

# Higher-Order Statistics based Modulation Classification using Hierarchical Approach

Afan Ali<sup>1</sup>

<sup>1</sup>School of Electronics and Information  
Northwestern Polytechnical University  
Xi'an, China  
afanali85@yahoo.com

Fan Yangyu<sup>2</sup>

<sup>2</sup>Faculty of Electronics and Information  
Northwestern Polytechnical University  
Xi'an, China  
fan\_yangyu@nwpu.edu.cn

**Abstract**—Hierarchical based digital modulation classifier is designed using feature-extraction based method with AWGN channel. A characteristic parameter of the received information samples is used to separate between amplitude and angular modulated signals. M-ary ASK signals are separated using instantaneous amplitude of the received samples. Combination of higher-order cumulants up to order eight are computed to classify between M-ary PSK modulated signals. A new feature is proposed in the decision tree of the classifier to separate QPSK and 8PSK modulation. Simulation results are used to verify that this approach is robust under low SNR.

**Keywords**—Classification, digital modulation, higher order statistics.

## I. INTRODUCTION

Classification of modulation format has a significant importance in non-cooperative communication. It has very useful applications in both military and civilian applications. Likelihood function based [1], [2] and Feature-extraction based [3]-[6] are the two main methods in signal classification. In the likelihood function based method, likelihood-function of the received signal is taken and a decision is made by comparing likelihood ratio with a threshold. Probability of false classification can be reduced by this method but it can add computational complexities. In the feature-based approach, decision is made based on the values of extracted features. This method can help to achieve near optimal results if properly designed and is usually less complex [6]. The features can be time or frequency domain. A detailed overview of feature based classification can be found in [3]. Many types of features can be used for modulation classification. Some common used features in literature are instantaneous frequency, amplitude and phase [14], different transform of wavelets [15] and higher order cumulants and moments [16-18].

A seven characteristics parameters decision theory based recognition method to achieve a result of 90% classification at 10 dB SNR is proposed in [6]. Another method in [9] introduces five new characteristics features which improved the recognition rate greatly but performance declined considerably under 5dB SNR.

This paper strives to detect modulation format of amplitude and angular modulation defined by the set  $I_1 = \{2\text{ASK}, 4\text{ASK},$

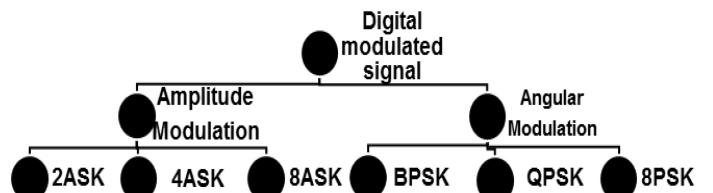


Fig. 1. Hierarchical based Classification Model.

$8\text{ASK}\}$  and  $I_2 = \{\text{BPSK}, \text{QPSK}, 8\text{PSK}\}$  respectively as shown in Table I. The classification method used is hierarchical and feature-based. Feature-vectors that have been used comprises of the instantaneous features and higher order statistics (HoS) of the incoming modulated signals. Hierarchical classification tree of incoming modulated signal has been shown in Fig. 1.

## II. SYSTEM MODEL

Two sets of digital modulations are defined as  $I_1$  and  $I_2$  respectively. The system model is defined as:

$$s(t) = x(t) + n(t) \quad (1)$$

where  $n(t)$  is Additive White Gaussian Noise (AWGN) and  $x(t)$  is the received signal.

Expressions used for the received modulated sequence  $x(t)$  are as follows:

$$x_{\text{ASK}} = A \operatorname{Re} \left[ \sum_k A_k e^{j2\pi f_c t} g(t - kT_s) \right] \quad (2)$$

$$x_{\text{PSK}} = A \operatorname{Re} \left[ \sum_k C_k e^{j2\pi f_c t} g(t - kT_s) \right] \quad (3)$$

where  $x_{\text{ASK}}$  =ASK modulated received sequence,  $x_{\text{PSK}}$  =PSK modulated received sequence  $A_k=2i-M-1$ ,  $i=0,1,2,\dots,M-1$ ,  $C_k=e^{j2\pi i/4}$ ,  $i=0,1,2,\dots,M-1$ ,  $A$  is amplitude power of the received signal,  $f_c$  is the carrier frequency ,  $M$  is the modulation level,  $A_k$  and  $C_k$  map the transmitted symbols,  $T_s$

is the symbol period, and  $g(t)$  is the finite energy signal with a  $T_s$  duration.

### III. CUMULANTS AND MOMENTS

Some definitions of higher order statistics have been introduced in this section. Description involves both stochastics and deterministic signals. If  $\{x(k)\}, k=0,\pm 1,\pm 2,\dots$  is a stationary discrete time signals and its moments upto order  $n$  exists, then

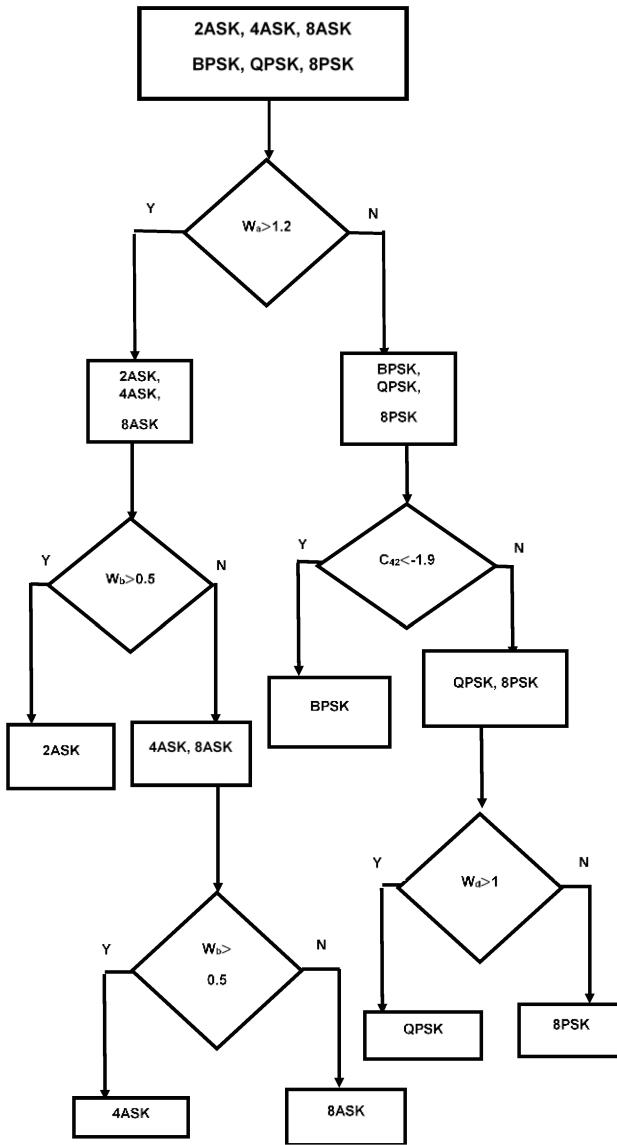


Fig. 2. Flow Chart of Hierarchical classifier

$$m_n^x(\tau_1 + \tau_2, \dots, \tau_{n-1}) \cong E[X(k)X(k + \tau_1) \dots X(k + \tau_{n-1})] \quad (4)$$

represents the  $n$ th order moment function of the stationary signal. It only depends on time difference  $\tau_1, \tau_2, \dots, \tau_{n-1}$ . Now the following relationship between the cumulants and moments exist for order upto 4 [11]:

1<sup>st</sup> order cumulants:

$$c_1^x = m_1^x = E[X(k)] \text{ (mean value)} \quad (5)$$

2<sup>nd</sup> order cumulants:

$$c_2^x = m_2^x(\tau_1) - (m_1^x)^2 \text{ (covariance sequence)} \quad (6)$$

Definition of 3rd- and 4th-order cumulants of a zero-mean, stationary random process is as follows

3<sup>rd</sup> order cumulants:

$$c_3^x(\tau_1, \tau_2) \cong E[X(k)X(k + \tau_1)X(k + \tau_2)] \quad (7)$$

4<sup>th</sup> order cumulants:

$$\begin{aligned} c_4^x(\tau_1, \tau_2, \tau_3) &= E[X(k)X(k + \tau_1)X(k + \tau_2)X(k + \tau_3)] \\ &- c_2^x(\tau_1)c_2^x(\tau_2 - \tau_3) - c_2^x(\tau_2)c_2^x(\tau_3 - \tau_1) \\ &- c_2^x(\tau_3)c_2^x(\tau_2 - \tau_1) \end{aligned} \quad (8)$$

In this paper, typical 2nd, 4th and 8<sup>th</sup> order cumulant are computed as follows:

$$C_{20}(s) = \text{cum}[s, s] = \frac{1}{N} \sum_{n=1}^N s^2(n)$$

$$C_{21}(s) = \text{cum}[s, s^*] = \frac{1}{N} \sum_{n=1}^N |s(n)|^2$$

$$C_{40}(s) = \text{cum}[s, s, s, s] = M_{40} - 3M_{20}^2$$

$$C_{42}(s) = \text{cum}[s, s, s^*, s^*] = M_{42} - |M_{20}|^2 - 2M_{21}^2$$

$$\begin{aligned} C_{80}(s) = \text{cum}[s, s, s, s, s, s, s, s] &= M_{40} - 35M_{40}^2 - \\ &28M_{60}M_{20} + 420M_{40}M_{20}^2 - 630M_{20}^4 \end{aligned}$$

(9)

$M$  stands for moment of signal,  $M_{pq} = E[s(n)^p s^*(n)^q]$  and  $N$  are number of samples. Abbreviations and Acronyms

### IV. PROPOSED HIERARCHICAL CLASSIFIER

The flow chart of the proposed hierarchical classifier has been shown in Fig. 2.

Instantaneous amplitude and angular modulation (two main digital communication groups) have been used to classify the modulated signal in this paper. These two digital communication groups are broken down as follows:

#### A. Angular/Ampitude modulation test

First, the classifier decides whether the incoming signal is amplitude or angular modulated signal. This is done by evaluating the instantaneous amplitude of the signal and calculating a feature which has been called  $W_a$ , ratio of the second-order cumulant, as follows:

TABLE I. SIGNAL GROUP AND THEIR ASSOCIATED CANDIDATE MODULATION TYPE

Signal Group	Candidate modulation type
Amplitude modulation	2ASK,4ASK,8ASK
Angular modulation	BPSK,QPSK,8PSK

$$W_a = \frac{\hat{C}_{20}}{\hat{C}_{21}} \quad (11)$$

where  $\hat{C}_{20} = \frac{1}{N} \sum_{n=1}^N |a_{cn}(n)|^2$ ,  $\hat{C}_{21} = \frac{1}{N} \sum_{n=1}^N |a_{cn}(n)|^2$ ,  $N$  is

the number of samples,  $a_{cn}(n) = a_n(n) - 1$ ,  $a_n(n) = \frac{a(i)}{m}$ ,

$a(i)$  is the received modulated sequence which can be complex for angular modulation and real for amplitude modulation,  $m$ =sample mean.

If the value of  $W_a$  is greater than 1.2, it is angular modulation and if its value is lower than 1.2, it is amplitude modulation.

#### B. 2ASK/4ASK/8ASK test

In order to distinguish between 2ASK/4ASK and 8ASK, a hypothesis test based on two features has been introduced. The two features are defined as follows:

$$W_b = \sum_{n=1}^N |w_1(n)| \quad (12)$$

where  $w_1(n) = \frac{|a_{cn}(n)|}{M_1}$  and  $M_1 = (\frac{1}{N} \sum_{n=1}^N |a_{cn}(n)|) - 1$

$$W_c = \sum_{n=1}^N |w_2(n)| \quad (13)$$

where  $w_2(n) = \frac{|w_1(n)|}{M_2}$  and  $M_2 = (\frac{1}{N} \sum_{n=1}^N |a_1(n)|) - 1$

Now, the hypothesis test used to distinguish the 2ASK, 4ASK and 8ASK modulation comprises of two steps:

$$1. W_b > \begin{matrix} \text{ASK4,8} \\ \text{ASK2} \end{matrix} \gamma \quad (14)$$

$$2. W_c > \begin{matrix} \text{ASK8} \\ \text{ASK4} \end{matrix} \gamma \quad (15)$$

where  $\gamma = 0.45$  is threshold.

#### C. BPSK/QPSK/8PSK test

M-ary PSK modulation is classified using higher order cumulants. In [7], a hierarchical based decision tree based on fourth-order cumulant has been proposed but it suffers from frequency offset. This problem has been solved by converting frequency offset to fixed offset by calculating cumulant of  $z(n) = x^*(n-1)x(n)$  [10]. Theoretical values of  $z(n)$  are

given in Table II. These values are calculated in noise free constellation and calculating ensemble averages using (9).

TABLE II.  $Z(N)=S(N-1)S(N)$

Digital modulation	$ C_{20} $	$C_{40}$	$C_{42}$	$C_{80}$	$W_d$
BPSK	1	-2	-2	-272	68
QPSK	0	1	-1	-34	34
8PSK	0.02	0	-1	-0.95	0.95

Using Table II, a new decision tree based on fourth and eighth-order cumulant has been proposed. BPSK is separated from QPSK and 8PSK using fourth-order cumulant. Then, QPSK and 8PSK has been classified by introducing a new feature  $W_d$ , defined as follows:

$$W_d = \frac{|C_{80}|}{C_{42}^2} \quad (16)$$

From Table II, it can be seen that the new feature introduced helps to separate QPSK from 8PSK.

#### V. SIMULATION

Simulation results of the proposed classifier are discussed in this section. To distinguish between the set  $I_1 = \{2ASK, 4ASK, 8ASK\}$  and  $I_2 = \{BPSK, QPSK, 8PSK\}$ , each signal is tested for  $N=1000$  samples. The carrier frequency  $f_c$  and sampling frequency  $f_s$  have been chosen 150 kHz and 500 kHz respectively. AWGN channel has been assumed with SNR varying from 0dB to 35dB. The transmitted pulse is rectangular pulse shaped.

Calculate

$W_a, W_b, W_c, W_d, C_{20}, C_{21}, C_{40}, C_{42}, C_{80}, \hat{C}_{20}, \hat{C}_{21}$

If  $W_a > 1.2 \rightarrow BPSK, QPSK, 8PSK$

If  $C_{40}$  and  $C_{42} < -1.9 \rightarrow BPSK$

If  $C_{40}$  and  $C_{42} > -1.9 \rightarrow QPSK, 8PSK$

If  $W_d > 1 \rightarrow QPSK$

else  $\rightarrow 8PSK$

Else if  $W_b < 1.2 \rightarrow 2ASK, 4ASK, 8ASK$

If  $W_b < 0.5 \rightarrow 2ASK$

If  $W_b \geq 0.5 \rightarrow 4ASK, 8ASK$

If  $W_c < 0.5 \rightarrow 4ASK$

If  $W_c \geq 0.5 \rightarrow 8ASK$

end

end

Fig. 3. Algorithm for proposed classifier

Monte Carlo simulation with  $n=100$  trials has been carried out for calculating percentage of correct classification (PCC). The results are illustrated on the basis of PCC and input SNR. In each test, characteristic feature has been computed using 100 independent runs at above range of SNR and then average is calculated.

From Fig. 4 it is quite evident that the parameter  $W_a$  can be used to separate the angular modulation from amplitude modulation with increasing SNR. That is equivalent of saying that set  $I_1 = \{2\text{ASK}, 4\text{ASK}, 8\text{ASK}\}$  and  $I_2 = \{\text{BPSK}, \text{QPSK}, 8\text{PSK}\}$  are separated from one another by this feature.

In Fig. 5 and Fig. 6, parameter  $W_b$  and  $W_c$  are used to separate the modulation set  $I_1 = \{2\text{ASK}, 4\text{ASK}, 8\text{ASK}\}$ . It can be seen from Fig. 5 that there is no overlap of parameter  $W_b$  between 2ASK and 4ASK/8ASK. This becomes clearer with SNR greater than 5dB. In Fig. 6, it can be seen that there is no overlap of parameter  $W_c$  between 4ASK and 8ASK. However, in this case tendency becomes more stable with SNR greater than 10dB.

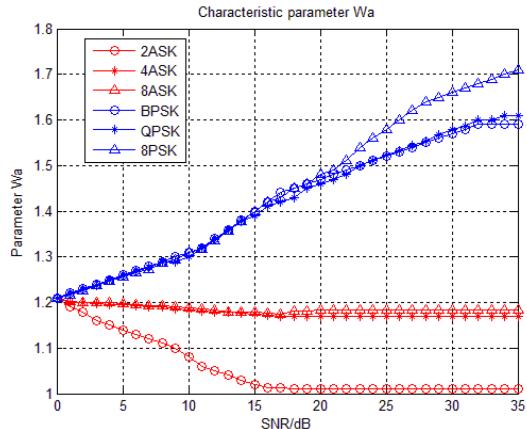


Fig. 4. Characteristic parameter  $W_a$

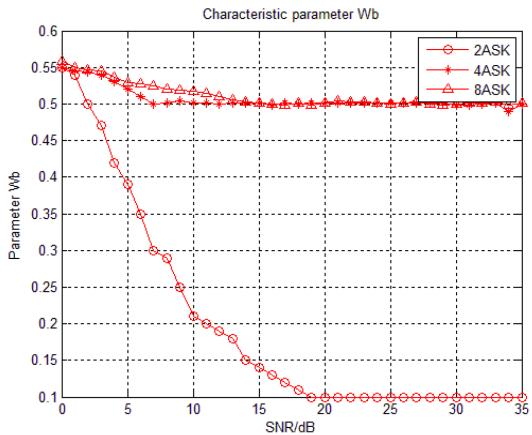


Fig. 5. Characteristic parameter  $W_b$

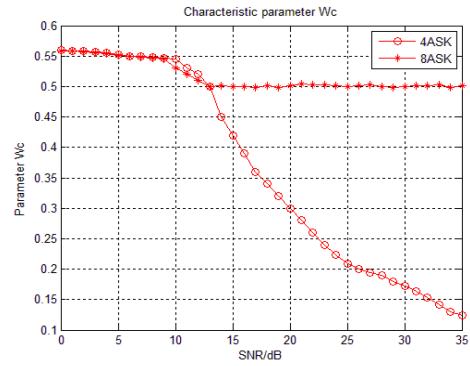


Fig. 6. Characteristic parameter  $W_c$

In Fig. 7, fourth-order cumulant of BPSK is compared to both QPSK and PSK8 with increasing SNR. It is very evident from the graph that  $C_{42}$  can be used to separate BPSK amongst set  $I_2 = \{\text{BPSK}, \text{QPSK}, 8\text{PSK}\}$  at a very low SNR of 0dB.

Fig. 8 depicts the graph of new parameter  $W_d$  with increasing SNR. It is depicted from the graph that there is no overlap of this parameter between QPSK and 8PSK. It can also be seen from the graph that results are very robust till 5dB SNR.

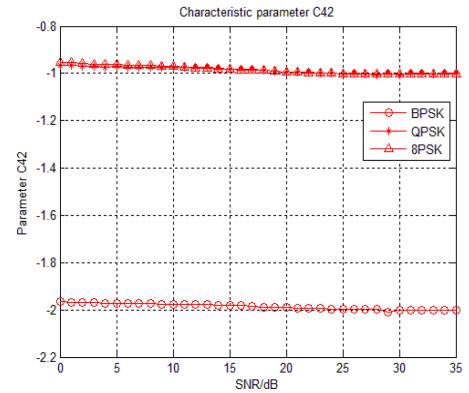


Fig. 7. Characteristic parameter  $C_{42}$

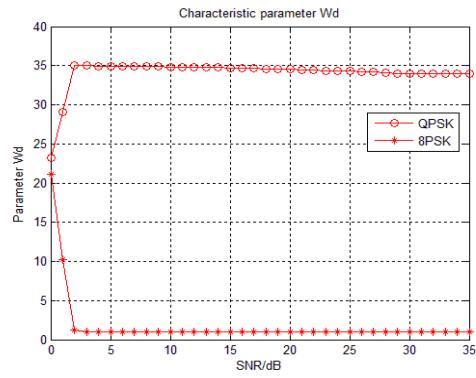


Fig. 8. Characteristic parameter  $W_d$

Table III. and IV. shows the percentage of correct classification for  $I_1 = \{2\text{ASK}, 4\text{ASK}, 8\text{ASK}\}$  and  $I_2 = \{\text{BPSK}, \text{QPSK}, 8\text{PSK}\}$  at SNR 10dB and 0dB respectively. For 10 dB SNR, QPSK gives a 98 percent of correct classification. For SNR 0dB, M-ary PSK signals achieves a percentage of above 95 for correct classification.

For M-ary ASK modulation set, recognition rate is not lower than 90 percentage at 10 dB SNR. However, recognition rate drops drastically for 4ASK and 8ASK at SNR 0dB.

TABLE III. PCC FOR SNR=10 DB USING HIERARCHICAL PROPOSED CLASSIFIER,N=1000 SAMPLES,N=100TRIALS

	Classifier output					
	BPSK	QPSK	8PSK	2ASK	4ASK	8ASK
BPSK	100	0	0	0	0	0
QPSK	0	98.92	1.08	0	0	0
PSK8	0	3.87	96.13	0	0	0
ASK2	0	0	0	95.4	4.6	0
ASK4	0	0	0	0	96.6	3.4
ASK8	0	0	0	0	7	93

TABLE IV. PCC FOR SNR=0 DB USING HIERARCHICAL PROPOSED CLASSIFIER,N=1000 SAMPLES,N=100TRIALS

	Classifier output					
	BPSK	QPSK	8PSK	2ASK	4ASK	8ASK
BPSK	100	0	0	0	0	0
QPSK	0	98.92	1.08	0	0	0
PSK8	0	3.87	96.13	0	0	0
ASK2	0	0	0	90.11	0.69	0.20
ASK4	0	0	0	0	51	49
ASK8	0	0	0	0	42	58

## VI. CONCLUSION

In this paper, hierarchical digital modulation approach has been used to classify two sets of digital modulated signal mainly:  $I_1 = \{2\text{ASK}, 4\text{ASK}, 8\text{ASK}\}$  and  $I_2 = \{\text{BPSK}, \text{QPSK}, 8\text{PSK}\}$  using AWGN channel. Instantaneous amplitude of the incoming signal has been used as a feature vector to classify between M-ary ASK signals whereas high-order cumulants have been used to classify M-ary PSK signals. Simulation results show that choice of features proposed in this paper are not only simple but also robust under noisy channel if designed properly.

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