control law can be employed to the actual interconnected vehicular system due to advantages such as the easy implementation of the actual system, the improvement of performance and stability, easier analysis and synthesis of the string stability, and the robustness against the uncertainties and disturbances. Unlike most of the other bidirectional control methods that only need the position and velocity of the neighboring vehicles, the proposed controller for each vehicle needs the acceleration information of the preceding and following vehicles. It should be noted that the acceleration can be acquired via filtering the information of the neighbor vehicles as mentioned before, which is also done in the simulation results. To verify these points more clearly, simulation results are provided in the next section.

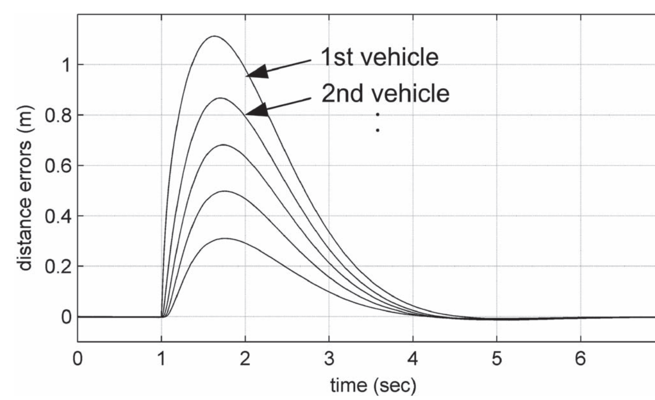


Fig. 4. Performance of the bidirectional strategy when the leader control is achieved without considering the following vehicle.

IV. Simulation Results

For the numerical simulations, we consider the interconnected vehicular systems in Fig. 1, each of which consists of six vehicles, including the leading vehicle, i.e., $n = 5$. To show that the platoon system using the proposed control law can guarantee the string stability and maintain the robustness against the model uncertainties and the disturbance, the following conditions are chosen. The desired distance between the neighboring vehicles x_i^d is 3 m, and the trajectory of the leading vehicle is $v_r(t) = 10$ m/s, which, of course, can be adjusted to other appropriate values without changing the performance of the proposed method. Although the trajectory velocity in the form of the unit step function may not be practical in the real world, we have chosen the trajectory velocity as such in order to show the performance of the proposed platoon algorithm more clearly. We choose the control parameters in the proposed control law as $\lambda = 1$, $k_i = 33$, $\bar{k} = 4$, and $q = 0.99$; the system parameters as $M_i = 1100$, $f_i = 0.001$, $c_i = 0.008$, and $\delta_i(t) = \sin(t)$; and the constant gains for the estimates of the uncertainties as $\gamma_i^c = 10^{-5}$, $\gamma_i^f = 10^{-5}$, $\gamma_i^M = 10^{-3}$, and $\gamma_i^D = 10^{-4}$. The initial conditions of the distance errors of all vehicles are set to be zero without a loss of generality since the effect of the propagated distance error does appear in this case as well. In addition, to estimate the velocity and acceleration of the preceding and following vehicles, we use a differentiator combined

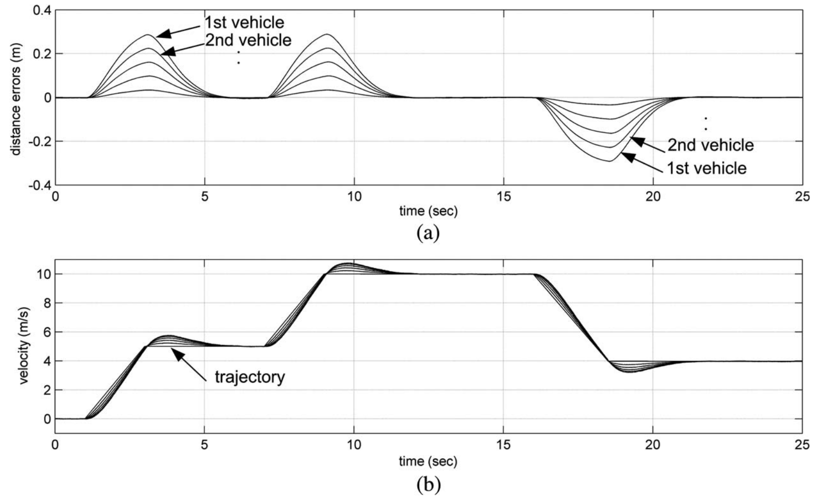


Fig. 5. Performance of the proposed bidirectional control law based on CSMC for the trajectory with abruptly changing acceleration and deceleration. (a) Distance errors. (b) Velocities.

with a low-pass filter described as $\alpha s/(s+\alpha)$, where α is a positive constant and can adjust the bandwidth of the low-pass filter. In this scenario, we choose $\alpha = 75$. Finally, we choose the initial conditions of the estimates of the system uncertainties as $\hat{c}_i(0) = 0.01$, $\hat{f}_i(0) = 0.003$, $\hat{M}_i(0) = 1000$, and $\hat{D}_i(0) = 1$.

To show the performance of the proposed platoon-control law, the simulation results can be compared with the results of the linear and nonlinear bidirectional platoon-control methods in [6], [16], and [17]. Fig. 2 shows the performance of the proposed method. We can see in Fig. 2(a) and (b) that the distance errors converge to zero and that the velocities of the vehicles follow the trajectory of the leading vehicle. To show the improved performance of the proposed control law, the results presented in [6], [16], and [17] can be compared. Even with the employment of the nonlinear control law, the errors of the bidirectional strategy in [16] and [17] oscillate and cannot converge to zero. From the comparison of the errors in Fig. 2(a) and (b) and the results in [6], [16], and [17], we can see that the performance of the platoon system can be improved by the proposed control law. Of course, the main objective of the work in [6] is to study the limitation of the distributed control in terms of the disturbance-to-output norm, instead of designing a better controller; here, we have referred to the results in [6] for a clearer comparison of the performance. In Fig. 3, the results of the estimation of the unknown parameters in the system model of the vehicles are presented. Fig. 3(a)–(d) shows the estimates of c_i, f_i, M_i, and D_i, respectively. The estimates in the figures converge to constants and are guaranteed to be bounded by the adaptive sliding mode control. To compare the performance of the proposed homogeneous control law and that of the role-dependent control law (i.e., the platoon control with the leader independent of the following vehicle), Fig. 4 shows the distance errors of the platoon system when the leader control is achieved without considering the following vehicle [3]–[12]. As shown in Figs. 2(a) and 4, the distance error values of the vehicles based on the proposed control law are smaller than those of the previous vehicle. From this comparison with the strategies of the leader control, it can be said that the proposed control scheme can reduce the maximum value of the magnitude of the distance errors.

In order to demonstrate the advantage of the proposed method, we have performed the additional simulation for a scenario with more abruptly changing acceleration, as in Fig. 5(b), which is similar to that used in [7]. As shown in Fig. 5(a), although the distance errors occur during the transient time due to the acceleration and deceleration of the reference trajectory, the string stability is maintained and the distance errors converge to zero. In addition, as shown in Figs. 2(b) and 5(b), acceleration control input u_i first becomes positive, then negative and, finally, zero. Fig. 6 shows the acceleration performance of the vehicles, in which the reference acceleration can be derived from the reference velocity in Fig. 5(b), and the appropriate acceleration and deceleration motion has been achieved to achieve the control objective. The vehicles are assumed to use both throttle and brake systems in such a way that the sign of the acceleration control input can be adjusted in an appropriate way, which is reasonable in practice. In addition, the maximum values of the velocities in Figs. 2(b) and 5(b) converge to a certain value along the vehicular stream. It can be concluded from the convergence of the overshoots of the velocities that the effect of the leading vehicles might be reduced, even when the index number increases; that is, the platoon system employing the proposed control law is scalable. Fig. 7(a)–(d) shows the estimates of c_i, f_i, M_i, and D_i, respectively, and the estimates converge to constants. Here, it should be noted that the convergence of the parameter estimates to the true estimates is well known to be not essential in obtaining the satisfactory performance of the adaptive control [21].

These results show that, when the proposed control law can be employed to the platoon system, we can ensure that the control objectives can be achieved, even in the presence of the acceleration and deceleration of the reference trajectory, in such a way that the performance of the bidirectional strategy can be improved and the magnitudes of the errors can be reduced.

V. Conclusion

We proposed the adaptive bidirectional platoon-control law based on CSMC for an interconnected vehicular system. The proposed platoon-control law could improve the performance

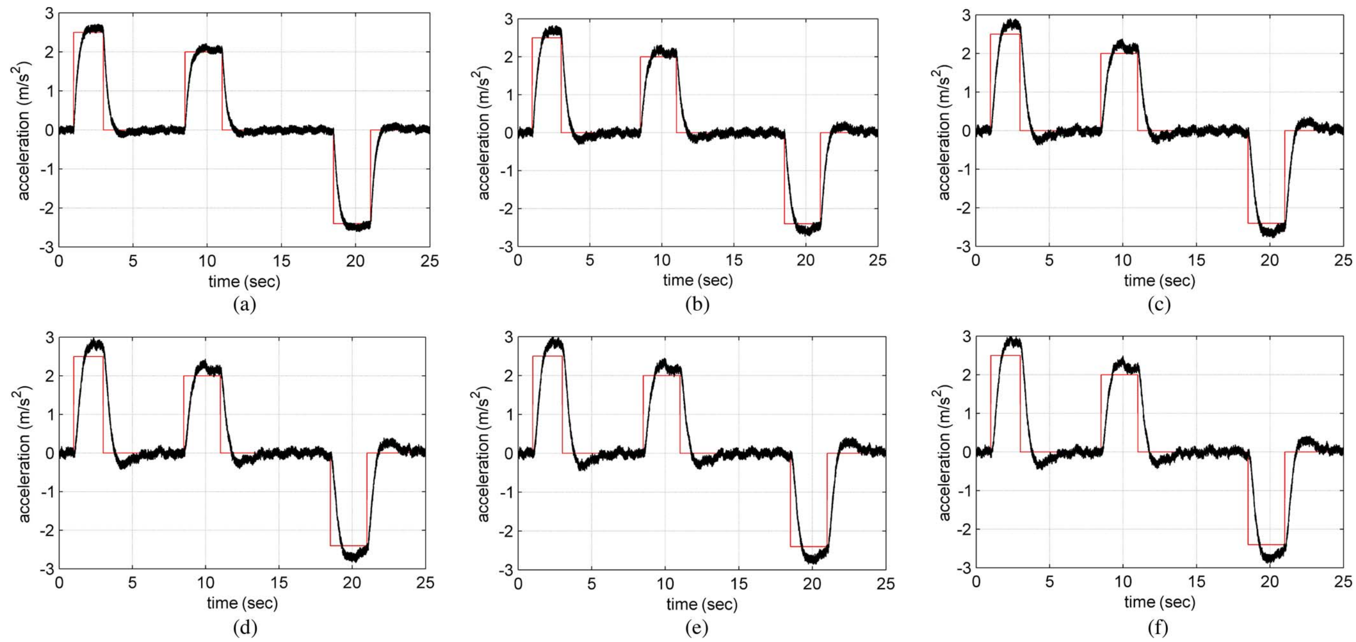


Fig. 6. Accelerations of the vehicles and the trajectory acceleration. (a) Leader vehicle. (b) First vehicle. (c) Second vehicle. (d) Third vehicle. (e) Fourth vehicle. (f) Fifth vehicle.

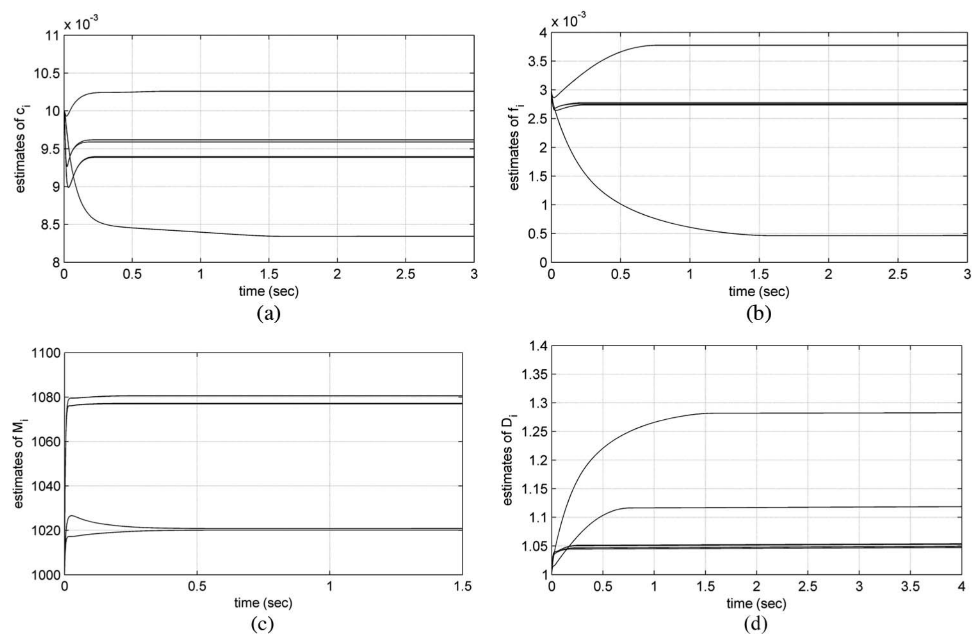


Fig. 7. Estimates of the parameter uncertainties and disturbances. (a) Estimate of c_i. (b) Estimate of f_i. (c) Estimate of M_i. (d) Estimate of D_i.

and stability of the bidirectional strategy. To these ends, we employed the adaptive CSMC method with the leader controlled, considering the following vehicle. From the proposed control law, we can ensure that the bidirectional strategy can achieve the high feasibility of the implementation of the actual interconnected vehicular system in virtue of the acquisition of the information using the onboard sensors, the homogeneity of the control law against the role of the vehicle, and the improvement of the degraded performance and scalability. The analysis of the stability and performance of the proposed control strategy, and simulation results, were provided. In future research work, the issue with regard to the solution to the degraded scalability of the bidirectional strategy can be pursued in order to employ the platoon-control laws to an actual interconnected vehicular system, such that the capability of the traffic system can be improved and traffic congestion and collisions can be prevented. Moreover, the issue with regard to the recognition of neighboring vehicles in the actual interconnected system using a vision system can be pursued in order to acquire the information of the adjacent vehicles. Of course, since the proposed method does not rigorously consider the time delay in the state measurement and estimation, a more analytic method may have to be developed considering a more practical situation, which can be pursued in the future. Finally, a formation control using the CSS that considers the formation stability in 2-D space can be studied and applied to a system with a more general polygonal structure through the extension of the presented platoon-control work to the mesh case, in which the practical issues can be pursued through real experiments involving actual vehicle systems.

References

[1] J. E. Naranjo, C. González, J. Reviejo, R. García, and T. D. Pedro, "Adaptive fuzzy control form inter-vehicle gap keeping," *IEEE Trans. Intell. Transp. Syst.*, vol. 4, no. 3, pp. 132–142, Sep. 2003.

[2] A. Ferrara and P. Pisu, "Minimum sensor second-order sliding mode longitudinal control of passenger vehicles," *IEEE Trans. Intell. Transp. Syst.*, vol. 5, no. 1, pp. 20–32, Mar. 2004.

[3] D. Swaroop and J. K. Hedrick, "String stability of interconnected systems," *IEEE Trans. Autom. Control*, vol. 41, no. 3, pp. 349–357, Mar. 1996.

[4] D. Swaroop, J. K. Hedrick, and S. B. Choi, "Direct adaptive longitudinal control of vehicle platoons," *IEEE Trans. Veh. Technol.*, vol. 50, no. 1, pp. 150–161, Jan. 2001.

[5] P. Seiler, A. Pant, and K. Hedrick, "Disturbance propagation in vehicle strings," *IEEE Trans. Autom. Control*, vol. 49, no. 10, pp. 1835–1841, Oct. 2004.

[6] H. Hao and P. Barooah, "Stability and robustness of large platoons of vehicles with double-integrator models and nearest neighbor interaction," *Int. J. Robust Nonlin. Control*, vol. 23, no. 18, pp. 2097–2122, Dec. 2013. [Online]. Available: http://onlinelibrary.wiley.com/doi/10.1002/rnc.2872/abstract

[7] G. Guo and W. Yue, "Autonomous platoon control allowing range-limited sensors," *IEEE Trans. Veh. Technol.*, vol. 61, no. 7, pp. 2901–2912, Sep. 2012.

[8] G. Guo and W. Yue, "Hierarchical platoon control with heterogeneous information feedback," *IET Control Theory Appl.*, vol. 5, no. 15, pp. 1766–1781, Oct. 2011.

[9] C.-M. Lin and C.-H. Chen, "Car-following control using recurrent cerebellar model articulation controller," *IEEE Trans. Veh. Technol.*, vol. 56, no. 6, pp. 3660–3672, Nov. 2007.

[10] S. S. Stankovic, M. J. Stanojevic, and D. D. Siljak, "Decentralized overlapping control of a platoon of vehicles," *IEEE Trans. Control Syst. Technol.*, vol. 8, no. 5, pp. 816–832, Sep. 2000.

[11] W. B. Dunbar and D. S. Caveny, "Distributed receding horizon control of vehicle platoons: Stability and string stability," *IEEE Trans. Autom. Control*, vol. 57, no. 3, pp. 620–633, Mar. 2012.

[12] P. Barooah and J. P. Hespanha, "Error amplification and disturbance propagation in vehicle string with decentralized linear control," in *Proc. IEEE Conf. Decision Control, Eur. Control Conf.*, Seville, Spain, Dec. 2005, pp. 4964–4969.

[13] A. Renfrew, A. Gonzalez-Villasenor, and P. Brunn, "Contact bridging dampers for back-up on automated highways," *IET Intell. Transp. Syst.*, vol. 1, no. 2, pp. 167–175, Jun. 2007.

[14] J. Zhou and H. Peng, "Range policy of adaptive cruise control vehicles for improved flow stability and string stability," *IEEE Trans. Intell. Transp. Syst.*, vol. 6, no. 2, pp. 229–237, Jun. 2005.

[15] L. Xiao and F. Gao, "Practical string stability of platoon of adaptive cruise control vehicles," *IEEE Trans. Intell. Transp. Syst.*, vol. 12, no. 4, pp. 1184–1194, Dec. 2011.

[16] R. H. Middleton and J. H. Braslavsky, "String instability in classes of linear time invariant formation control with limited communication range," *IEEE Trans. Autom. Control*, vol. 55, no. 7, pp. 1519–1530, Jul. 2010.

[17] A. Ghasemi, R. Kazemi, and S. Azadi, "Stable decentralized control of platoon of vehicles with heterogeneous information feedback," *IEEE Trans. Veh. Technol.*, vol. 62, no. 2, pp. 4299–4308, Nov. 2013. [Online]. Available: http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=6482670

[18] M.-S. Park and D. Chwa, "Swing-up and stabilization control of inverted-pendulum systems via coupled sliding-mode control method," *IEEE Trans. Ind. Electron.*, vol. 56, no. 9, pp. 3541–3555, Sep. 2009.

[19] M.-S. Park and D. Chwa, "Orbital stabilization of inverted-pendulum systems via coupled sliding-mode control," *IEEE Trans. Ind. Electron.*, vol. 56, no. 9, pp. 3556–3569, Sep. 2009.

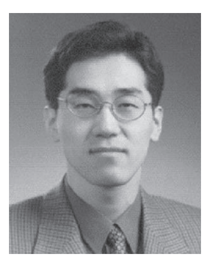
[20] D. Chwa and J. Y. Choi, "Adaptive nonlinear guidance law considering control loop dynamics," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 4, pp. 1134–1143, Oct. 2003.

[21] H. K. Khalil, *Nonlinear Systems*. New York, NY, USA: Macmillan, 1992.

[22] V. I. Utkin, J. Guldner, and J. Shi, *Sliding Mode Control in Electro-Mechanical Systems*, 2nd ed. Boca Raton, FL, USA: CRC Press, 1999.

Ji-Wook Kwon received the B.S., M.S., and Ph.D. degrees in electrical and computer engineering from Ajou University, Suwon, Korea, in 2005, 2007, and 2012, respectively.

In 2009 he was a Visiting Researcher with the Institute of Science and Technology of Yvelines, University of Versailles Saint-Quentin-en-Yvelines, Mante la Jolie, France. In 2012 he was a Senior Researcher with Korea Institute of Industrial Technology, Ansan, Korea. Since 2013 he has been with Yonsei Institute of Convergence Technology, Yonsei University, Incheon, Korea, where he is currently a Senior Researcher. His research interests include mobile robot control and its applications, multiple robot cooperation, formation control, mobile and aerial manipulation, and manipulation using multiagent systems.



Dongkyoung Chwa received the B.S. and M.S. degrees in control and instrumentation engineering, and the Ph.D. degree in electrical and computer engineering from Seoul National University, Seoul, Korea, in 1995, 1997, and 2001, respectively.

From 2001 to 2003 he was a Postdoctoral Researcher with Seoul National University, where he was also a Brain Korea 21 (BK21) Assistant Professor in 2004. In 2003 he was a Visiting Scholar with University of New South Wales, Sydney, Australia, at the Australian Defence Force Academy and the University of Melbourne, Melbourne, Australia, and in 2011 he was with University of Florida, Gainesville, FL, USA. Since 2005 he has been with the Department of Electrical and Computer Engineering, College of Information Technology, Ajou University, Suwon, Korea, where he is currently a Professor. His research interests include nonlinear, robust, and adaptive control theories and their applications to robotics; underactuated systems, including wheeled mobile robots; underactuated ships; cranes; and the guidance and control of flight systems.