A Behavioral Car-Following Model that Captures Traffic Oscillations

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Abstract

This paper presents a behavioral car-following model based on empirical trajectory data that is able to reproduce the spontaneous formation and ensuing propagation of stop-and-go waves in congested traffic. By analyzing individual drivers' car-following behavior throughout oscillation cycles it is found that this behavior is consistent across drivers and can be captured by a simple model. The statistical analysis of the model's parameters reveals that there is a strong correlation between driver behavior before and during the oscillation, and that this correlation should not be ignored if one is interested in microscopic output. If macroscopic outputs are of interest, simulation results indicate that an existing model with fewer parameters can be used instead. This is shown for traffic oscillations caused by rubbernecking as observed in the US 101 NGSIM dataset. The same experiment is used to establish the relationship between rubbernecking behavior and the period of oscillations.

1   Introduction

Stop-and-go driving, or simply traffic oscillation, has raised much concern in the literature due to its severe negative impacts: increased fuel consumption, greenhouse emissions and safety risks (Bilbao-Ubillos 2008; Zheng, Ahn and Monsere 2010). Although these traffic oscillations were first reported fifty year ago on the Lincoln tunnel (Edie, 1961), our understanding of the causes and mechanisms of its formation and propagation is still limited. Several studies have shown that one of the causes of oscillations is lane-changing (Ahn and Cassidy, 2006; Laval, 2006; Laval and Daganzo, 2006; Mauch and Cassidy, 2002; Zheng et al., 2011a; Zheng et al., 2011b), but it has become clear now that oscillations may form away from lane changes, e.g. in tunnels or the ones in Figure 1(a). Also unclear are the mechanisms that induce oscillation amplitude growth (i.e., vehicle speed inside the oscillation decreases persistently as it propagates upstream) and regular periods, which vary from 2-15min (Ahn, 2005; Ahn et al., 2004; Laval et al., 2009; Mauch and Cassidy, 2002).

Modeling efforts on the subject date back to the 50s and are still ongoing, see e.g. Laval and Leclercq (2010) for a review. Early theoretical studies (Chandler et al., 1958; Herman et al., 1958) analyzed the linearity of car-following models and attributed the propagation of oscillation to the over-reactions of car-following behaviors. Recently, partial success has been achieved by car-following stability analysis, which has shown that small disturbances can grow to fully fledged oscillations with realistic periods when model parameters are carefully chosen (Wilson, 2008; Wilson and Ward, 2011). While this has been proven for differential equation-type models (e.g. the Bando, Hasebe et al. (Bando et al., 1995) or the IDM model (Treiber et al., 2000)), it is expected that this result should hold more generally. Treiber and Kesting (2011) analytically calculated the characteristics of traffic oscillations (e.g., wavelength and growth rate) and argued that traffic oscillations are manifestation of convective instability. Additionally, they claimed that the stability class depends not only on the model but also on model parameters. Li and Ouyang (2011) developed a mathematical framework to quantify the period and magnitude of traffic oscillations and then analyzed these characteristics across the propagation of traffic oscillations. They found that the nonlinearity is critical in producing reasonable oscillation growth; i.e., bounded growth. This approach

==provides understandings of traffic oscillations from the mathematical perspective, however, we argue that it is limited in providing insights to better understand driver behavior under traffic oscillations.==

Conversely, Laval and Leclercq (2010) conjectured that the formation and propagation of traffic oscillations were due to the aggressive or timid driver behavior. Simulations showed that their model, referred to as the L-L model hereafter, could reproduce traffic oscillation period and amplitude as observed empirically. This study suggests a new perspective to explain the mechanism of formation and propagation for traffic oscillations. However, the behavioral model assumed in this model has not been verified empirically. Additionally, this model assumed that drivers behave homogeneously in equilibrium (i.e., following the same fundamental diagram) and consistently before and after traffic oscillations. These assumptions, however, were not validated by empirical observations.

Zheng et al. (2011a) used wavelet transform method to analyze vehicle trajectories and study traffic oscillation features such as amplitude and duration. To the best of our knowledge, Zheng et al. (2011b) is the only reference that uses trajectory data to investigate driver behavior during oscillations. However, their focus was on comparing driver behavior before and after oscillations, while a comprehensive dynamic description of car-following behavior across oscillations is still missing.

This paper aims to fill this void. The objective is to develop a behavioral model that describes driver behavior under congested traffic by analyzing vehicle trajectories undergoing traffic oscillations. Towards this end, Section 2 describes the trajectory data and Section 3 introduces background information. Section 4 presents the measurement methodology and formulates the new model. Section 5 conducts statistical analysis of model parameters and section 6 shows an application of the model in the case of rubbernecking. A discussion and outlook is provided in section 7.

2 Data Description

The trajectory data used in this paper comes from the Next-Generation Simulation project (NGSIM, 2006); i.e., the 2100-foot southbound US-101 segment in Los Angeles, California from 7:50 a.m. to 8:35 a.m. on June 15, 2005. Figure 1(a-b) shows the trajectories on the median lane from 7:50 a.m. to 8:20a.m and Figure 1 (c) is a sketch of the study site. Notice how traffic oscillations spontaneously appear every 2-3 minutes in the first 15min (Figure 1(a)) and also oscillations from downstream appear in the second 15min. For the spontaneously formed oscillations, after thorough examination of the data, the possibility of lane-changing in triggering traffic oscillations is eliminated. Thorough examination of the freeway design elements and the video data suggests that the most likely cause for the traffic oscillations seems to be rubbernecking. During that time at the same location where traffic oscillations begin, clean-up work was being performed in the median; see Figure 1(a) for a snapshot. Vehicles tend to slow down when they approached that spot. The thoughts about the cause will be tested and discussed in Section 6 and 7.

Figure 1 Illustration of vehicle trajectories and study site

(a) vehicle trajectory plot (for US 101 study site); (b) schematic of the study site: southbound US-101 in Los Angeles, California (NGSIM, 2006).

3 Background

3.1 Newell's car-following model

Newell's car-following model (Newell, 2002) gives the exact numerical solution of the kinematic wave model (Lighthill and Whitham, 1955; Richards, 1956) with a triangular fundamental diagram (referred to as the KWT model hereafter). In free-flow vehicles always travel at the free-flow speed, while in congestion a vehicle trajectory is a translation in time and space of the leader's by τ and δ, respectively; see Figure 2(a):

$$x_{i+1}(t) = \min\{\underbrace{x_{i+1}(t-\tau) + u\tau}_{\text{free-flow}}, \underbrace{x_i(t-\tau) - \delta}_{\text{congestion}}\}, \tag{1}$$

where $x_{i+1}(t)$ represents the position of vehicle $i+1$ at time t and u is the free-flow speed. The time and space shift (τ, δ) is assumed constant for a given vehicle but may vary as a random walk across different vehicles. This model implies a linear speed-spacing relationship in congestion; see Figure 2 (b):

$$S(v) = \delta + \tau.v, \tag{2}$$

where v is vehicle speed and $S(v)$ is the corresponding equilibrium spacing. Notably, parameters τ and δ can be written using the wave speed $-w$, and jam density κ in the KWT model, i.e.:

$$\tau = \frac{1}{w\kappa} \text{ and } \delta = \frac{1}{\kappa}. \tag{3}$$

Therefore, δ can be interpreted as the jam spacing and τ as the wave trip time between two consecutive congested vehicles.

Newell's car-following model is appealing for its simplicity and consistency with the KWT model. However, in this model disturbances will never amplify or decay.

Figure 2 Model illustration

(a) Newell's car-following model; (b) speed-spacing relation in Newell's car-following model; (c) examples of deviation from Newell trajectory (NGSIM, 2006); (d) the L-L model.

3.2 The L-L model

Laval and Leclercq (2010) observed that vehicle trajectories accord well with Newell' car-following model before they experience traffic oscillations. Namely, a follower's trajectory overlaps its leader's shifted by (τ, δ). In the sequel, this shifted trajectory is called a "Newell trajectory". However, deviations were significant during the oscillation; see Figure 2(c). To capture these, the L-L model allows vehicle trajectories to deviate from Newell trajectories by explicitly including a behavioral model via the term $\eta_i(t)$:

$$x_{i+1}(t) = \min\{\underbrace{x_{i+1}(t-\tau) + \min\{u\tau, \tilde{x}_{i+1}(t)\}}_{\text{free-flow}}, \underbrace{x_i(t - \eta_{i+1}(t)\tau) - \eta_{i+1}(t)\delta}_{\text{congestion}}\}, \tag{4}$$

where $\tilde{x}_{i+1}(t)$ is the desired distance travelled during τ resulting from a vehicle kinematics model that limits accelerations:

$$a(v) = a_m(1 - {v}/{u}) - gG, \tag{5}$$

where the acceleration $a(v)$ depends on the current speed v, maximum acceleration a_m, the gravitational acceleration $g = 9.8 \text{ m/s}^2$, and the percent grade 100G%. By solving (5) analytically, one can obtain the expression for $\tilde{x}_{i+1}(t)$:

$$\tilde{x}_{i+1}(t) = u\left(1 - \frac{gG}{a_m}\right)\tau - \frac{\left(1 - e^{-\frac{\tau a_m}{u}}\right)\left(u\left(1 - \frac{gG}{a_m}\right) - v_{i+1}(t-\tau)\right)}{a_m/u}. \tag{6}$$

The term $\eta_{i+1}(t)$ captures the non-equilibrium behavior often observed in the dataset; i.e., deviations from Newell's model. It is defined as the ratio of the actual steady-state spacing and the equilibrium spacing; i.e.:

$$\eta_{i+1}(t) = s_{i+1}(t)/S(v_i(t_i^*)), \tag{7}$$

where $s_{i+1}(t)$ is the steady-state spacing for vehicle $i+1$ at time t, $S(.)$ is the equilibrium spacing given by (2), and t_i^* is the time the characteristic line is emanated from the leader reaching the follower at time t; see Figure 2(d). Notice that the steady-state spacing is measured by assuming that the leader is stationary and driving at $v_i(t_i^*)$ until time t, and $S(v_i(t_i^*))$ is obtained from (2) with v equal to $v_i(t_i^*)$ (see Figure 2(d)). Clearly, when a driver is in equilibrium, $\eta_i(t) = 1$ and the follower's trajectory overlaps the Newell trajectory; otherwise, $\eta_i(t)$ deviates from 1. Non-equilibrium behavior in the follower is triggered in this model whenever the leader decelerates. The ensuing evolution of $\eta_i(t)$ is assumed to fall into one of three patterns, as illustrated in Figure 3: concave triangle, convex triangle, or constant. This gives rise to three user classes: timid, aggressive, and Newell drivers, respectively.

Figure 3 Behavior patterns

The physical meaning of the concave (convex) triangle pattern is that when drivers first perceive an oscillation they will tend to get farther (closer) to the leader -- relative to the equilibrium congested branch -- until some critical point where they will tend to come back to the equilibrium branch. The maximum degree of deviation from the equilibrium is denoted by α and the corresponding η_i-value, η_T. Namely, $\alpha = |\eta_T - 1|$. Clearly, the constant pattern implies that the driver will remain on their equilibrium branch throughout the oscillation. The rate $\varepsilon = \dot{\eta}_i(t)$ is assumed constant.

Notice that the driver behavior described by the L-L model was a conjecture inspired from observations, and it has not been verified yet.

4 Measurements

Up to this point, the behavioral model just described is in a state of conjecture. We now undertake the empirical measurement of $\eta_i(t)$, which is absent in the study of Laval and Leclercq (2010), and find new behavioral patterns that arise recurrently in our sample. Fortunately, these new patterns can be captured by a parsimonious model, of which the L-L model is a special case.

4.1 Methodology

The method for measuring $\eta_i(t)$ is illustrated in Figure 4, where it can be seen that $\eta_i(t)$ can also be written as:

$$\eta_i(t) = \tau_i(t)/\tau . \tag{8}$$

The term $\tau_i(t)$ is the actual wave travel time and τ is the equilibrium value given by Eqn. (3). It follows that the measurement of $\eta_i(t)$ boils down to measuring $\tau_i(t)$, which is much more convenient than measuring the steady-state spacing. To see this, recall that when the vehicle is accelerating/decelerating, its steady-state spacing is given by the term " $S(v_i(t_i^*))\eta_{i+1}(t)$ " in Figure 4, which is not easy to observe from the data.

Figure 4 Measurement of $\eta_i(t)$

Notice that the value of $\eta_i(t)$ depends on the product $w\kappa$ by virtue of Eqn. (3) and also on w alone since it defines the slope of characteristics. Fortunately, we have observed that within a wide range of values of w and κ, the general shape of $\eta_i(t)$ is maintained across different measurements; see e.g. Figure 5. In our analysis we use w = 16 km/h, as the case in many studies using NGSIM data (Durent et al., 2011; Laval and Leclercq, 2010), and τ is set to be the average of $\tau_i(t)$ in equilibrium before oscillations from all trajectories sampled.

Figure 5 Effects of model parameters

4.2 The behavior model

We observe that the driver behavior across traffic oscillations follows a very consistent pattern across the sample; see Figure 6(a) for a typical example. Generally, drivers maintain a constant $\eta_i(t)$ before the oscillatory state. When the deceleration wave arrives, $\eta_i(t)$ deviates from the constant level, which implies that they enter the non-equilibrium mode. Interestingly, after the deviation, $\eta_i(t)$ maintains a constant level again, although it is not necessarily the value before the non-equilibrium. Based on these observations, the "asymmetric behavioral model" is proposed to describe a driver's car-following behavior in congestion: (i) drivers enter the non-equilibrium mode when an oscillation is triggered; (ii) the ensuing dynamics of $\eta_i(t)$ in non-equilibrium, called "reaction pattern", follow one of three categories: concave triangle, convex triangle, and constant; and (iii) before and after the non-

equilibrium mode, drivers are in equilibrium. The model can be well described using the following parameters, as shown in Figure 6(b):

η_i^0 is the stable value of $\eta_i(t)$ <u>before</u> the non-equilibrium behavior,
η_i^1 is the stable value of $\eta_i(t)$ <u>after</u> the vehicle has reached equilibrium,
η_i^T is the value of $\eta_i(t)$ when the vehicle has the maximum deviation from η_i^0,
ε_i^0 is the average slope of $\eta_i(t)$ between η_i^0 and η_i^T, and
ε_i^1 is the average slope of $\eta_i(t)$ between η_i^T and η_i^1.

The moment the non-equilibrium behavior starts is the last essential component of the model. Based on our observations, in most cases the trigger towards non-equilibrium is the initial deceleration wave emanated from the oscillation. Therefore, we assume that drivers in equilibrium will switch to non-equilibrium mode as soon as they are forced to decelerate. As an example, the dynamics of $\eta_i(t)$ for the concave triangle pattern can be written as:

Before non-equilibrium: $\eta_i(t) = \eta_i^0$; $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (9 − 1)

During non-equilibrium: $\begin{cases} \eta_i(t) = \eta_i^0 + \varepsilon_i^0 * (t - t^0), \eta_i(t) \le \eta_i^T; & (9-2) \\ \eta_i(t) = \eta_i^T - \varepsilon_i^1 * (t - t^T), \eta_i^1 \le \eta_i(t) \le \eta_i^T; & (9-3) \end{cases}$

After non-equilibrium: $\eta_i(t) = \eta_i^1$; $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (9 − 4)

where t^0 denotes the starting point of non-equilibrium and t^T is the instance when η_i^T is achieved. The dynamics of convex pattern are identical except for the sign preceding ε_i^0 and ε_i^1.

Notice that in this model (i) drivers are heterogeneous and each driver has its own set of parameters $[\eta_i^0, \eta_i^1, \eta_i^T, \varepsilon_i^0, \varepsilon_i^1]$, (ii) η_i^0 is not necessarily equal to 1, and it may change after the driver passes the oscillation to η_i^1, which means that each driver has a unique equilibrium congestion branch before traffic oscillations and does not necessarily come back to that branch after oscillations, and (iii) the reaction patterns (i.e., convex and concave triangles) are not necessarily isosceles; i.e., $\varepsilon_i^0 <> \varepsilon_i^1$.

Figure 6 Behavior profile of the asymmetric behavioral model
(a) empirical plot of $\eta(t)$; (b) illustration of asymmetric behavioral model.
Figure 7 Driver categories
(a) physical meaning of driver categories; (b) histogram of η_i^0.

The physical meaning of η_i^0 is clear. In the context of the KWT model different values of η_i^0 correspond to different congested branches in the fundamental diagram; see Figure 7(a). If $\eta_i^0 \ll 1 (\eta_i^0 \gg 1)$, the driver prefers to maintain a spacing smaller (larger) than the average level, referred to as "originally aggressive" ("originally timid") driver, abbreviated as OA (OT); see point "1" ("3"). If η_i^0 is close to 1 (i.e., $\eta_i^0 \sim 1$), the driver behaves at the average level, named as "originally Newell" driver, abbreviated as ON; see point '2'. Hence, the η_i^0-value captures the characteristics of a driver in equilibrium *before* traffic oscillations, and similarly the η_i^1-value captures the characteristics in equilibrium *after* traffic oscillations. Notice that driver categories are determined by their characteristics in equilibrium before traffic oscillations, and that every driver category can exhibit any of the three reaction patterns. In contrast, the original L-L model assumes that drivers are homogeneous and their reaction to oscillations is symmetric; i.e., $\eta_i^0 = \eta_i^1$ and $\varepsilon_i^0 = \varepsilon_i^1$. Additionally, driver categories are determined by the reaction pattern. Clearly, the original L-L model is only a special case of the asymmetric behavior model. Table 3 provides a summary of the asymmetric behavior model and also the difference between this model and the original L-L model.

Table 1: Difference between original L-L Model and the asymmetric behavior model

The parameters of the asymmetric behavior model can be measured from the $\eta_i(t)$ plot obtained using the methodology introduced above. Particularly, the measured $\eta_i(t)$-function is approximated by a piecewise-linear function characterized by parameters $[\eta_i^0, \eta_i^1, \eta_i^T, \varepsilon_i^0, \varepsilon_i^1]$. This is illustrated in Figure 6(b), which also shows the corresponding speed profile. Time t^0 corresponds to the beginning of the deceleration cycle (i.e., starting point of the non-equilibrium mode) and t^T is the instance when η_i^T, the maximum or minimum value of $\eta_i(t)$ during the oscillation, is achieved. Time t^1 is selected to minimize the variation of linear regression on segment 3 and 4. The average level for segment 1 is η_i^0 and η_i^1 for segment 4. The slopes of segment 3 and 4 are ε_i^0 and ε_i^1, respectively.

5 Statistical Analysis

In this section we perform a statistical analysis on the collected sample. The main purposes are to (1) study the behavior characteristics of drives, and (2) examine the relationship between model parameters, which may help to further simplify the asymmetric behavior model.

Figure 8 Correlation between successive drivers

We exhaustively sampled lane 1 from 7:50a.m.-8:05a.m. and examined the correlation of driver characteristic (denoted by η_i^0) between successive drivers. Figure 8 shows the result from 123 trajectory pairs (with lane-changers excluded), where the horizontal and vertical axes are the η_i^0 value for the leaders and followers, respectively. The plot suggests no significant correlation between the value η_i^0 for two successive drivers. The same analysis for the remaining parameters yields the same result. This implies that driver behavior is a "personality" characteristic, independent of other drivers. As a result, in the sequel we use random sampling to select trajectory pairs from other lanes/time periods for detailed investigation.

We sampled trajectories from 7:50a.m.-8:20a.m. on lanes where traffic oscillations occurred, but did not include time period after 8:20a.m.because oscillations can hardly be distinguished thereafter. Particularly, the sample was split to two groups, period 1 and period 2 corresponding to the first and the second 15min. The reason for the splitting is that oscillations originating downstream of the study site appeared after about 8:03a.m.; see Figure 1 (a-b). We have obtained 44 trajectories pairs from period 1 and 67 from period 2. Investigation was conducted on this sample.

5.1 Statistics on drivers' behavior characteristics

Figure 7(b) is the histogram of η_i^0 for all 111 trajectory pairs. The threshold of ON drivers is set to be $0.9 \leq \eta i0 \leq 1.1$, and the lower (upper) regime corresponds to OA (OT) drivers. Notice from Figure 9 that the composition of driver categories is consistent in period 1 and period 2. Of course, the proportions may change if the threshold varies, but the magnitude should hold.

Figure 9 Vehicle categories

We find a correlation between the driver category and the reaction pattern: (i) OA drivers tend to have the concave triangle pattern, and OT drivers the convex pattern; (ii) ON drivers are equally likely to have any of the three patterns in period 1 but prefer to concave triangle pattern in period 2; see statistics in Figure 10. Figure 11 illustrates some common and typical examples for the evolution of $\eta_i(t)$, which account for roughly 90% of the sample. Result (i) is intuitive. OA drivers maintain a small spacing before the oscillation, and therefore, they have little room to further decrease the spacing in non-equilibrium. In contrast, OT drivers preserve more than enough room and may feel less pressure to react. In other words, OA drivers are less aggressive during oscillations and OT drivers are less timid. Result (ii), however, is surprising. Two reasons may contribute to this result: (1) Oscillations in period 1 spontaneously arose in the study site (at distance 400m, see Figure 1(a)) while originated from downstream in period 2 (see Figure 1(b)); (2) in period 1 traffic was moderately congested with speeds of about 30mph before the oscillations and recovered to 40~50mph after; while it was dense in period 2 with 20~30mph driving speed before the oscillations and below 30mph after. This can be seen in Figure 12. In dense traffic, the equilibrium spacing is much smaller, which forces ON drivers to adopt a relatively safe reaction pattern, in this case the concave triangle pattern. Under moderate congestion there is no particular reason to react in either way.

Figure 10 Behavior pattern statistics
Left: period 1; Right: period 2.

Figure 11 Examples of behavior patterns during oscillations (circled in dashed line)
(a) originally aggressive driver; (b) originally timid driver; (c) originally Newell driver.

Figure 12 Examples of trajectories and speed profiles in different periods.
(a): period 1; (b): period 2.

Figure 13 shows the histogram of all parameters for the whole sample because we did not observe significant differences between the distributions from periods 1 and 2. Table 2 shows the mean and coefficient of variation for all model parameters. The sample has been broken down according to the reaction patterns to avoid spurious results. In both periods, the coefficients of variation for the η's are around 20 -- 35%, which indicates mild variability. A very high variability is observed in the case of the ε_i^0 and ε_i^1, which may be due to the large variation in the duration of the oscillations.

Table 2: Descriptive statistics

Figure 13 Histogram of all parameters

5.2 Hypotheses tests on model parameters

In this section, we test the correlation between model parameters; i.e., all the five parameters are considered as random variables and the correlation of each pair is examined. We also perform the paired t-test to see whether or not variables are significantly different. These are important because (i) the correlation will determine whether we can assume independent distributions for the parameters in the model, and (ii) the significance of difference of paired differences will help to decide if parameters can be combined.

Tables 3 and 4 show the t-statistic from least squares regression for all parameter combinations. The tables also show the p-values from the Wilconxon signed-rank test, a non-parametric statistical hypothesis test used to compare two repeated measurements on a single sample to assess whether their population means differ. Notice that the sample size may be not large enough to estimate the distribution of a parameter. Therefore, non-parametric hypothesis tests were conducted, which did not require information about the variable distribution as the input. The main conclusions, at the 95% confidence level, are that in the two periods for both patterns (i)η_i^0, η_i^T, and η_i^1 are all correlated, and (ii) ε_i^0 and ε_i^1 are not significantly different. Result (i) implies that all pairs among parameters η_i^0, η_i^T, and η_i^1, are correlated. Additionally, differences exist between samples from the two periods: (iii) ε_i^0 and ε_i^1 have no (or very week) correlation with each other and with the η's except for the concave pattern sample in period 2; and (iv) η_i^1 and η_i^0 are not significantly different in period 2 but the difference is significant for convex pattern sample and the combined sample in period 1.

The correlation among the model parameters was expected. It indicates that drivers behave differently and that these differences are consistent across drivers. Notice that the correlation may be obscured if samples of the two patterns are combined. For example, the correlation between η_i^T and η_i^0 is significant for the separate samples, but not for the combined sample. Also expected was that $\varepsilon_i^0 = \varepsilon_i^1$ since there is no strong reason to believe that deviations to and from equilibrium should happen at different rates. But the controversial finding about the distribution of η_i^1 and η_i^0 comes to a surprise. The result that $\eta_i^1 = \eta_i^0$ is intuitive, because it implies that drivers stick to the same equilibrium branch before and after traffic oscillations, i.e., no change in drivers' behavior characteristics. However, that $\eta_i^1 \neq \eta_i^0$ indicates significant behavior changes imposed by traffic oscillations. Three potential explanations can be offered: (i) trajectories in period 1 are too short to capture the full recovery. Note that drivers experience long acceleration time after the oscillations in period 1 because the traffic state there is significantly less congested than before the oscillations; see Figure 12 (a). (ii) Lack of experience. We find that in period 2 drivers experienced at least two consecutive oscillations; see Figure 12(b). It is possible that drivers who experience oscillations for the first time, especially those react aggressively to oscillations (i.e., convex pattern), become more conservative after the oscillation passes; see the higher value of η_i^1 than η_i^0 for samples of period 1 in

Table 2. As drivers gain more experiences, they react promptly and are able to recover to the original equilibrium branch. (iii) Oscillations in period 2 are better developed, which allows drivers to anticipate the deceleration and acceleration cycle and therefore maintain a consistent equilibrium branch. Nevertheless, we think $\eta_i^1 = \eta_i^0$ is more common. Otherwise, it means that every time drivers experience a traffic oscillation, they change their preferred equilibrium branches completely, in this sense that they come from a different distribution. A change in the distribution may be caused by extreme environmental changes, e.g. rain or sharp changes in geometric design, which are not observed in our case.

Table 3: Parameter correlation for period 1 (t-statistics from least squares regression in lower diagonal and p-value from Wilcoxon signed-rank test in upper right, grey cells indicate significance at 5% significance level)

Table 4: Parameter correlation for period 2 (t-statistics from least squares regression in lower diagonal and p-value from Wilcoxon signed-rank test in upper right, grey cells indicate significance at 5% significance level)

6 Simulations

In this section we use simulation to evaluate the performance of the asymmetric behavior model at the macroscopic level and to compare its predictions with the L-L model. A simplified model to capture the rubbernecking phenomenon is introduced in Section 6.1 and the corresponding simulation results are analyzed in Section 6.2. The comparison with the original L-L model is presented in Section 6.3.

6.1 The experiment

Recall from Section 2 that there is strong indication that rubbernecking caused by the clean-up work on the median of US-101 may cause the oscillations in Figure 1(a). In support of this conjecture, we simulated for 32 minutes a 1.25km flat one-lane roadway with a triangular fundamental diagram where $u = 120$ km/h, $\kappa = 150$ veh/km and $w = 16$ km/h. The rubbernecking zone is located at $x \in [1,1.05]$ km (see the red bar in Figure 14(a)). When drivers enter this zone, they have a probability r to rubberneck, which will cause their speed to drop by a factor of $(1-p)$. Rubbernecking will occur at most once in this zone, if any. The roadway is empty at $t = 0$ and the inflow demand is set to 80% of capacity. Drivers are assumed to travel at free-flow speed when there is no congestion, and follow the asymmetric behavioral model when traffic is congested. Several combinations of the parameters a_m, r, and p are tested in the simulations.

According to the previous section the asymmetric behavior model is now reduced to three parameters per driver $[\eta_i^0, \eta_i^T, \varepsilon_i]$, in which $\varepsilon_i = \varepsilon_i^0 = \varepsilon_i^1$ and $\eta_i^1 = \eta_i^0$. Notice that some of the parameters are correlated and one should use their joint density function to generate the model parameters. However, as noted earlier, the sample size in this study may be not sufficient. Therefore, we resorted to use the sample enumeration method (Ben-Akiva and Lerman, 1985). Namely, each trajectory pair measured yields a set of $[\eta_i^0, \eta_i^T, \varepsilon_i]$ from the original measurement of $[\eta_i^0, \eta_i^1, \eta_i^T, \varepsilon_i^0, \varepsilon_i^1]$, in which η_i^1 is dropped and the value of ε_i is set to be the average of ε_i^0 and ε_i^1. In this way, the correlation, if any, among the three parameters of a parameter set $[\eta_i^0, \eta_i^T, \varepsilon_i]$ is preserved. The measurement of 111 pairs forms a parameter matrix. Each time a vehicle is generated, he/she will be assigned with a parameter set $[\eta_i^0, \eta_i^T, \varepsilon_i]$ randomly selected from the matrix. Notice that the matrix is a representation of the real measurement and no estimation of parameter distribution is required.

6.2 Simulation results

The vehicle trajectories from a few simulation runs are shown in Figure 14(b-d). Figures 13(b) and (c) suggest that the asymmetric behavior model has produced traffic oscillations that are very consistent with the empirical oscillations in Figure 1(a-b). Particularly, the oscillations produced have similar period (~3min), and they also exhibit precursor areas (circled by red dashed curves) that lead to fully grown oscillations, which are obvious in the empirical data. These two figures also illustrate how the period of oscillation is positively correlated with *p*. Notice that in both cases the precursor region is well reproduced and the amplitude grows similarly as in Figure 1(a). The

detailed view in Figure 14 (d) make apparent the large white voids that appear in the traffic stream. These voids are caused by OT drivers that prefer to maintain larger spacing.

Figure 14 Trajectories from simulations using asymmetric behavior model

(a) sketch of the highway segment in simulations; (b-d) trajectories from simulations (r=4%).

Figure 15 Relationship between oscillation period and parameters of asymmetric behavior model

Figure 15 shows the relationship between the period of oscillation and the parameters a_m, *r,* and *p*, predicted by the model. The period was measured using Fourier spectrum analysis (Li et al., 2009) to the speed time series collected at location $x = 500$ m. Each point in the figure represents the average for 5 simulation runs. Notice that the maximum acceleration has little effect on the period when the rubbernecker proportion is greater than 4% but a significant one when the proportion is small. Also notice how *r* is negatively correlated with the period and how *p* is positively correlated with it. This suggests that both the rubbernecker proportion and the speed reduction imposed by rubbernecking (denoted by 1-*p*) have negative impacts. Particularly, the relationship between period and *r* appear to be convex and rapidly approaching a stable value of about 2-3 min. The negative impact of *r* seems intuitive: the more rubberneckers, the more often (i.e., smaller interval) traffic oscillations may arise. Notice, however, that the period is not necessarily determined by the arrival of rubberneckers. The relationships are more complex. From Figure 14 (b) and (c) one can see that not every rubbernecker (denoted by a green trajectory) will lead to an oscillation. For example, in the precursor area labeled in Figure 14 (b) there are several rubberneckers that all contribute to the ensuing oscillation. We agree with Laval and Leclercq (2010) on the conjecture that traffic oscillation period may have connections with the discharge flow from previous oscillation and the approaching speed of incoming vehicles. As an illustration, we find that strong acceleration capability (i.e., large a_m-value) allows drivers to accelerate quickly to high speed and result in high discharge flow after oscillations; while with small a_m-value, the acceleration process is much longer and discharge flow is smaller. The effects are particularly significant when disturbance is small (e.g., *r* <4%, *p* >0.85). This may explain the impacts of *p* and a_m.

6.3 Model comparison

We run the same rubbernecking experiment using the original L-L model. The parameters used in the L-L model correspond to sample averages; i.e., ε is set to be the mean of all ε_i, and α equals to the average of $|\eta_i^T - \eta_i^0|$. We have obtained ε=215veh/h, α=0.27. The proportion of different driver categories is taken from the whole sample: 33% of OA drivers, 22% of OT drivers, and 45% of ON drivers. Of course, η_i^0 and η_i^1 are set to be 1 as assumed in the model.

Figure 16 and 17 present examples of trajectories from simulations and results of analysis, respectively. We find that (1) the original L-L model has produced similar magnitude of period and the negative (positive) effects of *r* (*p*) on oscillation period. However, (2) there are more large white voids in trajectories from asymmetric behavior model as shown by Figure 14(c) and Figure 16(c); and (3) the effect of a_m is negligible in the original L-L model; see Figure 17. Result (1) appears reasonable because the original L-L model can be considered as a representation of the *average* performance of drivers, which may result in some general features. Result (2) is probably caused by the different assumption of η_i^0-value in the two models. Recall that the original L-L model assumes that $\eta_i^0 = \eta_i^1 = 1$. Consequently, the white voids after traffic oscillations are due to the limited acceleration capability of drivers. By contrast, in the asymmetric behavior model, white voids can be created not only by limited acceleration capability but also by OT drivers (i.e., large η_i^0-value) because the model allows a wide range of $\eta_i^0(\eta_i^1)$. Result (3), however, is still puzzling. We suspect that the discharge flow from previous oscillations and approaching speed of drivers may play a role.

Figure 16 Trajectories from simulations using original L-L model with r=4% (green trajectories represent rubberneckers).

Figure 17 Relationship between oscillation period and parameters of the original L-L model

The performance of the two models suggests that the asymmetric behavior model should be used in order to capture the important features of traffic oscillations and to understand why and how the features are generated.

7   Discussions

Using empirical trajectory data this paper has unveiled the complete dynamic behavior profile of drivers experiencing traffic oscillations. This behavior can be described by the asymmetric behavior model with five parameters $[\eta_i^0, \eta_i^T, \eta_i^1, \varepsilon_i^0, \varepsilon_i^1]$ per driver, which could be drawn from a joint distribution function. We find that drivers exhibit different characteristics in congestion, reflected by their unique equilibrium branches, and that these characteristics are correlated with their reaction to traffic oscillations. To some extent, drivers' characteristics in equilibrium determine their response to traffic oscillations. In this sense, it seems that driver heterogeneity causes traffic oscillation to form and propagate. Because the heterogeneity is inherent to drivers, the formation and propagation of traffic oscillations seem inevitable as long as triggering factors exist. This implies that triggering factors would be a primary concern in the control and management of traffic oscillations.

The statistical analysis and simulation results revealed the model can be further reduced to three parameters per driver $[\eta_i^0, \eta_i^T, \varepsilon_i]$ without losing general macroscopic features. This is promising because it suggests that the simple asymmetric behavior model can serve as the basis of studies in traffic oscillations such as investigation of the formation and propagation mechanism and design of management policies that will help to reduce or mitigate the negative impacts of traffic oscillations. Of course, one should be cautious in transferring our results to different driver population, because the model parameters are correlated and may vary with driver populations. In applications, one may use the sample enumeration to generate the parameters as in this paper if a large sample is not available or assume a joint distribution estimated from a large sample size.

A controversial finding from the statistical analysis is that $\eta_i^1 \neq \eta_i^0$ in samples from period 1, which is probably due to the limited spatial extent of trajectories or lack of experience in traffic oscillations or different degree of development of oscillations. If the latter two reasons hold, they imply that traffic oscillations can significantly change driver behavior and one expects to see the changes of the distribution of η_i^0 occur quite often. Investigation of the cause is the subject of current research.

The simulations seem to confirm our hypothesis that that rubbernecking is the cause of the spontaneously formed oscillations observed on the study site. In particular, our simulation results show well-defined relationships between the percentages of rubberneckers (r), the speed reduction $(1 - p)$, and oscillation period. It is interesting to note that these relationships tend to approach a common lower bound of about 2-3 min, which is the same value as in the empirical data in Figure 1(a-b), and as in the upgrade simulation experiments in Laval and Leclercq (2010). This might be an indication that the minimum oscillation period that a bottleneck can create is within that range.

The original L-L model captures one ideal case of driver behavior but is far from sufficient to cover the general patterns unveiled in this paper. Simulation results indicate that it is able to produce the general macroscopic features of oscillations, but it may disregard some important microscopic features such as the white voids and the impacts of drivers' acceleration capability. The L-L model, which is more parsimonious, may be satisfying for macroscopic interests. However, for microscopic studies such as individual driver behavior and impact evaluation, the asymmetric behavioral model should be used. In particular, to quantify the impacts of traffic oscillations on safety risk, fuel consumption, and greenhouse emission, the asymmetric behavioral model should be applied. For the quantitative loss of parsimoniousness, further investigation is needed such as comparison of the impacts of traffic oscillations using the two different models. This research is on-going.

A more detailed examination of the data and the model is needed to improve our understanding of the relationships alluded to in the previous paragraph. In particular, we suspect that the time until the next oscillation is directly related to the outflow from the previous one. In fact, notice from Figures 1(a) and Figure 11 how the flow entering a precursor region appears to be lower (more white voids in the figures) than when the oscillations start to propagate. Further research is needed to investigate the mechanism of how the oscillation period is determined. Once the mechanism is clear, the parameters r and p in the model can be calibrated. To this end, a massive data collection plan is needed, which should include not only vehicle trajectory data but also video footage capturing driver head/eye movement to identify the rubberneckers and their speed reduction. Note that the Strategic Highway Research Program 2 (SHRP2) naturalistic driving study (2011) is collecting data on in-vehicle driving behavior, including continuous video of driver and vehicle movement, which can be used to examine driver's behavior and/or errors. Once these data become available, r and p can be estimated. Additionally, the asymmetric behavioral model should be validated using datasets from other sites. This was not accomplished in this study because currently we could not find another dataset that provides complete trajectory data, but it is a priority for future research.

Finally, driver behavior needs further investigations. For example, it is unclear whether the longitudinal behavior of drivers is consistent when they participate in more than one oscillation. Neither is it known whether

driver characteristic will affect the magnitude of oscillations. Additionally, the impacts of oscillations on traffic flow should be investigated; e.g., the connection between traffic oscillations and bottleneck capacity. With the impacts clear, studies on managing/mitigating traffic oscillations should follow. All the issues raised in this section are being currently investigated by the authors.

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