Cumulant Based Automatic Modulation Classification of QPSK, OQPSK, 8-PSK and 16-PSK

Dibyajyoti Das, Prabin Kumar Bora, Ratnajit Bhattacharjee
Department of Electronics and Electrical Engineering
Indian Institute of Technology Guwahati
Email: {d.dibyajyoti,prabin,ratnajit}@iitg.ernet.in

***Abstract*—An automatic modulation classification (AMC) method based on the fourth-order cumulants is proposed to classify QPSK, OQPSK, 8-PSK and 16-PSK. The method consists of two steps: (1) classification of QPSK and OQPSK using the fourth order zero-conjugate cumulant of backward differences of the samples of the received nosiy signal and (2) classification of QPSK, 8-PSK and 16-PSK using the fourth order zero-conjugate cumulant of the received noisy signal and their squares. Step 1 is guided by the observation that the fourth order zero-conjugate cumulants of backward differences are distinct for those two modulation formats. Step 2 is based on the fact that the squaring of an MPSK signal results in another MPSK signal with $\frac{M}{2}$ phase states. The Monte Carlo simulation results show that the proposed method is able to classify the afore-mentioned modulation schemes at moderately low signal to noise ratios with a small sample size. The proposed method is robust to the presence of carrier phase and frequency estimation errors.**

I. Introduction

The objective of automatic modulation classification (AMC) algorithms is to classify the modulation format of unknown communication signals of interest, with applications in both commercial and military system. The AMC operation is performed after signal detection and before data demodulation. It incorporates several aspects of classical communication theory together, namely signal detection, parameter estimation, channel identification and tracking. To blindly classify the modulation format of the unknown signal of interest, two different approaches available in literature, namely the *likelihood based* (LB) and the *feature based* (FB) approaches.

The LB approach requires prior information about the signal parameters and uses the likelihood function for classification. Some of the likelihood based approaches available in the literature are the average likelihood ratio test (ALRT) [1], the generalized likelihood ratio Test (GLRT) [2] and the hybrid likelihood ratio test (HLRT) [3]. In the case of the ALRT, the unknown signal and channel parameters are treated as random variables with known probability density functions (pdfs) and the likelihood functional is averaged over this pdf. The GLRT treats the unknown parameters as deterministic but unknown and the likelihood function is maximized with respect to them. In the hybrid likelihood ratio test (HLRT) [3], some of the parameters are treated as random and the rest as deterministic but unknown.

The FB approach consists of two steps, namely signal processing and classifier steps. The signal processing step extracts the features of unknown signals. The classifier then distinguishes the unknown signals using these features. Some of the widely used features employ the higher order statistics [4-8], the wavelet transform[9-10] , the spectral features [11], the signal constellations [12-13], the multi-fractals [14], the Radon transform [15] and the cyclostationary features [16]. In contrast to the LB approach, the FB approaches are easy to implement and can be robust to the presence of model mismatches.

In [4], Swami and Sadler have introduced the fourth-order cumulant based AMC method to distinguish different linear modulation schemes. In the same paper, it is shown that the particular method is effective when it is used in a hierarchical scheme and distinguishes the subclasses at a low signal-to-noise ratio (SNR) with a small sample size. In the case of classification of subclasses of MPSK, the method fails to classify $M > 4$ as they have the same fourth order cumulant values. The work also has not addressed the classification of QPSK and OQPSK. It may be noted that both QPSK and OQPSK have same fourth order cumulant values.

This paper considers the problem of classification of QPSK, OQPSK, 8-PSK and 16-PSK under the same framework of hierarchical digital modulation classification using fourth- order cumulants. The proposed method classifies the QPSK and the OQPSK using the fourth order zero-conjugate cumulant of backward differences of the samples of the received nosiy signal. It classifies the QPSK, the 8-PSK and the 16-PSK using the fourth order zero-conjugate cumulant of the received noisy signal and its squares.

The rest of this work is organized as follows. Section II describes the basic signal model. The essential features for classification of the modulation schemes under consideration are formulated in Section III. The results and simulations are discussed in the Section IV. The conclusion is drawn in section V.

II. Signal Model

The complex envelop of the received noisy signal is given by

$$y(t) = s(t) + w(t), 0 \leq t \leq NT_s \tag{1}$$

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where T_s is the symbol duration, N is the number of samples and $w(t)$ is the circularly complex white Gaussian noise.
The complex envelop of the transmitted signal is given by

$$s(t) = \sqrt{2S}\sum_{n=0}^{N-1} a_n^k p(t-nT_s)e^{j\phi_c} \tag{2}$$

where a_n^k is a complex random variables taking values on the alphabet set $A^k = \{a_1^k, ..., a_{M_k}^k\}$, k indicates a particular modulation scheme, M_k is the number of symbols in the k^{th} modulation scheme and ϕ_c is the carrier phase.

Features are extracted from $y(t)$ in FB modulation classification. The fourth order cumulant of the samples of y(t) have proved to be important fatures.

III. Fourth-order cumulant of a complex random variable

Fourth-order cumulants of a complex random variable are important statistical parameters and have been used as features for AMC. For zero mean random variables p, q, r and s, the fourth order cummulant are related to the fourth-order and the second order moments as [4]

$$\begin{aligned} cum(p,q,r,s) =& E(pqrs) - E(pq)E(rs) \\ &- E(pr)E(qs) - E(ps)E(qr) \end{aligned} \tag{3}$$

The second order moments of a zero-mean complex random variable y can be defined in two ways depending upon the placement of the conjugates [4].

$$C_{20}(y) = E[y^2] \tag{4}$$
$$C_{21}(y) = E[|y|^2] \tag{5}$$

Using Equation (3), the fourth-order zero conjugate cumulant of the zero mean random variable y is given by

$$C_{40}(y) = E(y^4) - 3[E(y^2)]^2 \tag{6}$$

Similarly, the fourth-order two conjugate cumulant of the zero mean random variable y is given by

$$C_{42}(y) = E(|y|^4) - |E(y^2)|^2 - 2(E|y|^2)^2 \tag{7}$$

The normalized fourth-order cumulant is defined as

$$\widetilde{c_{4k}}(y) = \frac{C_{4k}(y)}{C_{21}^2(y)} \tag{8}$$

The theoretical normalized cumulant values of some of the subclasses of the MPSK family is given in Table I.

A. Proposed Cumulant Feature to Classify QPSK and OQPSK

In QPSK signalling, the bit transitions of the even and odd bit streams occur at the same time instants. This allows the phase of the signal to jump by as much as 180^o at a instant. In OQPSK signalling, the even and odd bit streams are offset in their relative alignment by one bit period which does not allow the phase of the signal to jump by greater than 90^o.

Table I
THE THEORETICAL NORMALIZED FOURTH ORDER CUMULANT VALUES OF SOME OF THE SUBCLASSES OF MPSK FAMILY

	$BPSK$	$QPSK$	$8-PSK$	$16-PSK$
$\widetilde{c_{40}}(y)$	-2	-1	0	0
$\widetilde{c_{42}}(y)$	-2	-1	-1	-1

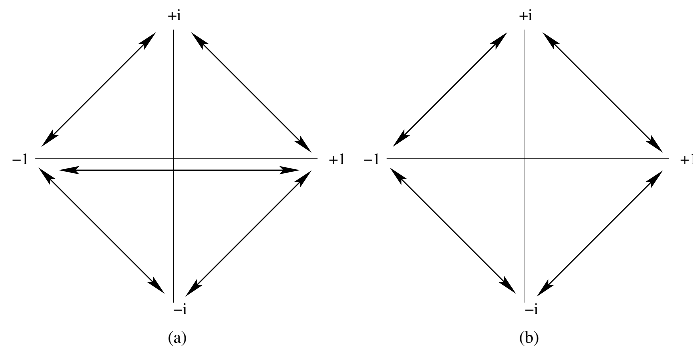

Figure 1. Constellations for (a) QPSK and (b) OQPSK

The fourth order cumulants of the QPSK and the OQPSK modulated signal are same as their constellations are similar. As a result, $\widetilde{c_{40}}(y)$ or $\widetilde{c_{42}}(y)$ can not be used as a discriminating feature to classify them. The constellation diagram of QPSK and OQPSK is shown in Figure 1. Here, it is proposed to use fourth-order zero conjugate cumumlant of the backward differences of the received noisy signal samples which is distinct for both QPSK and OQPSK. Let $y(n), n = 1, ..., N$ be the sample sequence obtained by sampling the received signal $y(t)$. We have considered one sample per symbol duration and the sample $y(n)$ can be written as

$$y(n) = s(n) + w(n) \tag{9}$$

where $s(n)$ is the transmitted symbol and $w(n)$ is complex white Gaussian noise with zero mean and variance σ_w^2.

We propose to use the finite difference of y(n) to discriminate between QPSK and OQPSK.

Consider the backward differences of the samples of the received signal given by

$$\nabla y(n) = y(n) - y(n-1), n = 1, ..., N-1 \tag{10}$$
$$= s(n) - s(n-1) + w(n) - w(n-1) \tag{11}$$

The possible backward differences of an ideal noise free constellation points of QPSK are $\{0, -2, 2, 2i, -2i, 1+i, 1-i, -1+i, -1-i\}$. The possible backward differences of an ideal noise free constellation points of OQPSK are $\{0, 1+i, 1-i, -1+i, -1-i\}$. The theoretical $|\widetilde{c_{40}}(\nabla y)|$ cumulants of the backward differences of noise free constellation points of QPSK and OQPSK using the equation (8) are given in Table

Table II
THEORETICAL NORMALIZED FOURTH ORDER CUMULANT OF THE BACKWARD DIFFERENCES OF QPSK AND OQPSK CONSTELLATION POINTS

	QPSK	*OQPSK*
$\lvert\tilde{c_{40}}(\nabla y)\rvert$	.5	2

II. It is clear that QPSK and OQPSK can be classified using $\lvert\tilde{c_{40}}(\nabla y)\rvert$

The estimates of second order and fourth-order zero conjugate cumulants of the backward differences of ∇y are given by

$$\hat{C_{20}}(\nabla y) = \frac{1}{N}\sum_{n=1}^{N} \nabla y_n^2 \tag{12}$$

$$\hat{C_{21}}(\nabla y) = \frac{1}{N}\sum_{n=1}^{N} |\nabla y_n|^2 \tag{13}$$

$$\hat{C_{40}}(\nabla y) = \frac{1}{N}\sum_{n=1}^{N} \nabla y_n^4 - 3\hat{C_{20}}^2 \tag{14}$$

$$\tag{15}$$

The corresponding normalized cumulant is given by

$$\hat{\tilde{c_{40}}}(\nabla y) = \frac{\hat{C_{40}}(\nabla y)}{\hat{C}_{21}^2(\nabla y) - 2\sigma_w^2} \tag{16}$$

where $2\sigma_w^2$ is variance of the noise difference of $w(n) - w(n-1)$. In this work, we have assumed that this variance is known to us. For recognising the modulation type of the received signal we compare $|c_{40}\hat{\tilde{(\nabla y)}}|$, with its theoretical value given in table II by using the following detection criterion

$$i^* = arg(\min_{i=1,2}|(|\tilde{c_{40i}}(\nabla y)| - |\tilde{c_{40}}\hat{(\nabla y)}|)|) \tag{17}$$

where $i = 1,2$ represent QPSK and OQPSK respectively, i^* represents the estimated modulation type of the received signal, $\tilde{c_{40i}}$is the theoretical feature of ith modulation type.

B. Classification of QPSK, 8-PSK and 16-PSK

Consider an MPSK signal where $M = 2^k$ and $k = 2,3,....$ The constellation of MPSK signal is given by

$$s_{MPSK}(n) = e^{\frac{i2\pi(n-1)}{M}}, n = 0,1,...,2^k - 1 \tag{18}$$

The squaring of this MPSK signal results in another MPSK signal with $\frac{M}{2}$ phase states.

$$s_{MPSK}^2(n) = e^{\frac{i4\pi(n-1)}{M}}, n = 0,1,...,2^k - 1 \tag{19}$$

$$= e^{\frac{i2\pi(n-1)}{\frac{M}{2}}} \tag{20}$$

From equation (20), it is clear that s_{MPSK}^2 is an MPSK signal with $\frac{M}{2}$ phase states.

Table I shows that QPSK can be discriminated from 8-PSK and 16-PSK using the fourth order cumulant of the received signal. But it fails to discriminate between 8-PSK and 16-PSK. Because of squaring, $\hat{\tilde{c_{40}}}(y^2)$ is distinctive of 8-PSK and 16-PSK as shown in Table III. To classify QPSK, OQPSK and 8-PSK, we can perform the following steps in the ideal noise free case:

Step 1: Using the $|\hat{\tilde{c_{40}}}(y)|$, find whether it is QPSK or MPSK with $M > 4$

Step 2: If $M > 4$, find $|\hat{\tilde{c_{40}}}(y^2)|$.

Step 3: If $|\hat{\tilde{c_{40}}}(y^2)| \simeq 1$ declare it as a PSK-8 otherwise if $|\hat{\tilde{c_{40}}}(y^2)| \simeq 0$, declare it as a PSK-16.

In the presence of noise, we compare $|c_{40}\hat{\tilde{(y^2)}}|$, with its theoretical value given in Table III using the following detection criterion

$$i^* = arg(\min_{i=1,2,3}|(|\tilde{c_{40i}}(y^2)| - |\tilde{c_{40}}\hat{(y^2)}|)|) \tag{21}$$

where $i = 1,2,3$ represent QPSK, OQPSK and 8-PSK respectively, i^* represents the estimated modulation type of the received signal, $\tilde{c_{40i}}$is the theoretical feature of ith modulation type. The hierarchical classification structure of [4] is extended as shown in Figure 2.

C. Effect of unknown phase rotation on the Proposed Feature

For complex valued constellation, the cumulant features $|C_{40}(\nabla y)|$ and $|C_{40}(y^2)|$ are unaffected by a phase rotation of the symbol set.

Let the phase rotation is ϕ, then the backward differences of the samples of the received signal is given by

$$\nabla y_\phi = e^{i\phi}[\nabla y_1 \nabla y_2 ... \nabla y_{N-1}] \tag{22}$$

Thus the modified fourth-order zero conjugate cumulant of ∇y_ϕ

$$\tilde{c_{40}}(\nabla y_\phi) = e^{i4\phi}\,\tilde{c_{40}}(\nabla y) \tag{23}$$

Similarly if the phase rotation is ϕ, then received sample sequence and its square are given by

$$\mathbf{y}_\phi = e^{i\phi}[y_1 y_2 ... y_N] \tag{24}$$

$$\mathbf{y}_\phi^2 = e^{i2\phi}[y_1^2 y_2^2 ... y_N^2] \tag{25}$$

Thus the modified fourth-order zero conjugate cumulant of $\mathbf{y}_\phi^2$ is given by

$$\tilde{c_{40}}(y_\phi^2) = e^{i8\phi}\,\tilde{c_{40}}(y) \tag{26}$$

It is clear from the equations (23) and (26) that $|C_{40}(\nabla y_\phi)|$ and $|C_{40}(y_\phi^2)|$ will be unaffected.

IV. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed method using the Monte Carlo simulation in terms of

Table III
THE FOURTH-ORDER ZERO CONJUGATE CUMULANT VALUES OF THE SQUARED NOISE FREE 8-PSK AND 16-PSK SIGNALS

	$8-PSK$	$16-PSK$
$\lvert\tilde{\tilde{c}}_{40}(y^2)\rvert$	1	0

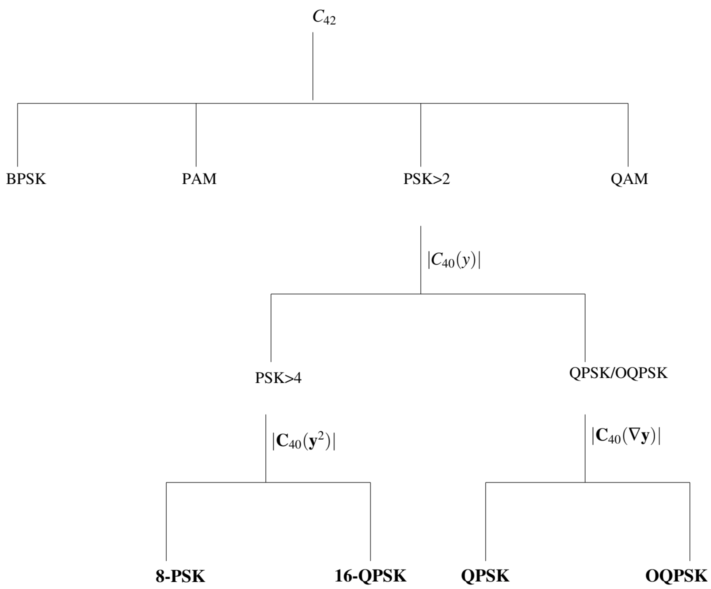

Figure 2. Hierarchical classification scheme based on HOS.

probability of the correct classification P_c versus SNR. Considering M modulation candidates for classification denoted as $\{M_1, ..., M_M\}$, the probability of correct classification P_c is given by

$$P_c = \sum_{i=1}^{M} P(M_i/M_i)P(M_i) \tag{27}$$

where $P(M_i)$ is the probability of occurrence of modulation candidate M_i, $P(M_i/M_i)$ is the probability of correct classification when M_i constellation was transmitted. We consider here four different sets of modulation candidates, namely $(i)\{QPSK, OQPSK\}$, $(ii)\{8-PSK, 16-PSK\}$, $(iii)\{QPSK, 8-PSK, 16-PSK\}$ and $(iv)\{QPSK, OQPSK, 8-PSK, 16-PSK\}$. We also assume that $P(M_i) = \frac{1}{M}, \forall i$. All the results are based on 1000 Monte Carlo trials, i.e., 2000 trials for the four-class problem, 3000 trials for the three-class problem and 4000 trials for the four-class problem.

Figure 3 depicts the probability of correct classification of $\{QPSK, OQPSK\}$ at different SNR values for sample sizes N=250, 500 and 2000. It is observed that the perfect classification can be achieved at SNR=6 dB for sample size N=250. Increasing the sample size to N=500, the perfect classification can be achieved at SNR=5 dB. Further, increasing the sample size to N=2000, the perfect classification can be achieved at SNR=2 dB.

Figure 4 shows the probability of correct classification of $\{8-PSK, 16-PSK\}$ at different SNR values for sample sizes

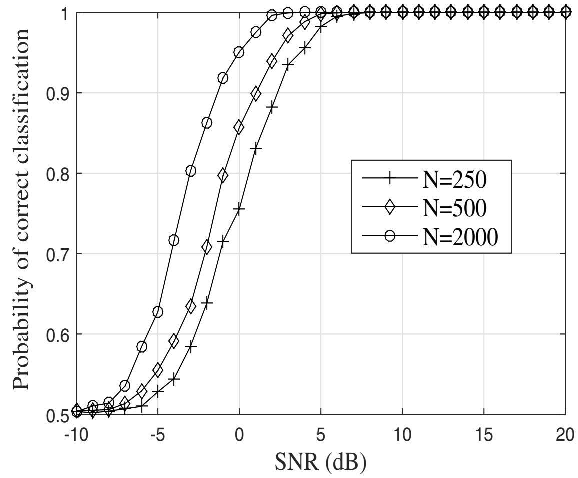

Figure 3. Classification of QPSK and OQPSK

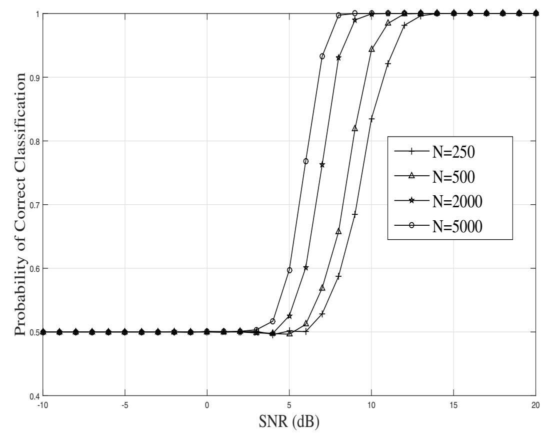

Figure 4. Classification of 8-PSK and 16-PSK

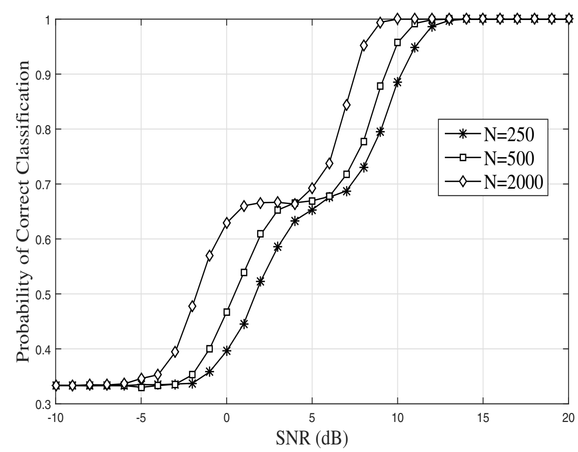

Figure 5. Classification of QPSK, 8-PSK and 16-PSK

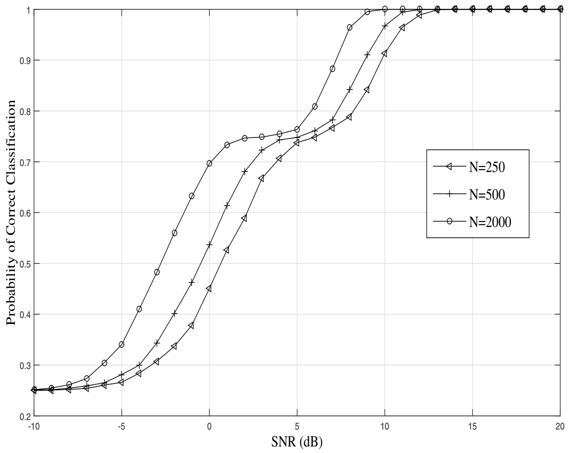

Figure 6. Classification of QPSK, OQPSK, 8-PSK and 16-PSK

N=250, 500, 2000 and 5000. It is observed that the perfect classification can be achieved at SNR=13 dB for sample size N=250. Increasing the sample size to N=500, the perfect classification can be achieved at SNR=12 dB. For sample size N=2000, the perfect classification can be achieved at SNR=10 dB . Further increasing the the sample size to N=5000, the perfect classification can be achieved at SNR=8 dB.

Figure 5 shows the probability of correct classification of $\{QPSK, 8-PSK, 16-PSK\}$ at different SNR values for sample sizes N=250, 500 and 2000. It is observed that the perfect classification can be achieved at SNR=12 dB for sample size N=250. Increasing the sample size to N=500, the perfect classification can be achieved at SNR=11 dB. Further increasing the the sample size to N=2000, the perfect classification can be achieved at SNR=9 dB.

Figure 6 shows the probability of correct classification of $\{QPSK, OQPSK, 8-PSK, 16-PSK\}$ at different SNR values for sample sizes N=250, 500 and 2000. It is observed that the perfect classification can be achieved at SNR=12 dB for sample size N=250. Increasing the sample size to N=500, the perfect classification can be achieved at SNR=11 dB. Further increasing the the sample size to N=2000, the perfect classification can be achieved at SNR=9 dB.

For the figures $4-6$, the classification of $\{8-PSK, 16-PSK\}$,$\{QPSK, 8-PSK, 16-PSK\}$ and $\{QPSK, OQPSK, 8-PSK, 16-PSK\}$ are done using the cumulant features $|\hat{\tilde{c}}_{40}(\nabla y)|$ and $|\hat{\tilde{c}}_{40}(y^2)|$ jointly. The feature $|\hat{\tilde{c}}_{40}(\nabla y)|$ is immune to the Gaussian noise. But the $|\hat{\tilde{c}}_{40}(y^2)|$ will not remain immune to the Gaussian noise as there is loss of Gaussianity during the squaring of the noisy signal. Also, with decreasing the sample size N, the variance of the feature$|\hat{\tilde{c}}_{40}(y^2)|$ increases in general [4]. The cumulative effect results in degraded performance at the low SNR regime when the sample size is small.

V. Conclusion

In this work, we proposed a cumulant based Automatic Modulation Classification (AMC) method to classify QPSK, OQPSK, 8-PSK and 16-PSK over AWGN channel. The method performs reasonably well at moderately low signal to nosie ratio with a small sample size. The method is also robust to the presence of carrier and phase estimation errors. The performance can be further improved by using spatial diversity techniques at the receiver.

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