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Abstract

We intend to define a continuous car-following model exclusively based on the time gap. A model of the interaction between a vehicle and its predecessor is produced by adjusting the time gap to a targeted safety time that is a function of speed. The model is defined by a differential system, to which a consistent numerical scheme is associated. The parameters of the model are statiscally estimated by maximum likelihood. In order to reproduce a heterogeneous traffic flow, vehicles are differentiated by type, and to recreate asymmetric longitudinal behaviour, acceleration phases are distinguished from deceleration phases. Introducing a reaction time, inducing a delay in the perception and processing of information about vehicles in interaction, can alter the stability of the flux through appearence of kinematic waves. By simulation, the types and domains of parameters which are asymptotically unstable are identified. The results reveal that, in the model, the statiscally estimated parameters form is strongly factor of instability.

Keyword : Traffic flow — Car-following model — Maximum likelihood estimation of the parameters — Simulation — Kinematic waves emergence

1 Introduction

A traffic flow, composed of numerous vehicles, can be apprehended as a macroscopic phenomenon. In a microscopic approach of modelling, a macroscopic phenomenon emerges from the interactions of a collective of agents : the traffic flow characteristics, complex and hardly visible, can be explained by the behavior of the drivers, more known, who compose it. The microscopic modelling of traffic flow is a field in which mathematics has been applied since the 1930s, the period when motor car ownership was becoming more widespread. The models developed are aimed at providing a better understanding of the underlying mechanisms that govern traffic flow. Questions concerning the stability of a stream of traffic, especially the emergence of traffic jams, cropped up early on.

Generally speaking, a longitudinal model of traffic takes an interactive car-following state and a free state into account. In this article, a car-following model, that can be extended to a free model, is defined. The paper is divided into four parts. To start with, section 2, a review of recognised models is undertaken in order to identify fundamental concepts that seem to be necessary in a car-following model. Then, section 3, a model exclusively based on time gap is presented. In the next part, section 4, a statistically estimation of the model parameters, using vehicles trajectories, is proposed. Lastly, section 5, the macroscopic stability of

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the model, especialy the emergence of kinematic waves, is studied by simulation according to the form of the model parameters.

Microscopic variables notes A vehicle is identified by a integer i; the jth predecessor (downstream) is shown as $i+j$. The acceleration, the speed and the central position of the vehicle i are shown for a given moment t as: $a_i(t) \in \mathbb{R}$, $v_i(t) \in \mathbb{R}_*^+$ and $x_i(t) \in \mathbb{R}$. The time gap is defined by $\mathcal{T}_i(t) = \frac{1}{v_i(t)}\{x_{i+1}(t) - x_i(t) - (\ell_i + \ell_{i+1})/2\}$, with $\ell \in \mathbb{R}^+$ the length of the vehicles.

2 Review and analysis

In a car-following state, a vehicle follows its predecessor at a speed that is generally less than a maximum desired speed. The situation is such that drivers are led to adapt their behavior as a function of the neighbouring vehicles. The reaction time $\mathcal{T}^r$ seems to be an essential parameter of the car-following situation. According to LEUTZBACH (1988), the reaction time is defined as the sum of several times: perception time (the time needed for a driver to recognise a coming event); decision time (during which the driver decides what action to take in response); and application time (the time needed for the driver to take action).

2.1 Bibliographical review

Models of the 1950-1970s Authors such as GREENSHIELDS (1935), CHANDLER *et al.* (1958), KOMETANI and SASAKI (1958), GREENBERG (1959), EDIE (1961), UNDERWOOD (1961), NEWELL (1961), HELLY (1961), GAZIS *et al.* (1961), BIERLEY (1963) or again PIPES (1967) developed microscopic traffic models. In the majority of them, the model entails defining an acceleration function. To start with, the model's definition is discrete, in steps equal to the reaction time. Two fundamental variables regarding a vehicle i and its predecessor $i+1$ quickly emerged: the difference of position $x_{i+1} - x_i$ and the difference of speed $v_{i+1} - v_i$. Each acceleration function corresponds to a certain strategy for speed regulation. A duality is established between the regulation of the difference of speed and the regulation of the difference of position. This duality is modelled by the use of these two variables, as defined in the model. For instance, the CHANDLER *et al.* model (1958) is based solely on the regulation of the difference of speed, whereas the (first) NEWELL model (1961) is based solely on the regulation of the difference of position. The remaining models generally result from regulating a function of these variables and the speed of the considered vehicle. GAZIS *et al.* (1961) define a general model in the form of:

$$a_i(t+\mathcal{T}^r) = \lambda^c \frac{v_i(t+\mathcal{T}^r)^l}{(x_{i+1}(t) - x_i(t))^m}(v_{i+1}(t) - v_i(t)) \tag{1}$$

where c designates the symbol + or -, λ^c varies depending whether the acceleration rate is positive (acceleration) or negative (deceleration), thereby enabling asymmetric acceleration behavior to be defined. l and m are parameters. Each model can be associated with a fundamental diagram of characteristic flow-density. One of the major difficulties of the first models is to meet the frontier conditions both in low and high densities, it seems necessary to distinguish between the free cases (low density) and the interactive cases (above a certain critical density threshold). Reciprocally, a microscopic safety function of distance, time or speed can be associated with each fundamental diagram. In the models of the 1950s to 1970s, this kind of function are generally not explicitly defined. The strategy for regulating instabilities and the target microscopic safety state are merged, the regulation strategy induced by the form of the acceleration function generates a certain safety state.

Models from the 1990s to the present day The development of computer technology resulted in the emergence of computer simulation. Microscopic approaches, often described as multi-agent, are used as a tool for understanding collective phenomena. A new wave of models appeared in the 1990s. From then on, models are defined by a system of differential equations which is associated with discretization schemes so that simulations can be performed. It then becomes necessary to distinguish reaction time and time steps in simulations. The work of TREIBER *et al.* (2006a) in particular clarify this aspect. In the recent models, a targeted microscopic safety state is generally parametered. GIPPS (1981) longitudinal model takes two states into account : free state and interactive (pursuit) state. The pursuit's model is based on the definition of a safety distance that is a function of the speed, calculated from distance of stopping. The *Optimal Velocity* model developed by BANDO and HASEBE (1995) is defined by applying a relaxation process to speed and to a model of optimal speed as a function of distance gap (that can be the first NEWELL' s model), noted $\mathscr{V}(d)$. The speed of a considered vehicle has to tend towards the speed of its predecessor for the situation to become stable : defining optimal speed as a function of the distance gap is tantamount to implicitly defining a targeted distance gap function of the speed. If one considers the speed of the predecessor to be constant, the targeted distance gap is defined by $\mathscr{V}^{-1}(v_{i+1})$. A version of *Optimal Velocity* model incorporates a reaction time (BANDO *et al.* (1998)). The LENZ model (1999) is a generalisation of the *Optimal Velocity* model in which the number of predecessors involved in the model is a parameter. The *Full Velocity Difference* model developed by JIANG *et al.* (2001) integrates two relaxation times, one applying to the optimal speed model and the other to the difference of speed. A duality weighted by the relaxation times is established between regulating the distance gap and regulating the difference of speed. The *Generalized Force*, model developed by HELBING and TILCH (1998) adopts a comparable approach. The model incorporates a *heaviside-type* function enabling asymmetric longitudinal behavior to be restored. The (second) model of NEWELL (2002) is a simple car-following rule in which the trajectory of a vehicle is a translation in space and time of its predecessor. The model, discretly defined, only depends to two parameters : a time and a minimum distance of spacing. Under certain assumptions on the parameters, the *Intelligent Driver* model developed by TREIBER *et al.* (2000) incorporates a targeted safety time. There are variants of the *Intelligent Driver* model in which the targeted safety time is variable, function of a local traffic context (TREIBER and HELBING (2003); TREIBER *et al.* (2006b)). AW *et al.* (2002) define a general model in the form of :

$$\frac{\mathrm{d}v_i}{\mathrm{d}t}(t) = C\frac{v_{i+1}(t) - v_i(t)}{(x_{i+1}(t) - x_i(t))^{\gamma+1}} + A\frac{1}{\tau}\left(\mathscr{V}\left(\frac{\ell}{x_{i+1}(t) - x_i(t)}\right) - v_i(t)\right) \tag{2}$$

where $\mathscr{V}$ is a targeted equilibrium speed and $C > 0$, $A > 0$ and $\gamma \geq 0$ are parameters. When A is zero, one finds the general form of GAZIS *et al.*, with $l = 0$ and $m = \gamma + 1$. When $A > 0$, a relaxation process is introduced and the vehicle's speed approaches an equilibrium speed $\mathscr{V}$. A parameter for the convergence speed is provided by relaxation time τ. Current models distinguish between two aspects : targeted microscopic safety state and regulation strategy. The targeted safety state is modeled by a certain targeted safety function that depends of speed, distance or time gap. For the regulation strategy, the majority of the models introduce relaxation process(es) to make attractive the targeted microscopic safety state.

Meta-models The basic idea of the WIEDEMANN (1974) longitudinal model is the assumption that a driver can be in one of four driving situations : (1) Free driving; when the distance gap is large. The driver's behavior is not influenced by the preceding vehicles. In this situation, the driver tries to reach and maintain a certain individually desired speed. (2) Regulating; the regulation strategies involve when a driver meets a slower predecessor. (3) Stable following; a driver tries to keep a safety distance more or less constant. Because of an imperfect control the speed difference oscillates around zero (*Action Point* car-following model, LEUTZBACH and WIEDEMANN (1986)). (4) Braking. The braking situation appears when a slower preceding vehicle is very close in front and, due to the immediate danger, the driver will apply high deceleration rates. Recently, TREIBER *et al.* (2006a) proposed a Human Driver (meta-) model that was applicable to all continuous microscopic models in which the acceleration function depends on the distance gap, the speed and the speed of the predecessor. This is an approach that comprises four elements : (1) a finite reaction time; when the numerical simulation time is not a multiple of the reaction time a linear interpolation formula is proposed. (2) A modelling of appreciation errors; some variables measured by a driver, like the speed of the predecessor or the distance gap, are random. The noise involved is a function of environmental conditions. (3) A temporal anticipation; anticipation entails approximating the informations of the predecessor at time t from information at time $t - \mathcal{T}^r$, lagging by the reaction time. In the approximation, the speed of the predecessor is supposed constant. (4) A spatial anticipation; introducing several preceding vehicles makes it possible to estimate the information of the predecessor and to compensate for the lag induced by the reaction time. In the approximation, the speed of the predecessor is supposed variable.

Fundamental concepts retained In the light of this bibliographical review, it would seem that several concepts have to be introduced in a car-following model :

1. The existence of a strictly positive reaction time. The reaction time is introduced with the aim of delaying the perception and processing of information about the neighboring vehicles. A delay is assumed to exist between the driver's acquisition of information and the effective use of that information.

2. The explicit definition of a targeted microscopic safety state. This hypothesis can be justified by the existence of the reaction time which can generate collisions. The safety state is a stable state of car-following.

3. The definition of strategies for regulating instabilities. Regulation strategies take place when the distance (or time) gap is not satisfying or when the speed of the predecessor varies. The regulation stratigy allows to reach the targeted safety state. Statistical studies reveal the existence of characteristic regulation strategies (WANG *et al.* (2006)).

4. The existence of an anticipatory ability. The existence of anticipatory abilities is a result of the following paradox (TREIBER *et al.* (2006a)). A strictly positive reaction time engenders collisions; these can be avoided by the respect of safety gaps. Nevertheless, one finds gaps in moving traffic which are smaller than the gap that theoretically ensure the absence of collisions. It would seem then that drivers are capable of adopting anticipatory strategies designed to offset the reaction time.

5. The implementation of asymmetric longitudinal behavior. Drivers' behavior differs according to whether they are placed in a situation of acceleration or deceleration. Moreover, the mechanical means brought into play for accelerating or decelerating are not comparable.

2.2 Microscopic safety state

The function of targeted safety distance or time seems to be a key parameter in car-following models, and macroscopic performance indicators such as mean speed or flow volume are directly dependent on it. The function defines the security related behavior of a driver confronted to collision risks. A normative approach makes it possible to define a collision-free microscopic safety state.

Normative approach to the microscopic safety state In a safety aspect, the minimum safety time is calculated and make it possible to prevent collisions due to the reaction time and to a limited capacity of decelerating. Normative approaches have been developed in different studies (KOMETANI and SASAKI (1958); GIPPS (1981); CARRÉ *et al.* (1985)), generally to model driver behavior. One considers initial conditions in which two vehicles are travelling at the same speed v. The minimum rate of acceleration (the maximal rate of deceleration) is denoted as $a^{min} < 0$. The minimum time needed for the vehicle to stop is defined by $\mathcal{T}^{stop} = -v/a^{min}$. We are interested in the worst case, in which the predecessor decelerates at maximum capacity : $a_{i+1}(t) = a_{i+1}^{min}$, $0 < t \leq \mathcal{T}_{i+1}^{stop}$, and in which we assume that the following vehicle suffers a reaction time $\mathcal{T}^r$ that delays its braking : $a_i(t) = a_i^{min}$, $\mathcal{T}^r < t \leq \mathcal{T}^r + \mathcal{T}_i^{stop}$. The minimum initial time gap that will enable the vehicle considered i to avoid any collision is written :

$$\mathcal{T}_i^{min}(v) = \begin{cases} \frac{(\mathcal{T}^r)^2}{2v} \frac{a_{i+1}^{min} a_i^{min}}{a_{i+1}^{min} - a_i^{min}} & \text{if} \quad a_i^{min} < a_{i+1}^{min} \quad \text{and} \quad v > \mathcal{T}^r \frac{a_{i+1}^{min} a_i^{min}}{a_{i+1}^{min} - a_i^{min}} \\ \mathcal{T}^r - \frac{v}{2} \frac{a_{i+1}^{min} - a_i^{min}}{a_{i+1}^{min} a_i^{min}} & \text{otherwise} \end{cases} \tag{3}$$

When deceleration capacities are equal, the minimum safety time gap is equal to the reaction time. If one considers the deceleration capacity of a predecessor to be greater than that of a given vehicle, the minimum safety time function is greater than the reaction time and growing. Conversely, if one considers the deceleration capacity of a predecessor to be less than a given vehicle, the minimum safety time function is lower than the reaction time and decreasing. These threshold values are theoretical. They depend on the dynamic capacities of a vehicle and the predecessor, which are vague data that have to be estimated by a driver. GIPPS (1981) assumes that the dynamic capacities of a vehicle are known precisely; on the other hand, the capacities of the predecessor have to be estimated.

Microscopic safety state and macroscopic performance of an homogeneous flow Ones denotes $\mathscr{V} : \mathbb{R}^+ \rightarrow [0, \vartheta]$ the targeted safety speed, monotonuous function of the distance gap, equal to a constant desired speed ϑ above a certain interaction distance denoted $\mathcal{D}$. When one considers the stable homogeneous state in which each vehicle travels at the same speed $v = \vartheta$ in a free case and $v = \mathscr{V}$ in an interactive case, an explicit analytical relationship linking macroscopic performance to the microscopic safety function can be defined. The density of a flow is written $\varrho \in [0, \varrho_{max}]$, with $\varrho_{max} = 1/\bar{\ell}$ the maximum density threshold ($\bar{\ell}$ is the mean vehicles length), the mean distance gap is $1/\varrho - \bar{\ell}$. The critical density threshold ϱ_c is reached for a mean distance gap equals to the interaction distance, one deduces $\varrho_c = 1/(\mathcal{D} + \bar{\ell})$. Since the positioning of the vehicles is homogeneous, if one assumes the function $\mathscr{V}$ to be monotonuous, the macroscopic speed and flow of a flux are defined by :

$$\begin{aligned} \mathcal{V}(\varrho) &= \mathscr{V}\left(1/\varrho - \bar{\ell}\right) \\ \mathcal{Q}(\varrho) &= \varrho \times \mathscr{V}\left(1/\varrho - \bar{\ell}\right) \end{aligned} \tag{4}$$

The maximum speed value is ϑ. The maximum flow value depends on targeted safety function and is not necessary reached at the critical density. Generally speaking, when the targeted safety speed function is convex on $d < \mathcal{D}$ (when the targeted safety time function of speed decreases), there is a drop capacity of flow past the critical density threshold. Conversely, a concave targeted safety speed function (a growing targeted safety time function), restores a progressive transition from free to interactive traffic. When the targeted safety speed function is linear (the targeted safety time function is constant) the fundamental diagram is triangular (KERNER and KONHÄUSER (1994); BANKS (1999)). As an example, let us consider a linear targeted safety time function (figure 1), a similar form was studied by KOMETANI and SASAKI (1961) :

$$\mathscr{T}(v) = \begin{cases} \alpha(v-\vartheta)+\beta & \text{if } \alpha < 0 \\ \beta & \text{if } \alpha = 0 \\ \alpha v + \beta & \text{otherwise} \end{cases} \tag{5}$$

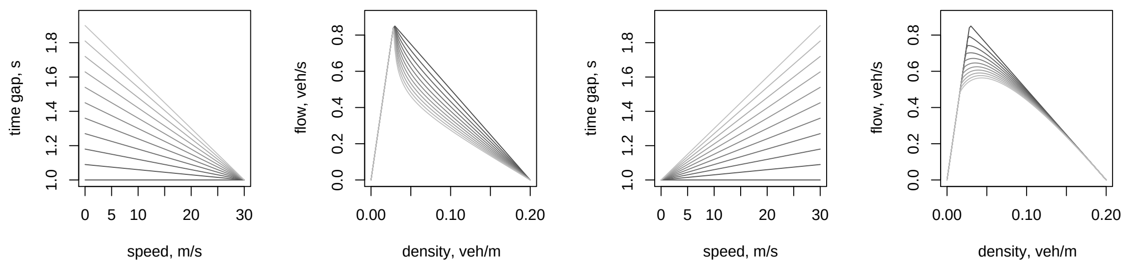

Figure 1: Targeted safety time as decreasing (at left) and increasing (at right) linear function of speed and corresponding fundamental flow-density diagram (α ranges from -0.03 to $0\,s^2/m$ and from 0 to $0.03\,s^2/m$ in steps of 0.003 s^2/m). $\beta = 1\,s$, $\vartheta = 30\,m/s$, $\bar{\ell} = 5\,m$.

3 Definition of an adaptive time gap car-following model

In a car-following context, the reaction time seems to be an essential parameter that defines a physiological delay. On the other hand the time gap defines a physical delay before a potential collision. The reaction time and the time gap seem to be complementary, consequently, the model proposed assumes a regulation of driver's time gap. This model is defined by the application of a relaxation process to the time gap and to a targeted safety time that is a function of speed. This approach can be justified by some statistical studies that suggest the existence of a finite memory function that can dictate the evolution of the time gap. In the figure 2, the mean autocorrelation function of the time gap is similar to the autocorrelation function of an autoregressive process like a relaxation process.

3.1 Definition of a basic model

Continuous definition In a continuous time case, the model is defined by the adaptive differential system :

$$\begin{cases} \frac{\mathrm{d}x_i}{\mathrm{d}t}(t) &= v_i(t) & (i) \\ \frac{\mathrm{d}\mathcal{T}_i}{\mathrm{d}t}(t) &= \lambda_i(t)\left(\mathscr{T}(v_i(t)) - \mathcal{T}_i(t)\right) + \sigma\mathcal{E}_i(t) & (ii) \end{cases} \tag{6}$$

with $v_i(t) = \frac{1}{\mathcal{T}_i(t)}\left\{x_{i+1}(t) - x_i(t) - (\ell_i + \ell_{i+1})/2\right\}$. If $\mathbf{I}$ vehicles are considered, the system is a nonlinear differential one with the unknown functions $(x_1(t), \mathcal{T}_1(t), \ldots, x_{\mathbf{I}}(t), \mathcal{T}_{\mathbf{I}}(t))$.

- The relationship (i), which models vehicles' movements, comes from the continuous physical models of kinematics. The speed vector of a vehicle is defined as the derivative of the position vector and the acceleration vector as the derivative of the speed vector.

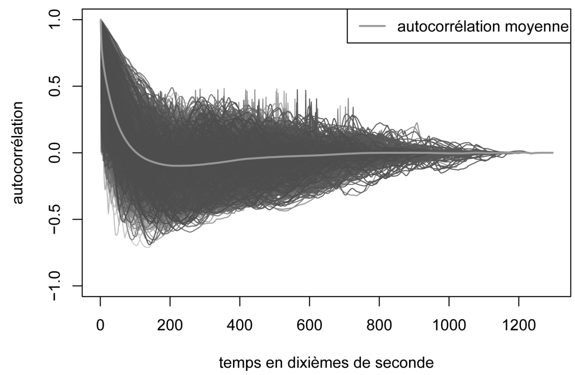

Figure 2: Autocorrelation and mean autocorrelation of time gaps calculated from a database generated by the American NGSIM project (*ngsim.fhwa.dot.gov*).

- The relationship (ii) defines the interaction between vehicles. The time gap $\mathcal{T}$ is regulated towards the targeted safety time gap $\mathscr{T}$ at an exponential speed. The fact that the function $\mathscr{T}$ depends on the speed involves an interaction between equations (i) and (ii). The parameter $\lambda_i > 0$, the inverse of a relaxation time, calibrates the convergence speed of the time gap, defining different types of acceleration behavior. A dynamic definition of parameter $\lambda_i(t)$, which is constant during the acceleration phases and the deceleration phases, enables asymmetric acceleration behavior to be restored. $\mathcal{E}_i$ is an additive symmetric stochastic noise that is zero in a deterministic case. One assumes that this noise is a gaussian white noise, so equation (ii) is more precisely written by :

$$\mathcal{T}_i(t) = \mathcal{T}_i(0) + \int_0^t \lambda_i(s)(\mathscr{T}(v_i(s)) - \mathcal{T}_i(s))\, ds + \sigma \mathcal{W}_i(t) \tag{7}$$

  with $(\mathcal{W}_i(t), i = 1, \dots, \mathbf{I})$ independent Brownian processes and σ a strictly positive parameter.

Through the relationship linking distance gap, time gap and speed, the regulating equation (ii) can be rewritten in the more conventional form :

$$\frac{\mathrm{d}v_i}{\mathrm{d}t}(t) = \frac{1}{\mathcal{T}_i(t)}\{v_{i+1}(t) - v_i(t)\} + v_i(t)\lambda_i(t)\left(1 - \frac{\mathscr{T}(v_i(t))}{\mathcal{T}_i(t)}\right) + \frac{v_i(t)}{\mathcal{T}_i(t)}\sigma\mathcal{E}_i(t) \tag{8}$$

This form is comparable with the standard defined by AW *et al.* (2002) and to the *Full Velocity Difference Model* with a speeds difference term regulated by the time gap. If the parameters λ and σ are nil, the time gap remains constant, equal to the initial time gap value, and the targeted safety time does not come into play. The model is then comparable to the generalized form defined by GAZIS *et al.* (1961) with $l = m = 1$.

The car-following model have to be completed by a free model to incorporate a maximum desired speed ϑ. The targeted safety time function is distinguished according to car-following and free cases :

$$\mathscr{F}(v_i(t), \mathcal{T}_i(t)) = \max\left\{\mathscr{T}(v_i(t)),\ \mathcal{T}_i(t)\frac{v_i(t)}{\vartheta}\right\} \tag{9}$$

The regulating equation (ii) is defined by :

$$\frac{\mathrm{d}\mathcal{T}_i}{\mathrm{d}t}(t) = \lambda_i(t)\left(\mathscr{F}(v_i(t), \mathcal{T}_i(t)) - \mathcal{T}_i(t)\right) + \sigma\mathcal{E}_i(t) \tag{10}$$

In the free case, for $\mathscr{F}(v_i(t),\mathcal{T}_i(t)) = \mathcal{T}_i(t)\frac{v_i(t)}{\vartheta}$, the regulating equation (ii) can be rewritten in the form :

$$\frac{\mathrm{d}v_i}{\mathrm{d}t}(t) = \frac{1}{\mathcal{T}_i(t)}\{v_{i+1}(t) - v_i(t)\} + v_i(t)\lambda_i(t)\left(1 - \frac{v_i(t)}{\vartheta}\right) + \frac{v_i(t)}{\mathcal{T}_i(t)}\sigma\mathcal{E}_i(t) \tag{11}$$

The interacting $\{v_{i+1}(t) - v_i(t)\}/\mathcal{T}_i(t)$ and stochastic terms tend towards zero when the gap increases. Notably, in a free case in which $\mathcal{T}_i(t) \gg 0$, the speed regulation equation is :

$$\frac{\mathrm{d}v_i}{\mathrm{d}t}(t) = v_i(t)\lambda_i(t)\left(1 - v_i(t)/\vartheta\right) \tag{12}$$

Definition of a discretization scheme Since the calculation time available to vehicles has to be limited, a first-order discretization scheme is implemented. Two resolution schemes are proposed : an implicit EULER scheme applied to (i) : $x_i(t+\delta t) = x_i(t) + \delta t\, v_i(t+\delta t)$ and an explicit EULER scheme applied to (ii) : $\mathcal{T}_i(t+\delta t) = \mathcal{T}_i(t) + \delta t\lambda_i(t)\left(\mathscr{F}(v_i(t),\mathcal{T}_i(t)) - \mathcal{T}_i(t)\right) + \sigma(\mathcal{W}_i(t+\delta t) - \mathcal{W}_i(t))$ where $\mathcal{W}_i(t+\delta t) - \mathcal{W}_i(t)$ is a centered gaussian random variable with variance equal to δt. With the help of the relationship linking speed, distance and time gap, the discretisation scheme can be written :

$$\left\{\begin{array}{lcl} x_i(t+\delta t) & = & x_i(t) + \delta t v_i(t+\delta t) \\ v_i(t+\delta t) & = & \frac{x_{i+1}(t) - x_i(t) - (\ell_i + \ell_{i+1})/2 + \delta t v_{i+1}(t+\delta t)}{\delta t + (1-\delta t\lambda_i(t))\mathcal{T}_i(t) + \delta t\lambda_i(t)\mathscr{F}(v_i(t),\mathcal{T}_i(t)) + \sigma(\mathcal{W}_i(t+\delta t) - \mathcal{W}_i(t))} \end{array}\right. \tag{13}$$

A condition appears on the smallness of the time step : $\delta t < \min_{t,i}\{1/\lambda_i(t)\}$. When the flux is composed of **I** vehicles travelling in a closed loop, the resolution of the system leads to the resolution of a linear system. The scheme calculates synchronically, through a inverse matrix calculation, the speed of vehicles at time $t + \delta$ knowing the state of the flux at time t.

Locally linear stability analysis We position ourselves in a simplified car-following case in which the speed of the predecessor $v_{i+1} < \vartheta$ is constant. The speed $v_i(t)$ of a vehicle in a car-following case has to converge towards the speed v_{i+1} of the predecessor and the time gap $\mathcal{T}_i(t)$ has to converge towards the safety time $\mathscr{T}(v_{i+1})$. Two aspects regarding the targeted state $\{v_{i+1}, \mathscr{T}(v_{i+1})\}$ deserve to be highlighted : its stability and its attractive properties. In the case where $\mathscr{T}(v)$ is constant, the equations (i) and (ii) of the system (6) are autonomous. In this case, the system is stable and attractive. In any given case, the relationship (ii) of the system (6) defines a first-order, non-homogeneous equation with variable coefficients. When one knows an initial condition, the system defines a Cauchy problem that admits of only one solution (Cauchy-Lipschitz). The attractiveness of the solution remains to be shown under conditions on the function $\mathscr{T}(v)$. A local linear analysis can be performed. The particular values of the jacobian matrix associated with the system have been measured according to different forms of the targeted safety time function (constant, linear and power type). The results show that, in the majority of cases, the targeted microscopic safety state is effectively attractive. Only special cases in which the targeted safety time function is strongly growing (power type, with exponents greater than 3) proved repulsive.

Characteristic examples of regulating instabilities Two characteristic situations enable us to illustrate the type of regulation strategy induced by the model : a deceleration situation (figure 3) and a catching up situation (figure 4). The speed of the predecesor and the targeted safety time are constant. According to the value of the λ parameter, the acceleration rate values reproduced are more or less high.

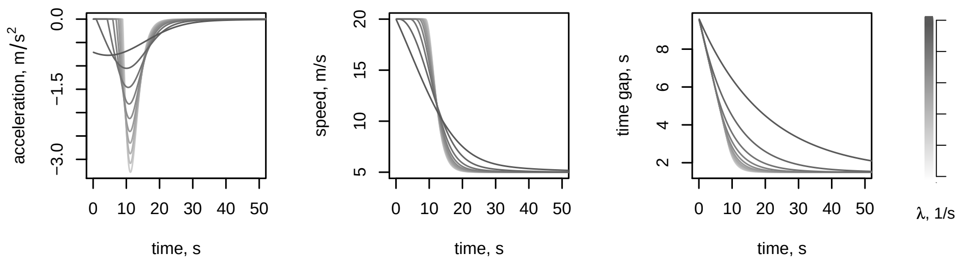

Figure 3: Example of a deceleration situation according to different values for the λ parameter. From left to right, acceleration rate, speed and time gap variables as a function of time respectively. Results obtained by simulation. λ varies from 0.05 to 0.5 s^{-1} by step of 0.05 s^{-1} getting ligth. $\vartheta = 20\,m/s$.

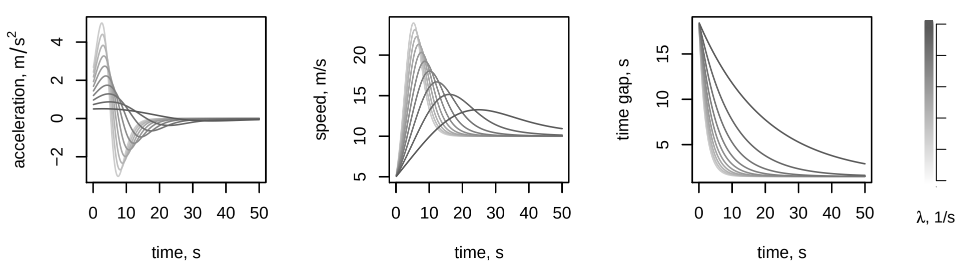

Figure 4: Example of a catching up situation according to different values for the λ parameter. From left to right, acceleration rate, speed and time gap variables as a function of time respectively. Results obtained by simulation. λ varies from 0.05 to 0.5 s^{-1} by step of 0.05 s^{-1} getting ligth. $\vartheta = \infty$.

3.2 Defining a model incorporating a reaction time

Generally, the reaction time is introduced in a car-following model by delaying all the observable variables (CHANDLER *et al.* (1961); HELLY (1961); NEWELL (1961); TREIBER *et al.* (2006a)). In order to integrate a reaction time $\mathcal{T}^r$, inducing a lag between the perception and the processing of information gathered from the neighboring vehicles, one chooses to delay only the variables of the precedessor (BANDO *et al.* (1998)). One defines the system :

$$\left\{ \begin{array}{lcll} \frac{\mathrm{d}x_i}{\mathrm{d}t}(t) & = & v_i(t) & (i) \\ \frac{\mathrm{d}\mathcal{T}_i^{(j)}}{\mathrm{d}t}(t) & = & \lambda_i(t)\left(\mathscr{F}(v_i(t), \mathcal{T}_i^{(j)}(t)) - \mathcal{T}_i^{(j)}(t)\right) + \sigma\mathcal{E}_i(t) & (ii) \end{array} \right. \tag{14}$$

in which $\mathcal{T}_i^{(j)}(t) = \frac{1}{v_i(t)}\left\{x_{i+1}^{(j)}(t) - x_i(t) - (\ell_i + \ell_{i+1})/2\right\}$ with $x_{i+1}^{(j)}(t) = x_{i+1}(t - \mathcal{T}^r) + \int_{t-\mathcal{T}^r}^{t} v_{i+1}^{(j)}(u)\,du$. For a considered driver i, the performances of its predecessor are unknow during the interval $]t-\mathcal{T}^r, t]$ and have to be estimated. The definition of $v_{i+1}^{(j)}(u)$ differs according to whether anticipatory abilities come into play or not. j is the number of predecessor(s) involved in the strategy.

Without anticipation Without anticipation, only the predecessor is taking into account, $j=1$. Its speed is supposed constant and is estimated by the last known value :

$$v_{i+1}^{(1)}(u) = v_{i+1}(t-\mathcal{T}^r),\ u \in]t-\mathcal{T}^r, t] \tag{15}$$

Hence, the position of the predecessor is linearly estimated (TREIBER *et al.* (2006a)) :

$$x_{i+1}^{(1)}(t) = x_{i+1}(t-\mathcal{T}^r) + \mathcal{T}^r v_{i+1}(t-\mathcal{T}^r) \tag{16}$$

One observes that $x_{i+1}^{(1)}(t) = x_{i+1}(t)$ if $\mathcal{T}^r = 0$ or if $v_{i+1}(u) = v_{i+1}(t-\mathcal{T}^r)$, $u \in]t-\mathcal{T}^r, t]$: there is only a lag if the reaction time is strictly positive and the speed of the predecessor is variable.

With anticipation With anticipation, a driver is assumed to be able to estimate the speed of the predecessor on the interval $]t-\mathcal{T}^r, t]$, taking the speed of preceding car $i+j$ as constant. $v_{i+1}^{(j)}$ is calculated as being a solution of the system Eq. (6) with (9) such that $v_{i+j}(u) = v_{i+j}(t-\mathcal{T}^r),\ u \in]t-\mathcal{T}^r, t]$ and $\sigma = 0$. The number of vehicles in interaction is higher the greater j is; the estimation is that much more accurate. When $j=1$, one finds the preceding case devoid of anticipation. When $j>1$, the speed of the predecessor is estimated recursively on the j predecessors by the formula :

$$v_{i+1}^{(j)}(t,\delta) = V_{i+1}(t,\delta, V_{i+2}(\ldots, V_{i+j-1}(t,\delta, v_{i+j}(t))\ldots) \tag{17}$$

where $V_i\left(t,\delta,v\right) = \frac{x_{i+1}(t)-x_i(t)-(\ell_i+\ell_{i+1})/2+\delta v}{\delta+(1-\delta\lambda_i(t))\mathcal{T}_i(t)+\delta\lambda_i(t)\mathscr{F}(v_i(t),\mathcal{T}_i(t))}$ is extracted from the deterministic ($\sigma=0$) discretization scheme. Hence, in a discrete case in which $\mathcal{T}^r$ can be divided by δt (or by using the approximation proposed by TREIBER *et al.* (2006a)), the position of the predecessor is defined by :

$$x_{i+1}^{(j)}(t) = x_{i+1}(t-\mathcal{T}^r) + \delta t \sum_{k=1}^{\mathcal{T}^r/\delta t} v_{i+1}^{(j)}(t-\mathcal{T}^r, k\times\delta t) \tag{18}$$

and can be approximated by the linear interpolation :

$$x_{i+1}^{(j)}(t) = x_{i+1}(t-\mathcal{T}^r) + \frac{\mathcal{T}^r+\delta t}{2} v_{i+1}^{(j)}(t-\mathcal{T}^r, \mathcal{T}^r) + \frac{\mathcal{T}^r-\delta t}{2} v_{i+1}(t-\mathcal{T}^r) \tag{19}$$

4 Statistical estimation of the parameters

In this part, the parameters attached to the car-following model are statistically estimated by the use of some pseudo-continuous sequences of trajectories observations. With the increase in databases on traffic observation and in particular on trajectories, more and more studies are being carried out. In the majority of cases, calibration methods entail minimising an error function. Measuring a convex function of the deviation of a certain objective variable between observed trajectories and simulated trajectories of a vehicle whose initial conditions and the trajectory of the preceding vehicle are real, is used as a function of error (PUNZO and SIMONELLI (2005); DURET *et al.* (2008)). The error function can be minimising through various non-linear algorithms of the genetic algorithm type (RANJITKAR *et al.* (2004); KESTING and TREIBER (2008a)) or the simplex method (BROCKFELD *et al.* (2004); OSSEN and HOOGENDOORN (2008)) for instance. Intra-driver variability can be evaluated by observing the error engendered on a sequence of a given individual, and inter-driver variability (OSSEN *et al.* (2008)) by observing the mean error by sequence engendered on the sample.

In this article, excepted for the targeted safety time function that is estimated in the light of least-square estimations, the parameters model are estimated by maximum likelihood using the expectation-maximisation algorithm. In order to restore both a variability of the response of a driver and a variability between the drivers, a two level statistical model is developed. Regulating parameters are assumed to be distributed across the sample of drivers while the noise parameter is supposed to be distributed across the time. The vehicles are differentiated by type and the pursuit situations are distinguished according to acceleration and deceleration phases.

4.1 Data used

The data used come from the NGSIM project (ngsim.fhwa.dot.gov). Every 10th of a second for 45 minutes they give the trajectory of the vehicles present on a section of around 640 meters of highway 101, a six-lane motorway in California, near Los Angeles (the Hollywood Freeway). The data were collected on 15 June 2005, between 7.50 am and 8.35 am. The numerical values were obtained by analysing the images provided by eight cameras. The data base contains observations of 6 101 trajectories. The context is interactive : it was rush hour and the speeds observed are relatively low (they rarely exceed 20 m/s). The vehicles are differentiated by type : motorcycles, cars and trucks.

4.2 Search for the form of the targeted safety time function

One of the basic assumptions of the model is that a driver tries to abide by a certain time gap in relation to the preceding car as a function of speed, described as the targeted safety time. In order to estimate that function, it seems logical to use the gross observed time gap according different classes of speed. However, the gross time gap is not always indicative of the targeted safety time : different cases have to be distinguished depending on whether a driver is in a free state, catches up with its predecessor, seeks to widen the gap or, what interests us, meets satisfactory safety conditions.

Non-parametric estimation When one estimates the density of the time gap by class of speed (estimations by gaussian kernel, figure 5), the distribution clearly proves to be dependent on speed. Estimations were produced on different samples from American motorways and urban environment (NGSIM project) or French motorway (ZELT) and on different lanes, revealing comparable results (figure 6). Some other statistical studies confirm this finding (BANKS (2003); NISHINARI *et al.* (2003); TREIBER *et al.* (2003), (2006b)). The interpretation varies, but drivers' risky behaviour, in which the time gap tends to decrease with speed, seems to be characteristic. In acceleration case, the values seems to be higher than in the deceleration case.

If one assumes that drivers are attracted by a certain satisfactory safety time as a function of speed, this guarantees a certain stability as regards their behaviors. In particular, it can result in a relatively low variance in the time gap once satisfactory conditions have been attained. For this reason, the sample is limited to observations for which a local (in time, $\pm 2\,s$) estimation of the standard deviation of the time gap is lower than a certain threshold ($\approx$1e-2 s). When the sample is confined to stable situations, the difference between the mean value and the modal value diminishes. The asymmetry of the dispersion of time gap decreases. The values obtained in acceleration are similar to the values obtained in deceleration case and it seems not necessary to distinguished the estimations. The difference observed on the complete sample can

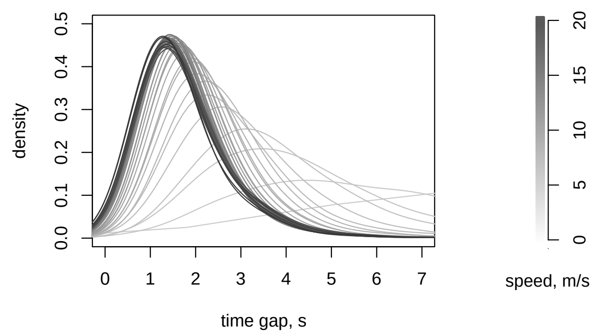

Figure 5: Estimate of densities of time gap with gaussian kernel by classes of speed. All types of vehicle taken together. Complete sample. The classes of speed varies varies from 0.5 to 20 m/s by step of 0.5 m/s getting dark.

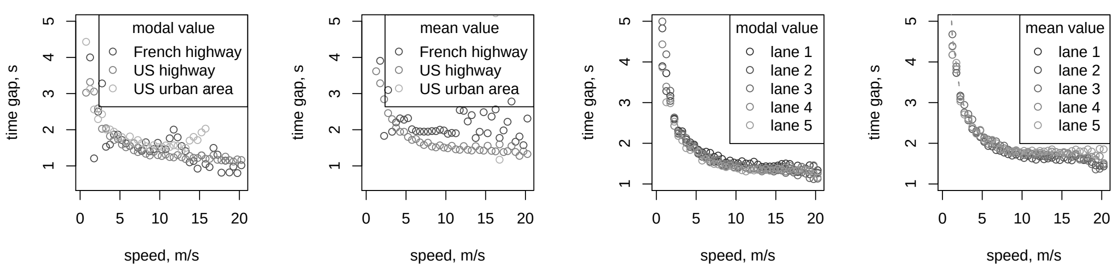

Figure 6: Modal and average estimation of the vehicles time gap by class of speed. All types of vehicle taken together. At left, according to samples of American and French motorways and an urban American environment and, at right, according to different lanes of the Hollywood Freeway 's datas. Complete sample.

be due to the reaction time intervening in unstable situations. When the types of vehicles are differentiated, one finds that the time gap decrease with speed in all cases but the values are unequal (figure 7).

Parametric estimation In the light of the non-parametric estimation by modal value, different functions are selected and compared to estimate $\mathscr{T}(v)$. The models selected depends on three parameters a time γ_1, a distance γ_2 and a speed γ_3 :

$$\mathscr{T}(v) = \gamma_1 + \gamma_2 \times \frac{1}{v} \ln(v/\gamma_3 + 1) \tag{20}$$

Parameters are estimated by least squares on the modal time gap values obtained through the non-parametric estimation for each class of vehicle (figure 8). In the case of cars, parametric estimation seems to correspond to non-parametric estimation. On the other hand, the cases of motorcycles and trucks are more approximate, for want of sufficient samples.

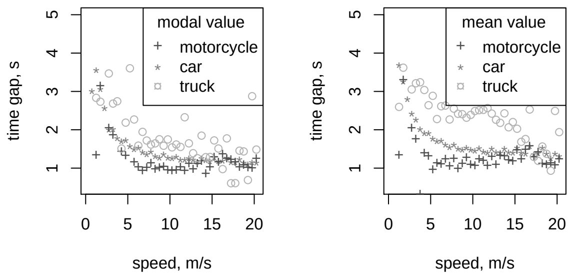

Figure 7: Modal and average estimations of the vehicles time gap by class of speed. From the left to the right : motorcycles, cars and trucks. Restricted sample.

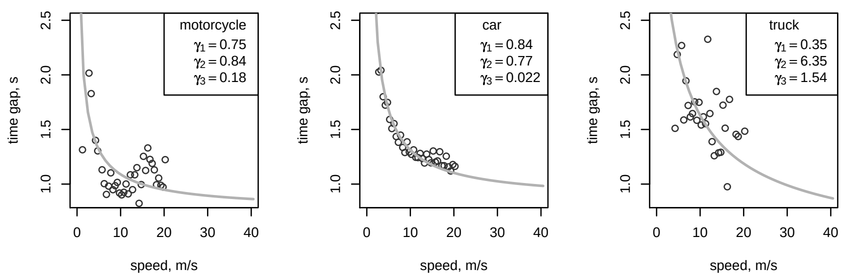

Figure 8: Estimation by least squares of parameters γ_1, γ_2 and γ_3 of the targeted safety function $\mathscr{T}(v) = \gamma_1 + \gamma_2 \times \frac{\ln(v/\gamma_3+1)}{v}$. From the left to the right : motorcycles, cars and trucks.

4.3 Research and estimate of the laws of regulation and noise parameters

In regulation situations in which the speed of the preceding vehicle varies, or in which the distance gap (or time) is unsatisfactory, speed is supposed to be regulated towards a targeted safety speed. In the model, the regulation strategy is parametered by λ, j, $\mathcal{T}^r$ and the noise $\mathcal{E}$ with variability σ^2. The parameter λ calibrates the speed of convergence that defines more or less flexible behaviour. The number j of predecessors involved in the anticipation process allows to palliate to the delay induced by the reaction time $\mathcal{T}^r$. The additive noise $\mathcal{E}$ makes it possible to evaluate a driver response variability and so the model applicability. In the statistical model proposed, the reaction time is supposed constant equal to $\mathcal{T}^r = 1\,s$ for each drivers. Parameters λ, j and $\mathcal{E}$, are supposed to be random variables distributed according to some distributions of probability. The λ and j parameters are assumed to be distributed across the sample of drivers, the $\mathcal{E}$ parameter is supposed to be distributed across the time. The approach involves making a statistical estimation at two levels (GOLDSTEIN (2003)) in which level 1, at the scale of an individual, corresponds to the estimation of the $\mathcal{E}$ parameter variance, and in which level 2, at the scale of the sample, corresponds to the estimation of the λ and j parameters distributions.

The sample is limited in order to arrive at an exclusive car-following case : the observations are restricted so that time gap is lower than 3 s (in order to guarantee an interactive situation). One ensures that instantaneous time gap effectively converges towards the targeted safety time. For each vehicle, only the longest trajectorie that are continuous in time is considered. N_i denoting the duration of the observation sequence of the ith individual and $\mathbf{I}$ the number of individuals observed. For the sequence of a given driver i, the development of the time gap is described by the autoregressive relationship :

$$\mathcal{T}_i^{(j)}(t+\delta t) = (1-\delta t\lambda_i)\mathcal{T}_i^{(j)}(t) + \delta t\lambda_i \mathscr{T}(v_i(t)) + \sigma(\mathcal{W}_i(t+\delta t) - \mathcal{W}_i(t)) \tag{21}$$

Ones denotes $n = [t/\delta t] \in \mathbb{N}$ and $\mathcal{E}_i(n) = \sigma(\mathcal{W}_i(n+1) - \mathcal{W}_i(n))$. Then the autoregressive relationship becomes :

$$\mathcal{T}_i^{(j)}(n+1) = (1-\delta t\lambda_i)\mathcal{T}_i^{(j)}(n) + \delta t\lambda_i \mathscr{T}(v_i(n)) + \mathcal{E}_i(n) \tag{22}$$

with $\mathcal{E}_i(n)$ a centered gaussian variable with variance $\sigma^2\delta t$. In the following, one assumes that $\delta t = \mathcal{T}^r = 1\,s$. The form of the function $\mathscr{T}(v)$ is given by the previous estimates.

Non-parametric estimation To start with, in order to assess the relevance of the partition applied to the sample and with the aim of having an *a priori* on the distributions, the parameters are estimated for each individual by least squares on its observation sequence. The λ parameter is estimated by supposing no anticipation strategy :

$$\tilde{\lambda}_i^{ls} = -\frac{\sum_{n=1}^{N_i}(\mathcal{T}_i(n-1) - \mathscr{T}(v_i(n-1)))(\mathcal{T}_i(n) - \mathcal{T}_i(n-1))}{\sum_{n=1}^{N_i}(\mathcal{T}_i(n-1) - \mathscr{T}(v_i(n-1)))^2} \tag{23}$$

The number j of predecessor involved is estimated by using the estimation $\tilde{\lambda}^{ls}$:

$$\tilde{j}_i^{ls} = \underset{j=\{1,\ldots,5\}}{\arg\min} \sum_{n=1}^{N_i} \Big(\mathcal{T}_i^{(j)}(n) - (1-\tilde{\lambda}_i^{ls})\mathcal{T}_i^{(j)}(n-1) - \tilde{\lambda}_i^{ls}\mathscr{T}(v_i(n-1))\Big)^2 \tag{24}$$

The unbiased empirical standard deviation is then used as an estimator of noise standard deviation:

$$\left(\tilde{\sigma}_i^{ls}\right)^2 = \frac{1}{N_i - 1}\sum_{n=1}^{N_i} \Big(\mathcal{T}_i^{(\tilde{j}_i^{ls})}(n) - (1-\tilde{\lambda}_i^{ls})\mathcal{T}_i^{(\tilde{j}_i^{ls})}(n-1) - \tilde{\lambda}_i^{ls}\mathscr{T}(v_i(n-1))\Big)^2 \tag{25}$$

The histograms of the estimations obtained for the cars in a deceleration case are presented below (figure 9). The distribution of parameter λ appears uni-modal. An asymmetric distribution emerges, which can be modelled by a beta distribution. The distribution of parameter j seems to be exponential and can be modelled by a geometric distribution and the noise $\mathcal{E}$ can be supposed gaussian.

Parametric estimation One assumes henceforth that $(\lambda_i, 1 \le i \le \mathbf{I})$ are independent, random, beta continuous distribution variables, $(j_i, 1 \le i \le \mathbf{I})$ are independent, random, geometric discrete distribution variables, and $(\mathcal{E}_i(n), 1 \le i \le \mathbf{I}, 1 \le n \le N_i)$ are independent, random, centred gaussian distribution variables :

$$\lambda \backsim \mathcal{B}(r,s) \qquad j \backsim \mathcal{G}(p) \qquad \mathcal{E} \backsim \mathcal{N}(0,\sigma^2) \tag{26}$$

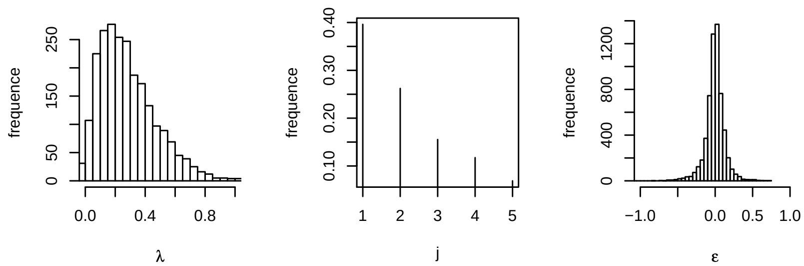

Figure 9: From the left to the right, histograms of estimates by least squares of parameters λ, j and residuals. Car type, deceleration case.

The parameter $\theta = (r, s, p, \sigma)$, of their respective distributions, is estimated with the help of the expectation-maximisation algorithm (DEMPSTER *et al.* (2004)). Since the noise is independent, the likelihood of a given individual i is a product written :

$$\mathcal{L}_i(\mathcal{T}_i, \lambda_i, j_i, \theta) = \frac{\lambda_i^{r-1}(1-\lambda_i)^{s-1}}{\beta(r,s)} p(1-p)^{j_i-1} \prod_{n=1}^{N_i} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\varphi_i(\mathcal{T}_i, \lambda_i, j_i, n)}{2\sigma^2}\right) \tag{27}$$

with $\varphi_i(\mathcal{T}_i, \lambda_i, j_i, n) = \left(\mathcal{T}_i^{(j_i)}(n) - (1-\lambda_i)\mathcal{T}_i^{(j_i)}(n-1) - \lambda_i \mathscr{T}(v_i(n-1))\right)^2$. The complete likelihood of the statistical model is written :

$$\mathcal{L}(\mathcal{T}, \lambda, j, \theta) = \prod_{i=1}^{\mathbf{I}} L_i(\mathcal{T}_i, \lambda_i, j_i, \theta) \tag{28}$$

$\mathscr{L}$ is the natural logarithm of the complete model's likelihood. In the context of the expectation-maximisation algorithm, the parameter θ are estimated sequentially through expectation and maximisation phases. $(\tilde{\theta}_n, n \in \mathbb{N})$ represents the successive estimations of θ. In the expectation phase, at an iteration $n > 0$, the expectation conditionally on the observables of the log-likelihood and to the estimation $\tilde{\theta}_{n-1}$ is calculated. The expectations are expressed with the aid of Bayes formulas. The integrals are approximate due to the numerical method of the rectangles. One denotes $\tilde{\mathbb{E}}_{\tilde{\theta}_{n-1}}(\mathscr{L})$ the approximation of the expectation conditionally on the observables of the log-likelihood. In the maximisation phase, parameters $\tilde{\theta}_n$ are estimated by maximising the quantity $\tilde{\mathbb{E}}_{\tilde{\theta}_{n-1}}(\mathscr{L})$. One finds an analytical value for $\tilde{\sigma}_n$ and $\tilde{j}_n$. On the other hand, $\tilde{r}_n$ and $\tilde{s}_n$ are estimated with the help of the *optim* function loaded with the stochastic SANN (BELISLE (1992)) scheme of the R freeware.

$\tilde{\theta}_0$, the initial estimation value, is given by the previous least squares method. The convergence of the parameters is relatively rapid when there is a large number of observations : 10 or so iterations are enough (car), 30 or so when the sample is small (motorcycle and truck). The findings are presented in figure 10. One finds small estimation of the variance of the noise $\mathcal{E}$ ($\approx 1e$-2) and the results seem to be similar for each sub-sample. Moreover, the variance of the noise is smaller in the expectation-maximisation than in the least squares estimation. One sees that, in an acceleration case, the distribution of parameter λ tends to shift to the right, depending on whether one is looking at trucks, cars or motorcycles respectively. In deceleration cases, one observes inverse behaviors. The estimations of the proportions of the anticipation strategy involved predecessors number are comparable for each sub-sample. One observes that in the majority of cases ($\approx 60\%$) only the direct predecessor is involved in the regulation process. In $\approx 25\%$ of cases, the second predecessor is taking into account, $\approx 10\%$ with the third, $\approx 4\%$ with the forth and $\approx 1\%$ with the fith or more.

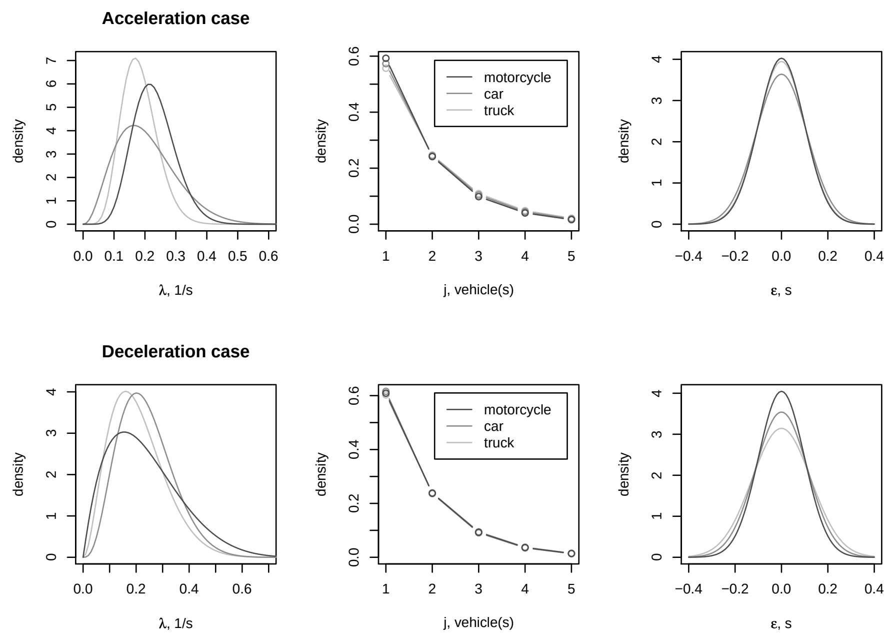

Figure 10: Densities of parameters λ, $\mathcal{E}$ and j estimated using the expectation-maximisation algorithm. Estimations differentiated according to acceleration and deceleration cases and to vehicle type.

5 Studies by simulation on a ring

We explore the macroscopic convergence of a flux whose environment is rectilinear and cyclic (with no frontier). In light of the results of the simulation, it seems that the system has two types of steady states :

- a homogeneous (stable) state in which vehicles' speeds are identical and safety,
- an heterogeneous (unstable) state in which one or several kinetic waves seem to spread out indefinitely.

Two experiments were carried out to evaluate the steady states stability. A single trajectory of vehicles travelling along cyclic stretch of road in an homogeneous state is observed after that the speed of one of the vehicles is reduced to zero during one time step. The experiment allow to observe how a perturbation propagate itself into the flow. In a second experiment, the mean results of several trajectories are measured. For different values of a parameter and different degrees of density, ten independent experiments, whose initial conditions are random, are conducted. The values of the time gap and targeted safety time, that can be very high when the speed is near zero, are limited to $20\,s$.

In the light of the simulation results, it seems that the stationary state reached by a system depends of the parameters of reaction time $\mathcal{T}^r$, targeted safety time function $\mathscr{T}$ and the ability to anticipate j. The regulating parameter λ seems to only interfere the time of the transition phase and do not influences the nature of the stationnary state reached.

5.1 $\mathscr{T}$ constant

When the targeted safety time is constant, three cases can be distinguished. For a targeted safety time strictly greater than the reaction time, with or without anticipation and whatever the initial conditions, the system seems to converge systematically towards a stable state. In the experiment 1, the perturbation is absorbed (figure 11). If the targeted safety time is constant and equal to the reaction time, with no anticipation, perturbations seem to spread indefinitely, without getting larger or smaller (figure 12). The wave of the perturbation propagation spreads backward at the speed of $-\ell/\mathcal{T}$ (independent to the density and the λ parameter). The implementation of some ability to anticipate (when $j > 1$) allow to absorb the perturbation. When the targeted safety time is constant and strictly lower than the reaction time, as expected in the normative approach, without anticipation, collisions take place. Anticipatory strategies is factor of macroscopic stability; when used, the number of collisions due to the reaction time reduces (figure 13). The smaller is the targeted safety time, provided the number of vehicles taking into account in the anticipation strategy is sufficient, no collision takes place. Some comparable results are presented by TREIBER *et al.* (2006a).

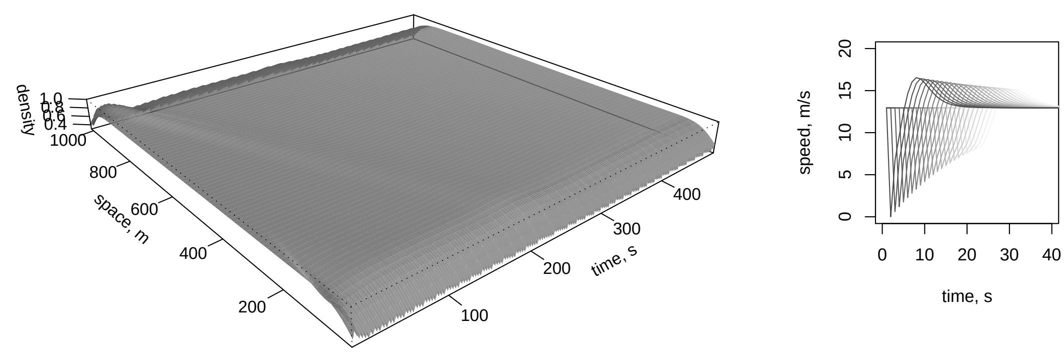

Figure 11: Experience 1. At left, space-time-density plots and, at right, speed of the vehicle perturbated and the 20 following vehicles according to the time. $\mathcal{T}^r = 1\,s$, $\lambda = 0.25\,s^{-1}$, $\mathscr{T} > \mathcal{T}^r$, $j = 1$, $\vartheta = 30\,m/s$, $\ell = 5\,m$.

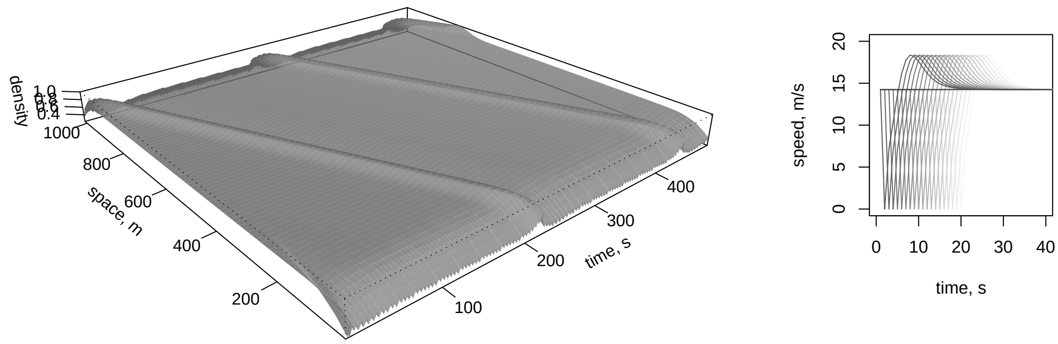

Figure 12: Experience 1. At left, space-time-density plots and, at right, speed of the vehicle perturbated and the 20 following vehicles according to the time. $\mathcal{T}^r = 1\,s$, $\lambda = 0.25\,s^{-1}$, $\mathscr{T} = \mathcal{T}^r$, $j = 1$, $\vartheta = 30\,m/s$, $\ell = 5\,m$.

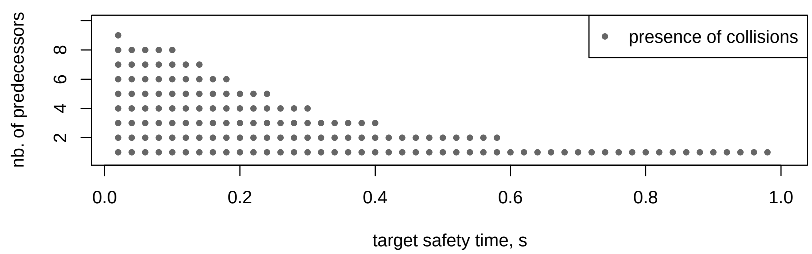

Figure 13: Experiment 2. Presence of collisions within the flux according to the targeted safety time value and the number of predecessors involved in the anticipation strategy. $\mathcal{T}^r = 1\ s$, $\lambda = 0.25\,s^{-1}$, $\vartheta = 30\,m/s$, $\ell = 5\,m$.

5.2 $\mathscr{T}(v)$ variable

The stability of the flow seems to depend heavily on the form of the targeted safety time function. A growing targeted safety time function is a factor of macrosocpic stability and, conversely, a decreasing targeted safety time function is a factor of macroscopic unstability. For instance, let us consider the linear targeted safety time function define in (5). It seems that, as soon as the slop become negative, the flux become instable (figure 14).

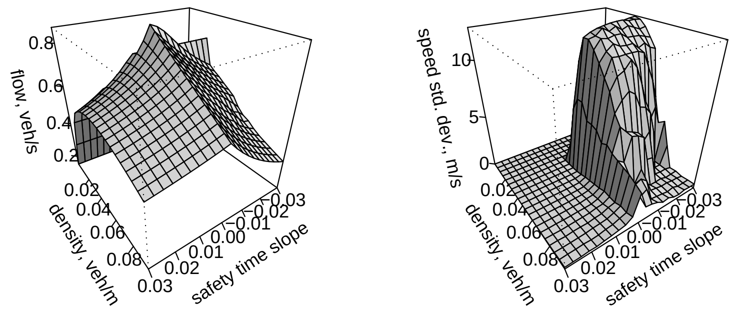

Figure 14: Experiment 2. Flow-density and variance of the speed-density plots. Results obtained according to different values for the slope parameter a of the targeted safety time function (α ranges from -0.03 to 0.03 s^2/m in steps of 0.003 s^2/m). $\mathcal{T}^r = 1\ s$, $\lambda = 0.25\,s^{-1}$, $\mathscr{T}(v)$ linear (definition (5) with $\beta = \mathcal{T}^r$), $j = 1$, $\vartheta = 30\,m/s$, $\ell = 5\,m$.

5.3 $\mathscr{T}(v)$ estimated statistically

Previously, one finds that the form of the targeted safety function obtained in the statistical estimation is strongly decreasing. Several statistical studies confirm this observation. Through simulation, one finds that this form is a powerful factor of macroscopic unstability. In the experiment 1, the flux proves to be very unstable and kinematic waves emerge easily (figure 15).

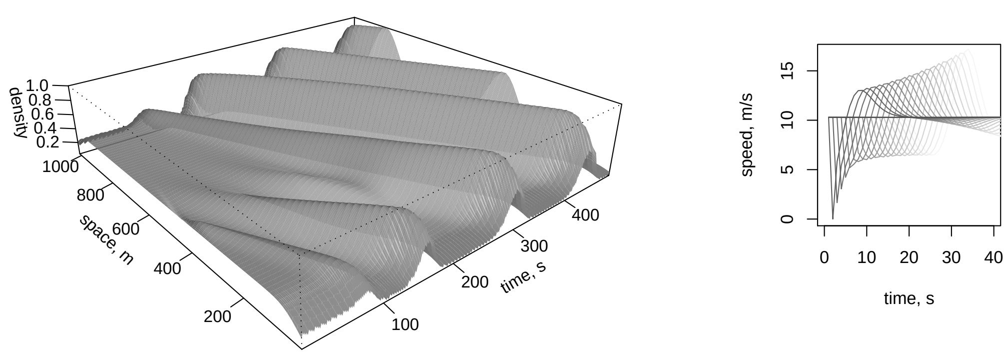

Figure 15: Experience 1. At left, space-time-density plots and, at right, speed of the vehicle perturbated and the 20 following vehicles according to the time. $\mathcal{T}^r = 1\,s$, $\lambda = 0.25\,s^{-1}$, $\mathscr{T}(v) = 2.5 + 0.75\frac{\ln(v+1)}{v}$, $j = 1$, $\vartheta = 30\,m/s$, $\ell = 5\,m$.

The system's convergence towards a state of homogeneity requires the inclusion of a minimum number (≈ 4) of predecessors taking into account in the anticipation strategy (figure 16). When no regulation strategy take place ($j = 1$), the system converge towards an heterogeneous state above a certain threshold value ($\approx 0.7\,s$) of the reaction time (figure 17). Some comparable results are presented by EISSFELDT and WAGNER (2003), TREIBER *et al.* (2007), KESTING and TREIBER (2008b).

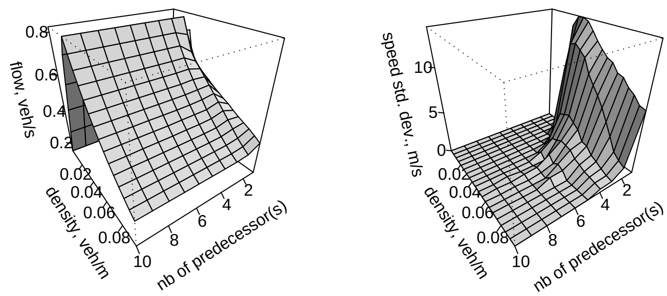

Figure 16: Experiment 2. Flow-density and variance of the speed-density plots. Results obtained according to different values for the number of vehicles involved in the anticipation strategy (j ranges from 1 to 10 vehicles). $\mathcal{T}^r = 1\,s$, $\lambda = 0.25\,s^{-1}$, $\mathscr{T}(v) = 2.5 + 0.75\frac{\log(v+1)}{v}$, $\vartheta = 30\,m/s$, $\ell = 5\,m$.

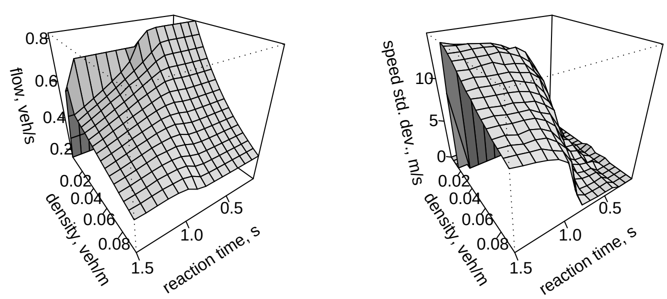

Figure 17: Experiment 2. Flow-density and variance of the speed-density plots. Results obtained according to different values for the reaction time ($\mathcal{T}^r$ ranges from 0.1 to 1.5 s in steps of 0.1 s). $\lambda = 0.25\,s^{-1}$, $\mathscr{T}(v) = 2.5 + 0.75\frac{\log(v+1)}{v}$, $j = 1$, $\vartheta = 30\,m/s$, $\ell = 5\,m$.

6 Conclusion

The car-following model proposed is defined in continuous time by a system of differential equations that describes, in addition to the usual laws of kinematics, the interaction of each vehicle with its predecessor by a relaxation equation that shifts time gaps towards a targeted safety times that is a function of speed. Four quantities are necessary for the car-following model to work with : the targeted safety time function, a relaxation quantity, the reaction time and the number of predecessors involved in the anticipation process.

The statistical findings highlight the differences in behaviour within a traffic flow. They reveal significant differences between vehicle types and also between acceleration and deceleration situations. The results reveal risky behaviour in which time gap decreases with speed for all vehicle types, although it seems that trucks maintain greater time gaps than cars and motorcycles. The estimation throw up a low variance of the noise, attesting to the model's applicability. The estimations of the regulation parameter proved different in acceleration cases and deceleration cases, justifying the asymmetry of driver's longitudinal behaviour. One finds that motorcycles tend to accelerate more sharply than cars and trucks. These findings reflect the greater acceleration capabilities of motorcycles in relation to cars, and even more so relative to trucks. On the other hand, in deceleration cases, it is trucks that prove to decelerate more intensely than motorcycles or cars. The estimations of the number of predecessors involved in the anticipation process are the same on each sub-sample, approximatively exponentially decreasing. In the majority of cases, only the direct predecessor is taking into account by a driver. A non negligeable part of the observations involved several predecessors that do not generally exceed five.

In the model, kinematic waves is generated by the integration of a reaction time. A compensatory effect comes into play between the reaction time, the targeted safety time and the ability to anticipate. The microscopic parameters form allows to control the macroscopic stability of a flow. One observes that a targeted safety time decreasing with speed, observed statistically, is a source of kinematic waves emergence. The simulation experiments showed that anticipatory abilities enable this effect to be offset and a state of homogeneity to be attained, sometimes at the price of a large number of vehicles involved in the anticipation process.

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References

AW, A., KLAR, A., MATERNE, T., RASCLE, M., 2002. Derivation of continuum traffic flow models from microscopic follow-the-leader models. *SIAM J. Applied Mathematics* **63(1)**, 259-278.

BANDO, M., HASEBE, K., 1995. Dynamical model of traffic congestion and numerical simulation. *Physical Review E* **51(2)**, 1035-1042.

BANDO M., NAKANISHI, K., NAKAYAMA, A., 1998. Analysis of Optimal Velocity Model with explicit delay. *Physical Review E* **58(5)**, 5429-5435.

BANKS, J. H., 1999. Investigation of some characteristics of congested flow. *Transportation Research Record* **1678**, 128-134.

BANKS, J. H., 2003. Average time gaps in congested freeway flow. *Transportation Research Part A* **37(6)**, 539-554.

BELISLE, C. J. P., 1992. Convergence theorems for a class of simulated annealing algorithms on $\mathbb{R}^d$. *Journal of Applied Probability* **29(4)**, 885-895.

BIERLEY, R. L., 1963. Investigation of an inter vehicle spacing display. *Highway Research Record* **25**, 58-75.

BROCKFELD, E., KUHNE, R. D., WAGNER, P., 2004. Calibration and validation of microscopic traffic flow models. *Transportation Research Record* **1876**, 62-70.

CARRÉ, J.-R., LASSARRE, S., LIGER, M., 1985. Fluidité - Sécurité : faut-il choisir? (2^e partie). *T.E.C.* **69** Mars-Avril, 8-15.

CHANDLER, F. E., HERMAN, R., MONTROLL, E. W., 1958. Traffic dynamics : studies in car following. *Operations Research* **6(2)**, 165-184.

DEMPSTER, A., P., LAIRD, N. M., RUBIN, D. B., 1977. Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society B* **39(1)**, 1-38.

DURET, A., BUISSON, C., CHIABAUT, N., 2008. Estimating individual speed-spacing relationship and assessing ability of Newell's car-following model to reproduce trajectories. *Transportation Research Record* **2088**, 188-197.

EDIE, L. C., 1961. Car-following and steady state theory for non-congested traffic. *Operation Research* **9(1)**, 66-76.

EISSFELDT, N., WAGNER, P., 2003. Effects of anticipatory driving in a traffic flow model. *The European Physical Journal B* **33(1)**, 121-129.

GAZIS, D. C., HERMAN, R., ROTHERY, R. W., 1961. Non-linear follow-the-leader models of traffic flow. *Operations Research* **9(4)**, 545-567.

GIPPS, P.G., 1981. A behavioural car-following model for computer simulation. *Transportation Research Part B* **15(2)**, 105-111.

GOLDSTEIN, H., 2003. *Multilevel Statistical Models. Third edition*. London, Edward Arnold.

GREENBERG, H., 1959. An analysis of traffic flow. *Operations Research* **7(1)**, 79-85.

GREENSHIELDS, B. D., 1935. A study of traffic capacity. *Highway Research Board Proceedings* **14(1)**, 448-477.

HELBING, D., TILCH, B., 1998. Generalized force model of traffic dynamics. *Physical Review E* **58(1)**, 133-138.

HELLY, W., 1961. Simulation of bottlenecks in single lane traffic flow. *Proceedings of the Symposium on Theory of Traffic Flow*, Herman R. C. Ed., 207-238.

JUANG, R., WU, Q., ZHU, Z., 2001. Full velocity difference model for a car-following theory. *Physical Review E* **64(1)**, 017101.1-017101.4.

KERNER, B. S., KONHÄUSER, P., 1994. Structure and parameters of clusters in traffic flow. *Physical Review E* **50(1)**, 54-83.

KESTING, A., TREIBER, M., 2008a. Calibrating car-following models using trajectory data : methodological study. *Transportation Research Record* **2088**, 148-156.

KESTING, A., TREIBER, M., 2008b. How reaction time, update time and adaptation factor influence the stability of traffic flow. *Computer-Aided Civil and Infrastructure Engineering* **23(2)**, 125-137.

KOMETANI, E., SASAKI, T., 1958. On the stability of traffic flow. Report n° 1. *J. of Operations Research Japan* **2(1)**, 11-26.

KOMETANI, E., SASAKI, T., 1961. Dynamic behavior of traffic with a nonlinear spacing. *Proceedings of the Symposium on Theory of Traffic Flow*, Herman R. C. Ed., 105-119.

LENZ, H., WAGNER, C. K., SOLLACHER, R., 1999. Multi-anticipative car-following model. *The European Physical Journal B* **7(2)**, 331-335.

LEUTZBACH, W., WIEDEMANN, R., 1986. Development and applications of traffic simulation models at the Karlsruhe Institut fur Verkehrwesen. *Traffic Engineering and Control* **27(5)**, 270-278.

LEUTZBACH, W., 1988. *An introduction to the theory of traffic flow*. Springer-Verlag, Berlin.

NEWELL, G. F., 1961. Nonlinear effects in the dynamics of car following. *Operations Research* **9(2)**, 209-229.

NEWELL, G. F., 2002. A simplified car-following theory : a lower order model. *Transportation Research Part B* **36(3)**, 195-205.

NISHINARI, K., TREIBER, M., HELBING, D., 2003. Interpreting the wide scattering of synchronized traffic data by time gap statistics. *Physical Review E* **68(2)**, 067101.1-067101.4.

OSSEN, S., HOOGENDOORN, S. P., GORTE, B. G., 2007. Inter-driver differences in car-following : A vehicle trajectory based study. *Transportation Research Record* **1965**, 121-129.

OSSEN, S., HOOGENDOORN, S. P., 2008. Validity of trajectory-based calibration approach of car-following models in the presence of measurement errors. *Transportation Research Record* **2088**, 117-125.

PIPES, L. A., 1967. Car-following models and the fundamental diagram of road traffic. *Transportation Research* **1(1)**, 21-29.

PUNZO, V., SIMONELLI, F., 2005. Analysis and comparision of microscopic flow models with real traffic microscopic data. *Transportation Research Record* **1934**, 53-63.

RANJITKAR, P., NAKATSUJI, T., ASANO, M., 2004. Performance evaluation of microscopic flow models with test track data. *Transportation Research Record* **1876**, 90-100.

TREIBER, M., HENNECKE, A., HELBING, D., 2000. Congested traffic states in empirical observations and microscopic simulations. *Physical Review E* **62(2)**, 1805-1824.

TREIBER, M., HELBING, D., 2003. Memory effects in microscopic traffic models and wide scattering in flow-density data. *Physical Review E* **68(4)**, 046119.1-046119.8.

TREIBER, M., KESTING, A., HELBING, D., 2006a. Delays, inaccuracies and anticipation in microscopic traffic models. *Physica A* **360(1)**, 71-88.

TREIBER, M., KESTING, A., HELBING, D., 2006b. Understanding widely scattered traffic flows, the capacity drop, and platoons as effects of variance-driven time gaps. *Physical Review E* **74(1)**, 016123.1-016123.10.

TREIBER, M., KESTING, A., HELBING, D., 2007. Influence of reaction times and anticipation on the stability of vehicular traffic flow. *Transportation Research Record* **1999**, 23-29.

UNDERWOOD, R. T., 1961. Speed volume and density relationships : Quality and theory of traffic flow. *Bureau of Highway Traffic*, Yale University, 141-188.

WANG, J., DIXON, K. K., LI, H., OGLE, J., 2006. Normal Deceleration Behavior of Passenger Vehicles at Stop Sign-Controlled Intersections Evaluated with In-Vehicle Global Positioning System Data. *Transportation Research Record* **1937**, 120-127.

WIEDEMANN, R., 1974. Simulation des Straenverkehrsflusses. *Schriftenreihe des instuts für verkehrswesen*, Universität Karlsruhe, Heft **8**.

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