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# A Comparision of Spacing and Headway Control Laws for Automatically Controlled Vehicles[1]

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# A Comparision of Spacing and Headway Control Laws for Automatically Controlled Vehicles [1]

D. SWAROOP*, J.K. HEDRICK,* C. C. CHIEN**, and P. IOANNOU**

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## SUMMARY

This paper investigates two different longitudinal control policies for automatically controlled vehicles. One is based on maintaining a constant spacing between the vehicles while the other is based upon maintaining a constant headway (or time) between successive vehicles. To avoid collisions in the platoon, controllers have to be designed to ensure string stability, i.e the spacing errors should not get amplified as they propagate upstream from vehicle to vehicle. A measure of string stability is introduced and a systematic method of designing constant spacing controllers which guarantee string stability is presented. The constant headway policy does not require inter-vehicle communication to assure string stablity. Also, since inter-vehicle communication is not required it can be used in systems with mixed automated-nonautomated vehicles, e.g for AICC (Autonomous Intelligent Cruise Control ). It is shown in this paper that for all the autonomous headway control laws, the desired control torques are inversely proportional to the headway time.

## 1. INTRODUCTION

Recently Intelligent Vehicle Highway Systems ( IVHS ) have become a topic of considerable interest. Such systems are being developed as a safe and efficient means of travel on congested roadways. The California Partners for Advanced Transit and Highway (PATH) have been developing automated vehicle control systems (AVCS) required for IVHS. In this paper, longitudinal controller design and analysis of AVCS is presented. A lot of research has been done in the longitudinal control area of AVCS. Hedrick, [2] and Sheikholeslam, [5] have proposed controllers using a spacing control strategy and Ioannou, [3], has proposed controllers using a headway control strategy. In order to eliminate collisions, string stability should be assured, i.e the spacing errors should not be amplified in the platoon. In order to assure string stability for constant spacing control policy, lead vehicle velocity or relative position is required. The advantage of a headway control strategy is that string stability is guaranteed without the use of lead vehicle information or preceding vehicle acceleration information. The conditions required to ensure string stability in [5] are modi-

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fied and a measure of string stability is introduced. This paper also proves that the longitudinal controller developed in [2] assures string stability.

In this paper, a vehicle model based on Cho and Hedrick, [1] is developed and validated. Control algorithms that explicitly address the issue of string stability for spacing and headway control stategies are developed, based on this model. Finally, using the traffic flow estimates computed by Shladover, [6], and Hedrick, [11], the two platoon control strategies are compared.

In section 2, the vehicle model is presented and validated. Section 3 simplifies the model developed in section 2 for control purposes and discusses the performance specifications for both strategies. Sections 4 and 5 present the Spacing and Headway control algorithm respectively and analyze them. Section 6 evaluates the effectiveness of these two strategies in terms of traffic flow estimates. In section 7, the results are summarised and directions for future research are suggested.

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## 2. MODELLING AND VALIDATION

In this section, a three state variable lumped parameter longitudinal model of a vehicle is developed for simulating the vehicles in the platoon. It is based on the following assumptions :

1. Ideal gas law holds in the intake manifold.
2. Temperature of the intake manifold is constant.
3. The drive axle is rigid.
4. The torque converter is locked.
5. The brakes obey first order linear dynamics.

A simple model for the intake manifold dynamics is given by :

$$\dot{m}_a = \dot{m}_{ai} - \dot{m}_{ao} \tag{1}$$

$$P_m V = m_a R T_m \tag{2}$$

where $m_a$ is the mass of air in the intake maniold and $\dot{m}_{ai}, \dot{m}_{ao}$ are the mass flow rates through the throttle valve and into the cylinders, respectively. $P_m$, $V$, $T_m$ are the intake manifold pressure, volume and temperature respectively. R is the gas constant for air. Assumptions 1 and 2 enable us to obtain an algebraic relationship between the manifold pressure, $P_m$ ( which is sensed ) and the mass of air in the manifold, $m_a$. The empirical relationship used for $\dot{m}_{ai}$ , [1] is :

$$\dot{m}_{ai} = MAX.TC.PRI\left(\frac{P_m}{P_a}\right) \tag{3}$$

where $MAX$ is a constant dependent on the size of the throttle body. $TC(\alpha)$ is the throttle characteristic which is the projected area the flow sees as a function of $\alpha$. $PRI$ is the pressure influence function which describes the choked flow

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relationship which often occurs through the throttle valve. $P_a$ is the atmospheric pressure. $\dot{m}_{ao}$ is the mass air flow rate into the combustion chamber and is a nonlinear function of the intake manifold pressure $P_m$ and the engine speed $w_e$. For vehicle position control applications, the intake manifold dynamics is much faster than the engine speed dynamics, so that we can assume

$$\dot{m}_{ao}(w_e, m_a) = \dot{m}_{ai} = MAX.TC(\alpha).PRI\left(\frac{P_m}{P_a}\right) \tag{4}$$

Assumptions 3 and 4 ensure that the wheel speed is proportional to the engine speed, $w_e$. The rotational dynamics of the engine is described by :

$$\dot{w}_e = \frac{T_{net} - R(hF_{tr} + T_{br})}{I_e} \tag{5}$$

where $T_{net}$ is the net combustion torque (indicated torque - friction torque). It is also a nonlinear function of $w_e$ and $P_m$. $\dot{m}_{ao}$ and $T_{net}$ are provided as tabular functions by the engine manufacturers. $R$ is the gear ratio, h is the tire radius and $I_e$ is the effective rotational inertia of the engine when the inertia of the wheel is also referred to the engine side. $T_{br}$ is the brake torque at the wheels and $F_{tr}$ is the tractive force. The tractive force $F_{tr}$ is given by [9]:

$$F_{tr} = K_r sat\left(\frac{i}{i_{max}}\right) \tag{6}$$

where $K_r$ is the longitudinal tire stiffness and $i$ is the slip between the tires and the ground. It is defined as:

$$i = 1 - \frac{v}{Rhw_e} \tag{7}$$

where $v$ is the longitudinal velocity of the vehicle. The dynamics of the brake is given by:

$$\dot{T}_{br} = \frac{T_{bc} - T_{br}}{\tau_b} \tag{8}$$

where $T_{bc}$ is the commanded brake torque and $\tau_b$ is the time constant for the brake actuator. Finally, the equation for longitudinal vehicle velocity is given by:

$$\dot{v} = \frac{F_{tr} - c_a v^2 - F_f}{M} \tag{9}$$

where $c_a$ is the drag coefficient, $F_f$ is the force due to rolling resistance and M is the effective mass of the vehicle.

The model developed above is used for simulation. We apply the same throttle input to the actual vehicle and the simulation model. The maneuvers that

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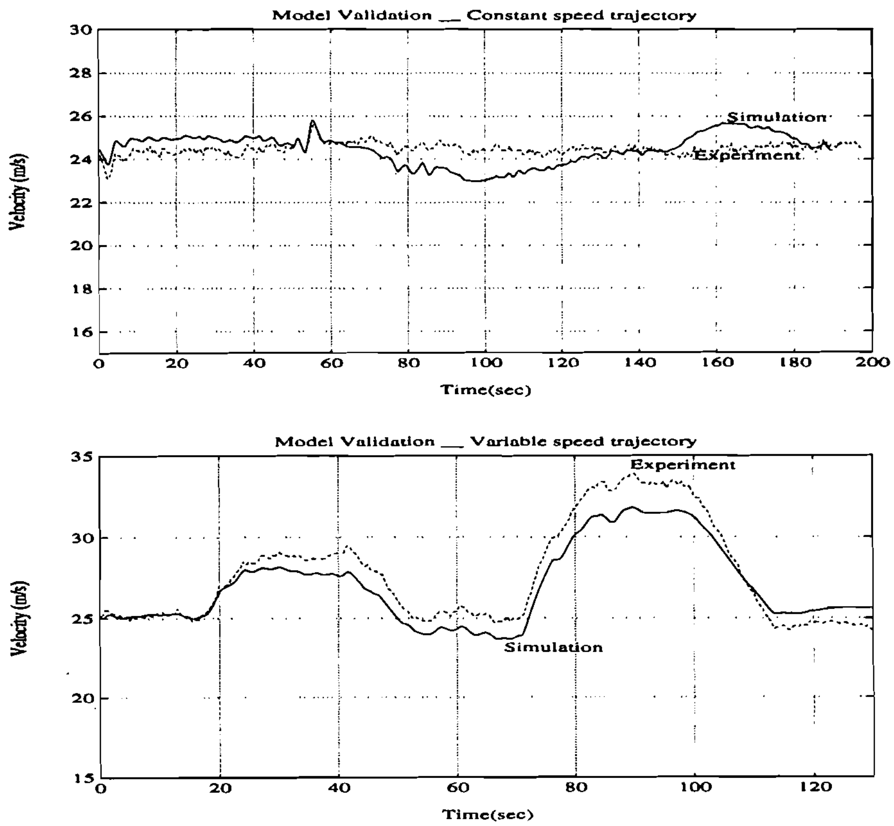


Fig. 1. Simulation Model Validation.

have been chosen to validate the simulation model do not require braking. Hence, the simulation model is only partially valid. The simulated and experimental responses for the constant speed and variable speed trajectory tests are shown in Figure 1. The two responses agree quite well.

## 3. SIMPLIFIED MODEL FOR CONTROL AND PERFORMANCE OBJECTIVES

The design of the controller is made simpler by a "NO SLIP" assumption, i.e $v = Rhw_e$. With this assumption, equations 5 and 9 reduce to

$$\dot{w}_e = \frac{T_{net} - c_a R^3 h^3 w_e^2 - R(hF_f + T_{br})}{J_e} \tag{10}$$

$J_e$ is the effective rotational inertia of the engine when the vehicle mass and the wheel inertias are referred to the engine side. Equations 4, 8 and 10 describe the model for the controller.

Algorithms for spacing and headway control strategies have the same feedback structure except for the information that is fed back. The feedback structure for both algorithms is developed using the I/O Linearization technique, [8]. I/O Linearization is best suited for this problem considering the nonlinearities in the engine model. It is assumed that every controlled vehicle has access to its internal signals like velocity, brake torque, acceleration etc., and that the parameters like aerodynamic drag, rolling resistance friction, effective engine inertia, gear ratios and tire radii etc., are known exactly. The desired output, "$y$", is the longitudinal position of the j-th following vehicle, $x_j$.

$$y = x_j \tag{11}$$

$$\dot{y} = \dot{x}_j = v_j = (Rhw_e)_j \tag{12}$$

$$\ddot{y} = \ddot{x}_j = \left(\frac{Rh}{J_e}[T_{net} - c_a R^3 h^3 w_e^2 - R(hF_f + T_{br})]\right)_j \tag{13}$$

Choose

$$(T_{nd})_j = [c_a R^3 h^3 w_e^2 + R(hF_f + T_{br})] + \frac{J_e}{Rh} u_j \tag{14}$$

where $(T_{nd})_j$ is the desired net engine torque and $u_j$ is chosen to make the closed loop system satisfy certain performance objectives. Knowing the desired net engine torque and the actual speed, the desired manifold pressure, $P_{md}$ can be found from the table-look up map. Using equation 4, the desired throttle angle, $\alpha_d$ can be calculated as follows:

$$\{\alpha_d = TC^{-1}\left[\frac{\dot{m}_{ao}(w_e, P_{md})}{MAX.PRI(\frac{P_m}{P_a})}\right] \tag{15}$$

We can simplify computations by combining $T_{net}(w_e, P_m)$ and equation 4 to yield $T_{net}(w_e, \alpha)$. If $\alpha \leq \alpha_0$, the minimum allowable throttle angle, then braking should occur, in which case, the desired brake torque $T_{bd}$ is given by

$$T_{bd} = \frac{J_e}{R^2 h} u_j - \frac{c_a R^3 h^3 w_e^2 + RhF_f}{R} \tag{16}$$

Define another synthetic output $y_b$ so that

$$y_b = T_{br} - T_{bd} \tag{17}$$

$$\dot{y}_b = \dot{T}_{br} - \dot{T}_{bd} = \frac{T_{bc} - T_{br}}{\tau_b} - \dot{T}_{bd} \tag{18}$$

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Choose $T_{bc}$ such that

$$T_{bc} = T_{br} + \tau_b(\dot{T}_{bd} - \lambda_b(T_{br} - T_{bd})) \tag{19}$$

To simplify implementation, $\dot{T}_{bd}$ is obtained by numerically differentiating the desired brake torque signal. $\lambda_b$ is chosen sufficiently high so that use of throttle and brake control approximates the vehicle (plant) model as

$$\ddot{x}_j = u_j \tag{20}$$

### 3.1. Platooning Specifications/Objectives

The following assumptions are made for analyzing the closed loop system:
1. All the vehicles are identical and are moving in a straight lane of a highway.
2. Prior to the lead vehicle maneuver, all the vehicles in the platoon were moving at the same steady speed ( $v_l(0-)$).
3. The lead vehicle takes only a finite amount of time ($t_f$) to perform a maneuver before reaching a steady speed.

The following are the platoon objectives :
1. The closed loop system should be asymptotically stable. This is a requirement characterising the stability of every vehicle in the platoon.
2. The spacing error is defined as the deviation from the desired position. The steady state spacing errors of all the vehicles in the platoon should be zero.
3. The transient errors should not amplify with vehicle index due to any lead vehicle maneuver. One way to satisfy this requirement is that the maximum absolute spacing error of j-th vehicle should be less than or equal to that of the j-1 st vehicle. This requirement characterises string stability in the strong sense. Weaker string stability requires that the maximum spacing error in the j-th following vehicle be less than or equal to that of the first following vehicle. In this paper, we require string stability in the strong sense.

External measurements that may be fed back:
1. Sensor Measurements: The speed and distance of the controlled vehicle relative to its preceding vehicle is sensed by onboard sensors like radar or sonar.
2. Broadcast Measurements: The lead vehicle velocity and acceleration may be broadcast ( by radio or other communication devices ) to all the following vehicles in the platoon. If necessary, every vehicle in the platoon broadcasts its velocity, acceleration and sensor measurements to its following vehicles. Hence, the preceding vehicle acceleration information can be treated as a broadcast measurement.

If any controlled vehicle requires its position relative to the lead vehicle, it can be obtained by numerically integrating its velocity relative to the lead vehicle. An alternative way to obtain this information is to have every vehicle

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in the platoon broadcast its position relative to the lead vehicle to all its following vehicles. Hence, the position of the j-th vehicle relative to the lead vehicle can be obtained by summing the position of j-th vehicle relative to the j-1 st vehicle (which is available from sensors like radar) and the position of j-1 st vehicle relative to the lead vehicle. We plan to update the former estimate with the latter one to get a better estimate of controlled vehicle's position relative to the lead vehicle. For autonomous control applications (control with onboard sensors only), we may require the preceding vehicle acceleration information to improve the performance of the platoon. Since the preceding vehicle acceleration information is not available, it can be obtained by numerically processing the controlled vehicle acceleration and its speed relative to the preceding vehicle.

## 4. SPACING CONTROL STRATEGY

Define the spacing error $\varepsilon_j$ of the j-th vehicle for this strategy to be

$$\varepsilon_j := x_{j-1} - x_j - L_j \tag{21}$$

$$\dot{\varepsilon}_j = \dot{x}_{j-1} - \dot{x}_j \tag{22}$$

$\varepsilon_j$ and $\dot{\varepsilon}_j$ are obtained from the onboard sensors which measure the position and velocity of the j-th (controlled ) vehicle relative to its predecessor. Define

$$w_l(t) := v_l(t) - v_l(0-) \tag{23}$$

where $v_l(t)$ is the velocity of the lead vehicle.

Consider the following control law:

For $j > 1$

$$\begin{aligned} u_j = {} & k_p\varepsilon_j + k_v\dot{\varepsilon}_j + k_l a_l - k_1(\dot{x}_i - \dot{x}_i(0-)) + k_a a_{j-1} \\ & - c_p(x_j(t) - x_l(t) + \sum_1^j L_i) - c_v(\dot{x}_j(t) - \dot{x}_l(t)) \end{aligned} \tag{24}$$

and

$$u_1 = (k_p + c_p)\varepsilon_1 + (k_v + c_v)\dot{\varepsilon}_1 + (k_a + k_l)a_l - k_1(\dot{x}_1 - \dot{x}_1(0-)) \tag{25}$$

where $a_l$ is the lead vehicle acceleration, $L_i$ is a constant distance the i-th vehicle desires to maintain from its predecessor, $a_{j-1}$ is the acceleration of the j-1 st vehicle ( j-th vehicle's predecessor ), $x_l(t)$ is the lead vehicle's position. The parameters $k_p, c_p, k_v, k_a, k_1, k_l$ have to be determined to suit the specifications. If any information is not available, then the corresponding design constant (gain) is set to zero.

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**DEFINITION:** If $f(t)$ is any function, then

$$||f||_1 := \int_0^\infty |f(t)|dt \tag{26}$$

$$||f||_\infty := \sup_{t\geq 0} |f(t)| \tag{27}$$

**FACT 1:**

Consider a strictly causal, stable LTI system. Let $h(t)$ be its impulse response, $\hat{h}(s)$ be its transfer function from the input to the output and $y(t)$ be the output corresponding to a bounded input $u(t)$. Then

$$||h||_1 \geq |\hat{h}(0)| \tag{28}$$

$$||h||_1 = |\hat{h}(0)| \Longleftrightarrow h(t) \geq 0 \quad or \quad h(t) \leq 0 \tag{29}$$

$$||y||_\infty \leq ||h||_1 ||u||_\infty \tag{30}$$

**FACT 2:**

If $h(t) = c\delta(t) + f(t)$ and $f(t)$ is absolutely integrable, where $\delta(t)$ is the Dirac-delta function, then

$$||h||_1 = |c| + ||f||_1 \tag{31}$$

Based on the equations 20, 23-25, define the following transfer functions:

$$\hat{g}(s) := \frac{\hat{\varepsilon}_1}{\hat{w}_l}(s) = -\frac{(k_a + k_l - 1)s - k_1}{s^2 + (k_v + c_v + k_1)s + (k_p + c_p)} \tag{32}$$

$$\hat{h}(s) := \frac{\hat{\varepsilon}_i}{\hat{\varepsilon}_{i-1}}(s) = \frac{k_a s^2 + k_v s + k_p}{s^2 + (k_v + c_v + k_1)s + (k_p + c_p)} \tag{33}$$

The transfer function $\hat{g}(s)$ describes the performance of the first follower in the platoon. It describes how the spacing error of the first follower changes due to a change in the lead vehicle speed from its previous steady state value. The transfer function $\hat{h}(s)$ describes the way the spacing errors are amplified/attenuated in the platoon. The design problem, therefore, is to minimise $||g||_1$ and $||h||_1$ over the admissible set of gains to attain the best possible performance. Equation 28 sets a lower bound on the minimum possible values of $||h||_1$ and $||g||_1$. Equation 29 gives us the condition when $||h||_1$ and $||g||_1$ achieve their respective lower bounds. Noting that $\hat{g}$ and $\hat{h}$ are second order transfer functions, both of them should have real and stable poles in order to achieve their minimum possible values. Define

$$\gamma := \min_K ||h||_1 \tag{34}$$

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where K is the admissible set of gains. $\gamma$ is a measure of string stability. From equation 30, $\gamma$ is required to be less than or equal to unity in order to ensure string stability. The smaller the value of $\gamma$ is, the smaller the value of maximum spacing error of the j-th vehicle is compared to that of the j-1 st vehicle.

**Remark 1:**

Given the transfer function $\hat{h}(s)$ in equation 33, and that $\phi(s) = s^2 + (k_v + c_v + k_1)s + (k_p + c_p)$ is Hurwitz, $\gamma = k_p/(k_p + c_p)$ is achievable with filtering if the following three conditions hold :

$$k_a \varepsilon \left[0, \frac{k_p}{k_p + c_p}\right) \tag{35}$$

$$\phi(s) = (s + \beta_1)(s + \beta_2), \quad \beta_2, \beta_1 \varepsilon R_+, \beta_2 \geq \beta_1$$

$$\frac{c_v + k_1}{1 - k_a} \quad \textit{lies between} \quad \beta_1 + \beta_2 \quad \textit{and} \quad \beta_1 + \frac{c_p}{(1 - k_a)\beta_1}$$

This remark is proved in the Appendix.

**Remark 2:**

To satisfy platoon objectives (1) and (2),

$$k_v + c_v \geq 0.0; \quad k_p + c_p \geq 0.0; \quad k_1 = 0. \tag{36}$$

Henceforth, it will be assumed that the platoon objectives (1) and (2) have to be satisfied.

**Remark 3:**

If the lead vehicle information cannot be fed back, consider the following semi-autonomous (in addition to onboard sensors, only the preceding vehicle acceleration information is required) control law:

$$u_j = k_p \varepsilon_j + k_v \dot{\varepsilon}_j + k_a a_{j-1} - k_1(\dot{x}_j - \dot{x}_j(0-)) \tag{37}$$

Then $\gamma = 1$ iff $\hat{h}(s) = 1$ and $\hat{g}(s) = 0, k_a = 1$. Potentially, string stability could be assured without the use of lead vehicle information. It cannot be done autonomously, since the preceding vehicle acceleration information is essential. To implement this control law, preceding vehicle acceleration has to be estimated perfectly which is not practically possible. Signal processing lag, [4], results in the amplification of spacing errors for low frequency lead vehicle maneuvers.

**Remark 4:**

If the only information available is the lead vehicle acceleration, string stability performance of the platoon cannot be improved, since the acceleration information does not affect $\hat{h}(s)$ and hence $||h||_1$. The lead vehicle acceleration can be used to improve the performance of the first follower in the platoon.

**Remark 5:**

In addition to the preceding vehicle acceleration information, if only the lead vehicle velocity information is available, consider the following control law: For $j > 1$,

$$u_j = k_p\varepsilon_j + k_v\dot{\varepsilon}_j + k_a a_{j-1} - c_v(\dot{x}_j(t) - \dot{x}_l(t)) \tag{38}$$

$$u_1 = k_p\varepsilon_1 + (k_v + c_v)\dot{\varepsilon}_1 + k_a a_l \tag{39}$$

Then, by Remark 1, $\gamma = 1$ is achievable if $k_a \varepsilon [0, 1)$, $\phi(s)$ has real and stable roots $\beta_2$ and $\beta_1$ with $\beta_2 \geq \beta_1 > 0$ and $c_v/(1 - k_a)$ lies between $\beta_1$ and $\beta_1 + \beta_2$.

**Remark 6:**

If the lead vehicle acceleration and velocity and the preceding vehicle acceleration are available for feedback, consider the following law: For $j > 1$,

$$u_j = k_p\varepsilon_j + k_v\dot{\varepsilon}_j + k_a a_{j-1} + k_l a_l - c_v(\dot{x}_j(t) - \dot{x}_l(t)) \tag{40}$$

$$u_1 = k_p\varepsilon_1 + (k_v + c_v)\dot{\varepsilon}_1 + (k_a + k_l)a_l \tag{41}$$

Then $k_l$ is chosen to improve the performance of the first following vehicle in the platoon by making $\hat{g}(s) = 0$ ( i.e decoupling the dynamics of the spacing error of the first following vehicle in the platoon from any variation in the lead vehicle velocity).

Remark 5 presents a method of choosing the gains when lead vehicle velocity information is available. The first step is to choose stable and real poles based on actuator bandwidths and sampling considerations etc., the second step is to choose $k_a$ based on the level of high frequency attenuation that is required. Using the conditions in Remark 5, $c_v$ is chosen. The other constants are found from the following equations:

$$\begin{aligned} k_p &= \beta_1\beta_2 \\ k_v &= \beta_1 + \beta_2 - c_v \end{aligned} \tag{42}$$

Finally, $k_l$ is selected to be $1 - k_a$ so that we have $\hat{g}(s) = 0, ||h||_1 = 1$.

In principle, $\gamma$ ( maximum $L_\infty$ to $L_\infty$ gain ) is achieved for some special inputs. It can be seen from Figure 2a that $||\varepsilon_i||_\infty < ||\varepsilon_{i-1}||_\infty$. Remark 5 suggests that we have to rely more on lead vehicle velocity information if the preceding vehicle's acceleration measurement is unavailable or unreliable.

**Remark 7:**

Consider the control law given in equations 24 and 25, then $\gamma = k_p/(k_p + c_p)$ is achievable if the conditions in Remark 1 hold with $k_1 = 0$.

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With the lead vehicle position information, $\gamma$ can be made less than unity. This means that the vehicles at the tail of the platoon experience smaller maximum spacing errors than the ones at the head of the platoon. A sliding controller, with lead vehicle velocity and acceleration information feedback, of the form given in [2], is such that $\hat{g}(s) = 0$ and $||h||_1 = 1$. A sliding controller without lead vehicle information satisfies $\hat{g}(s) = 0, \hat{h}(s) = 1$.

**Remark 8:**

If $\gamma > 1$, number $N$ for which the maximum spacing error of *Nth* vehicle (due to a maximum error $\varepsilon_0$ in the first following vehicle) is greater than the desired spacing is given by:

$$N = \min_i \quad i \geq 1 + \frac{log(\frac{l_i}{\varepsilon_0})}{log(\gamma)} \tag{43}$$

where $l_i = L_i - length \quad of \quad the \quad vehicle$, is the inter-vehicle distance that has to be maintained. The number $N$ is an upper bound on the maximum number of vehicles that can be packed in a platoon for safe operation.

**Remark 9:**

If every vehicle knows its index (i.e the number of vehicles preceding it in the platoon), and if $\gamma > 1$, collisions can be avoided by choosing $l_i = \gamma^{i-1} l_1$.

**Remark 10:**

If every vehicle knows its index, then consider the following semi-autonomous control law:

$$u_j = k_{v_j}\dot{\varepsilon}_j + k_{p_j}\varepsilon_j + k_a\ddot{x}_{j-1} \tag{44}$$

with the understanding that $\ddot{x}_0 = a_l$. Then, the transfer functions corresponding to 31 and 32 are:

$$\hat{g}(s) = \frac{(k_a - 1)s}{s^2 + k_{v_1}s + k_p} \tag{44}$$

$$\hat{h}(s) = \frac{k_a s^2 + k_{v_{j-1}}s + k_{p_{j-1}}}{s^2 + k_{v_j}s + k_{p_j}} \tag{45}$$

Comparing $\hat{h}(s)$ with the corresponding transfer function discussed in Remark 1, $\gamma$ can be made less than or equal to unity ( i.e string stability can be assured ) by choosing gains which increase with the vehicle index. In fact, this holds true for the autonomous case also, i.e when $k_a = 0$.

Remarks 9 and 10 emphasise that string stability can be ensured at the

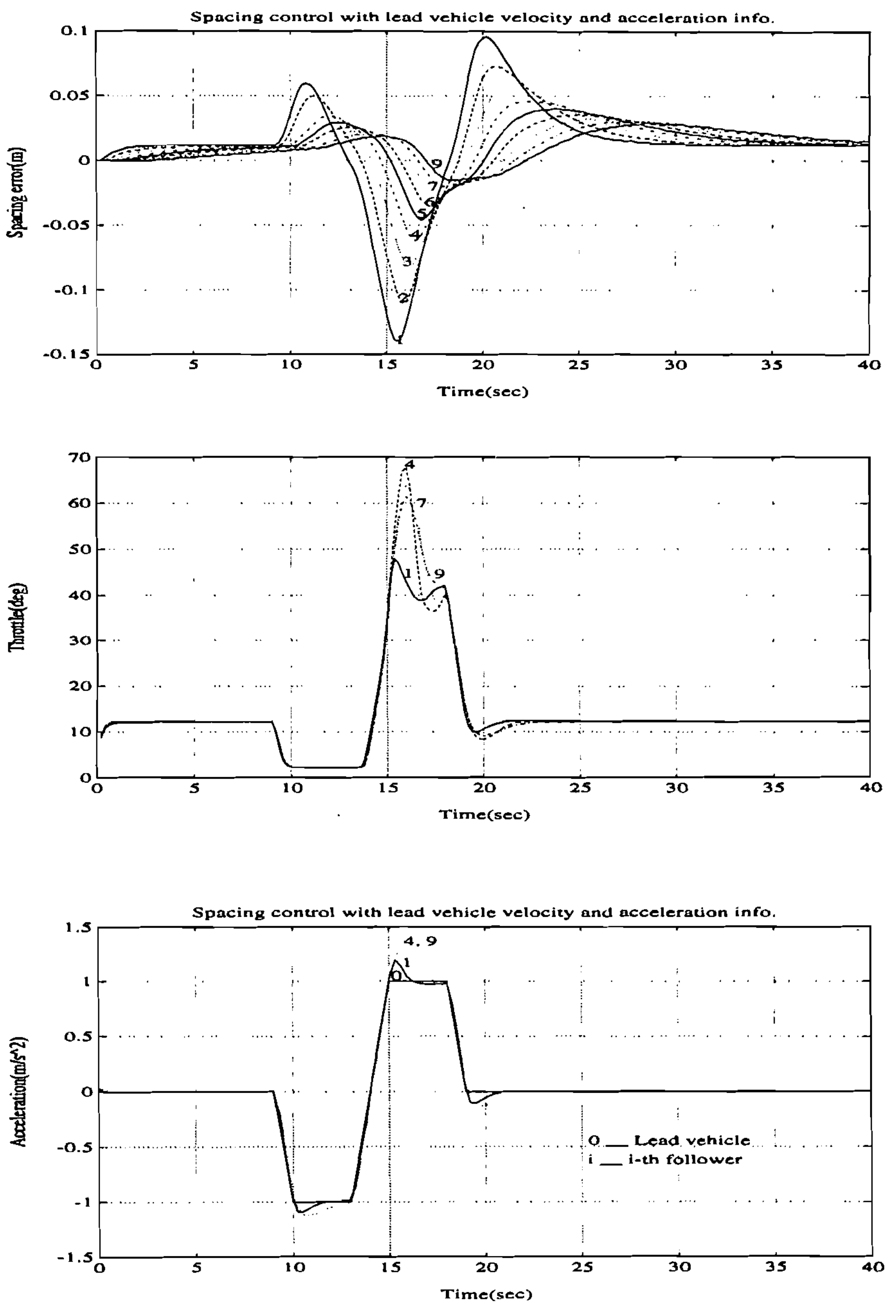


Fig. 2. Spacing control with lead vehicle velocity and acceleration information.

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expense of a) higher control efforts for the vehicles at the tail of the platoon and b) lower traffic capacity. The schemes in Remarks 9 and 10 limit the maximum number of vehicles that can be packed in a platoon. The justification of $\gamma$ being a measure of string stability is clear from the above remarks, since it indicates how well the maximum spacing errors are attenuated. Simulation plots with lead vehicle velocity and acceleration feedback are shown in Figure 2. For simulation plots shown in Figures 2–6, a 10 vehicle platoon is considered. In the simulation plots shown in Figures 2–6, the number $n$ on the plot represents the $n$ th following vehicle in the platoon. All the vehicles start with a velocity of 24.5 m/s and they are positioned in such a way that the initial error in spacing is zero. The gains used for the simulation plot shown in Figure 2 are: $k_p = 0.5$, $k_v = 1$, $k_a = k_l = 0.5$, $c_v = 0.5$. Simulation plots with lead vehicle velocity, position and acceleration feedback is shown in Figure 3. The gains used for this simulation are: $k_a = k_l = .5$, $k_p = 0.5$, $c_p = .25$ $k_v = 1.0$, $c_v = 0.75$. All the simulation results are obtained after imposing a throttle saturation rate of $1000°/sec$ and a brake saturation limit of $8000N - m$. It can be seen that the maximum spacing error of the second following vehicle is less than two thirds the maximum spacing error of first. Also, the spacing error of the vehicles at the tail of the platoon is negligible compared to that of the first vehicle in the platoon.

**Remark 11:**

If the control objective (2) is relaxed, then $k_1 \neq 0$. For the semi-autonomous control law, by Remark 1, $\gamma = 1$ and is achievable if $k_1/(1 - k_a)$ lies between $\beta_1$ and $\beta_1 + \beta_2$. It would be interesting to note that velocity feedback of the j-th vehicle in equations 24 and 25 would produce a steady state spacing error to any step change in the lead vehicle's velocity. The steady state spacing error is equal to $(k_1/k_p)\Delta_v$, where $\Delta_v$ is the step change in the lead vehicle's velocity from its previous steady state value. At the same time, it would ensure string stability for the case when no lead vehicle information or preceding vehicle acceleration is available. In the autonomous case, the zero steady state spacing error and the string stability requirements are at odds with each other. Therefore, for the autonomous case when there is no amplification of spacing errors, a new spacing error definition has to be made.

## 5. HEADWAY CONTROL STRATEGY

Define the spacing error to be

$$\varepsilon_j := x_j - x_{j-1} + L_j + h_w \dot{x}_j \tag{47}$$

where $h_w$ is the headway time [2] that is being controlled.

[2]Headway time is defined as the time it takes the vehicle j to cover a distance $x_j - x_{j-1} - L_j$

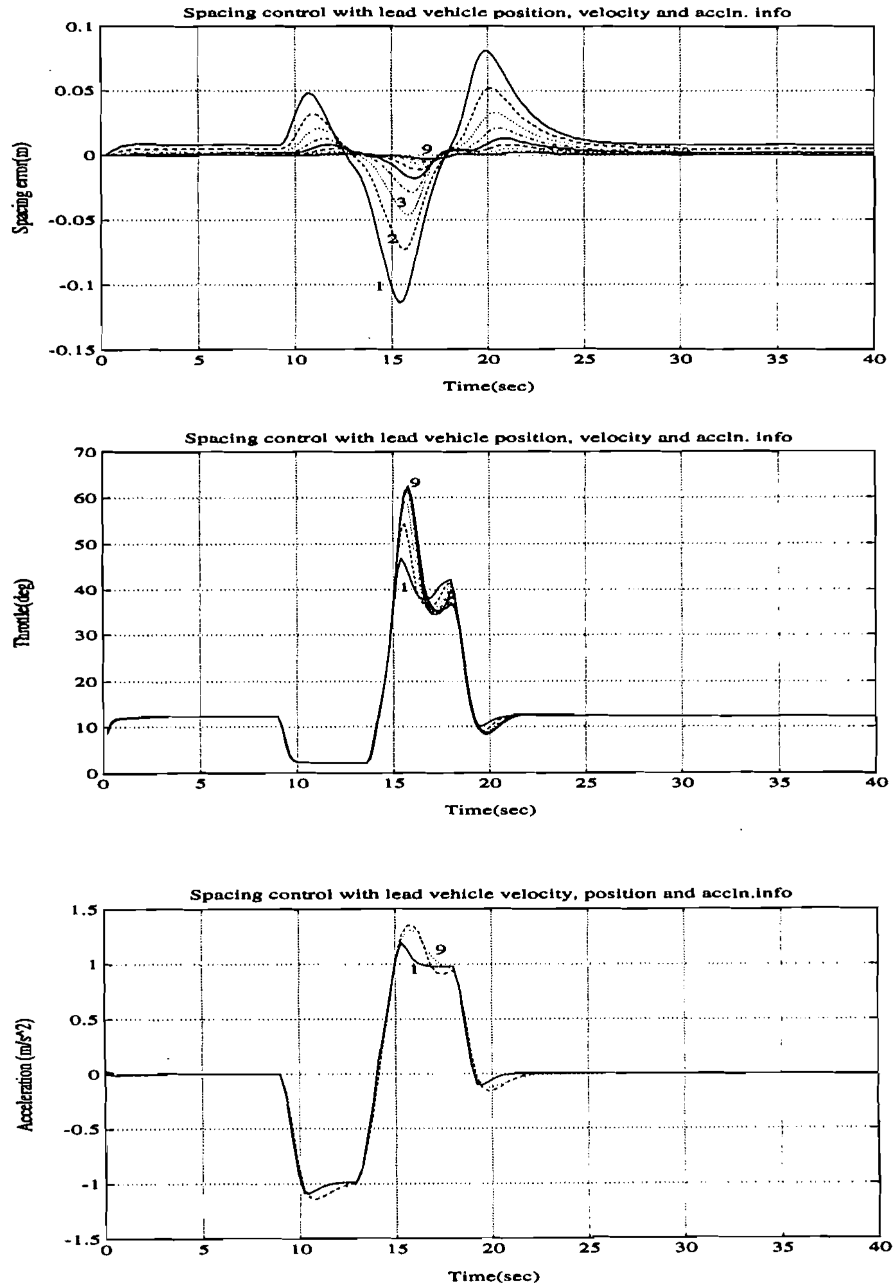


Fig. 3. Spacing control with lead vehicle position, velocity and acceleration information.

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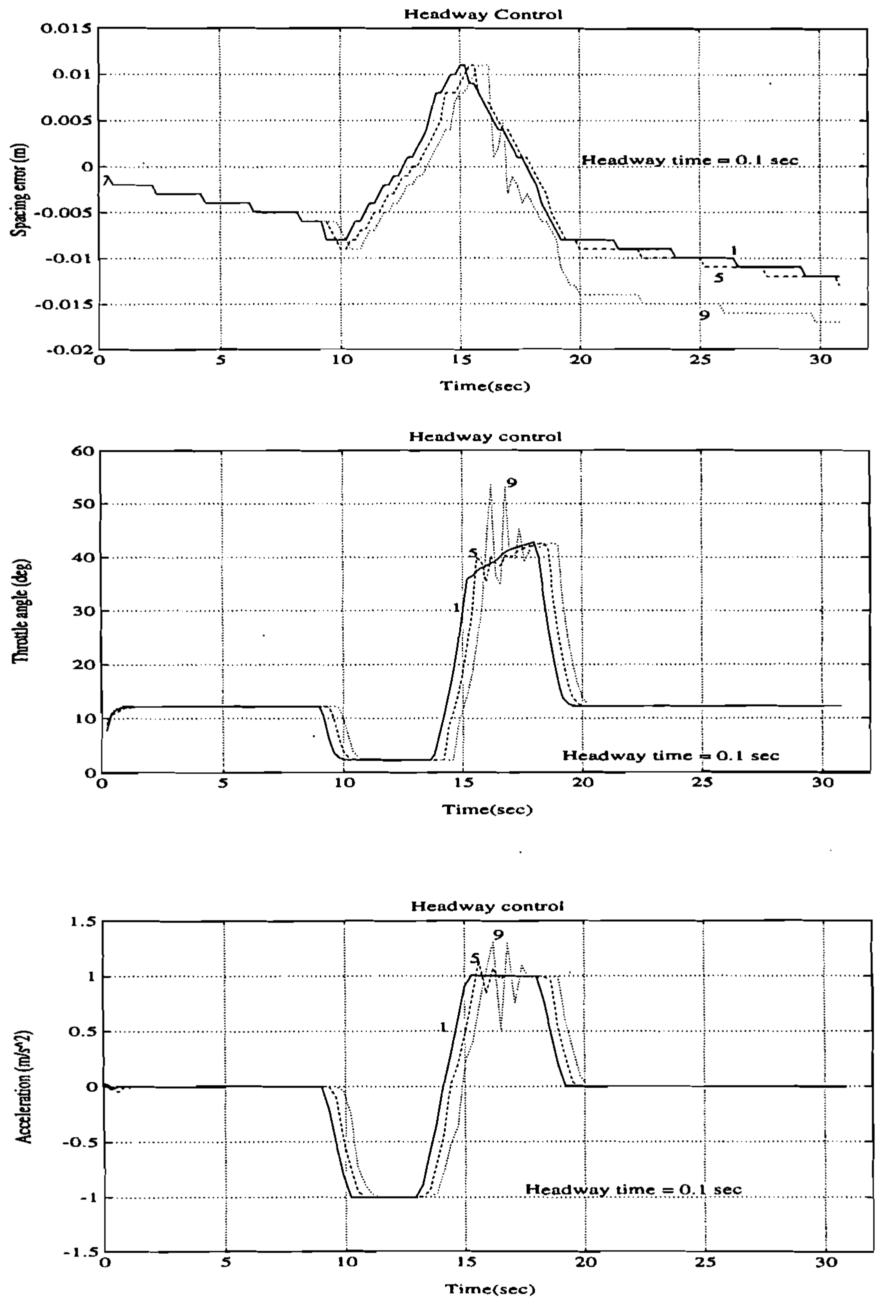


Fig. 4. Headway Control with a headway time of 0.1 sec.

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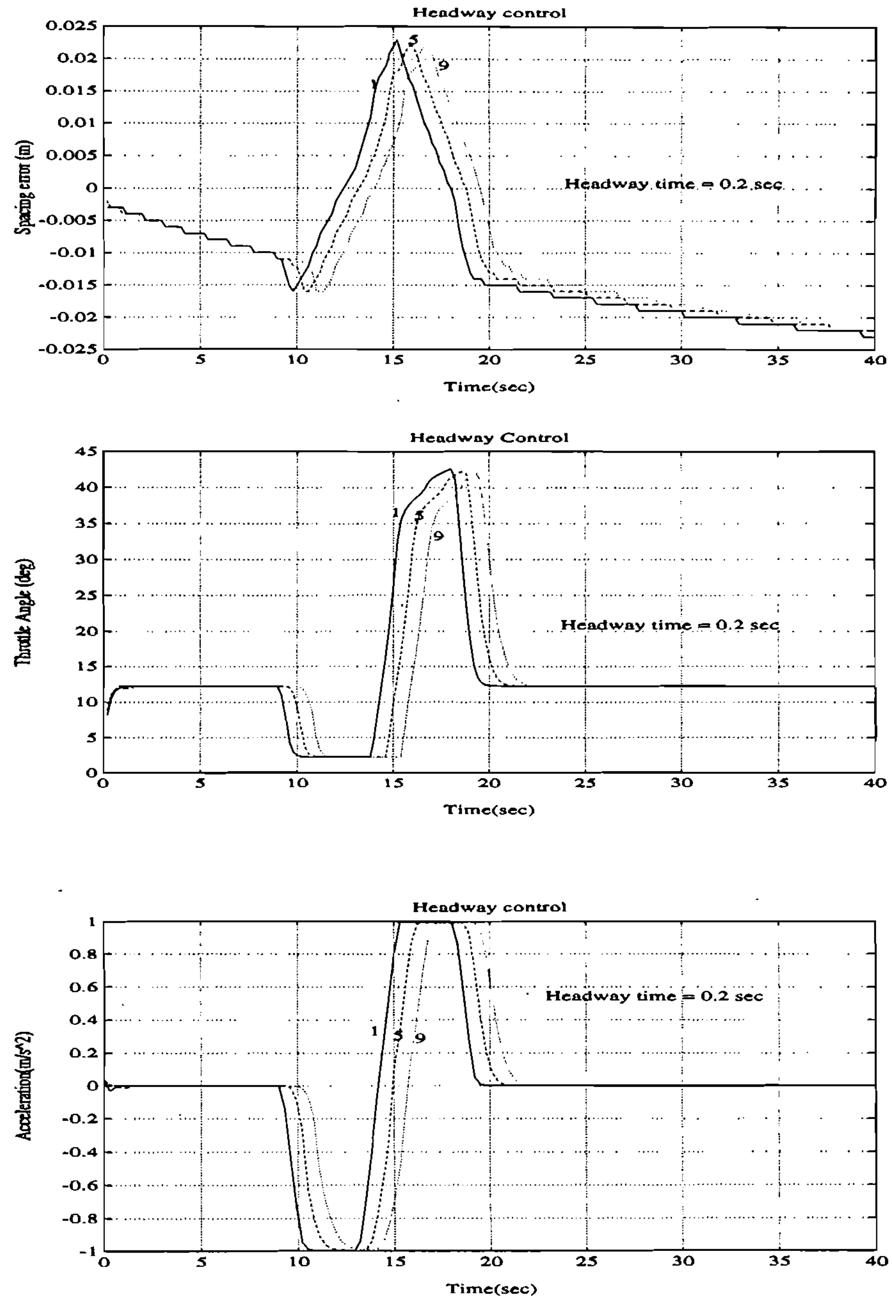


Fig. 5. Headway control with a headway time of 0.2 sec.

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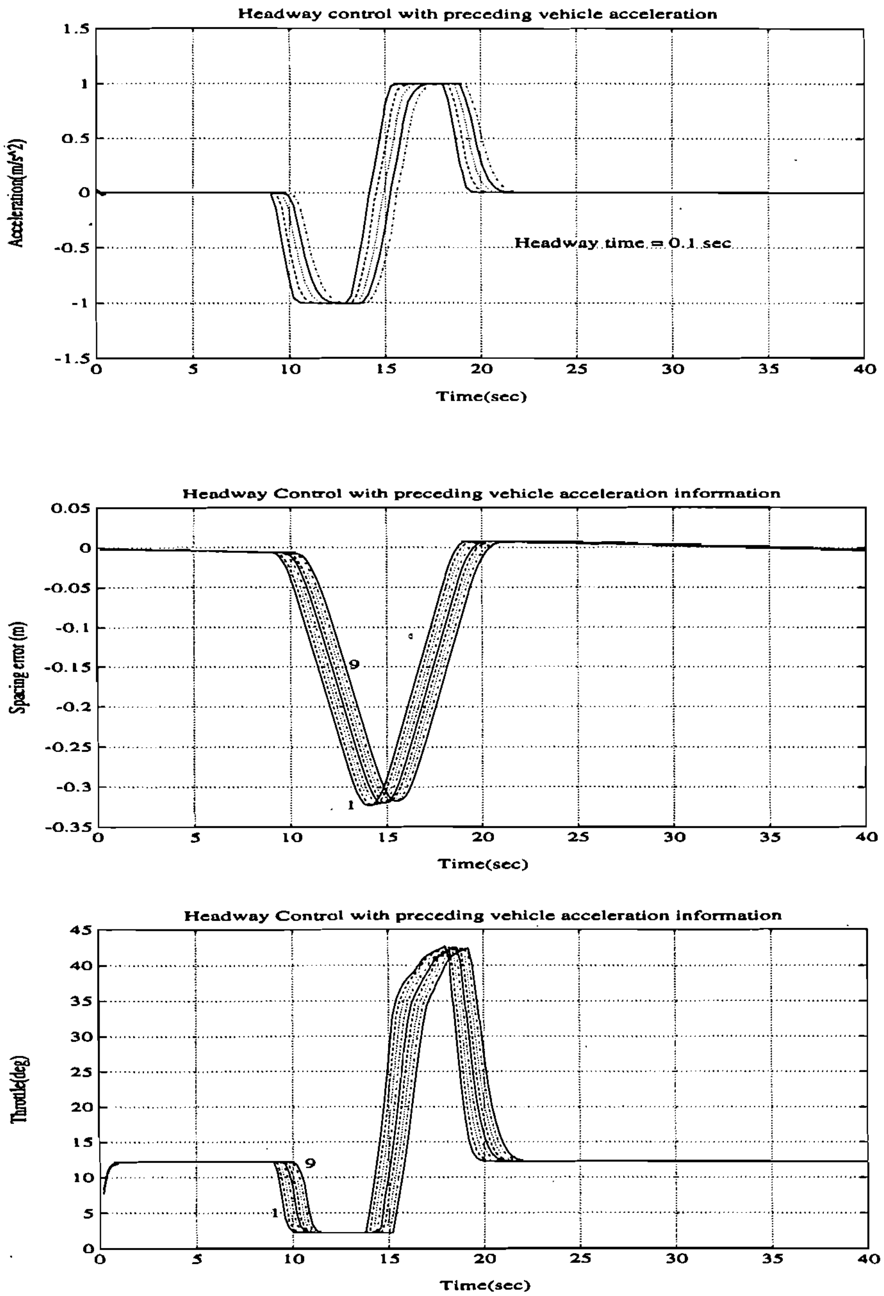


Fig. 6. Headway Control with preceding vehicle acceleration information.

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Consider the following law which does not require lead vehicle information

$$u_j = \frac{\dot{x}_j - \dot{x}_{j-1} + \lambda\varepsilon_j}{h_w} \tag{48}$$

Equations 45 and 20 yield

$$\dot{\varepsilon}_j + \lambda\varepsilon_j = 0 \tag{49}$$

In order to satisfy performance objectives 1) and 2), $\lambda$ has to be chosen to be greater than 0. Actuator bandwidth and sampling considerations place an upper bound on $\lambda$. It can be found that the transfer function $\hat{h}(s) := (\hat{\varepsilon}_j/\hat{\varepsilon}_{j-1})(s)$ is

$$\hat{h}(s) = \frac{1}{h_w s + 1} \tag{50}$$

Clearly, string stability can be assured for all $h_w$. This is a very attractive feature of this strategy considering that no lead vehicle information needs to be fed back. From equations 14 and 48, the desired net engine torque can be written as

$$T_{nd_j} = c_a R^3 h^3 w_e^2 + RhF_f + \frac{J_e}{Rh}\left[\frac{\dot{x}_j - \dot{x}_{j-1} + \lambda\varepsilon_j}{h_w}\right] \tag{51}$$

Similarly, the desired brake torque is given by

$$T_{bd} = \frac{J_e}{R^2 h}\left[\frac{\dot{x}_j - \dot{x}_{j-1} + \lambda\varepsilon_j}{h_w}\right] - \frac{c_a R^3 h^3 w_e^2 + RhF_f}{R} \tag{52}$$

It can be seen that the desired brake torque and the desired net engine torque are inversely proportional to the headway time. An important consideration in choosing a headway time is that it should not only improve the traffic capacity, but it also should be such that input saturation does not occur. Two such input saturation constraints are: Saturation of the throttle and the brake and rate saturation of the throttle actuator. Simulation results indicate that smaller headway time (less than 0.1 sec) results in saturation of throttle angle or throttle angle rate or both. Simulation results for a headway time of 0.1 sec is shown in Figure 4. It can be seen from the acceleration plot that the ride quality of the vehicles at the tail of the platoon deteriorates with an increase in vehicle index. Simulation results for a headway time of 0.2 sec is shown in Figure 5. Since the transfer function $\hat{h}(s) \to 1$ as $h_w \to 0$, the maximum spacing errors of all the vehicles have approximately the same magnitude unlike the spacing control. It should be noted that saturation can occur for a more demanding maneuver or for a vehicle with smaller saturation limits, even for $h_w > 0.20 sec$.

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As the headway time becomes smaller, the desired inter-vehicle distance does not change very much with the speed of the platoon (i.e it tends to become constant spacing control strategy, in the limiting case). Since string stability cannot be assured for a constant spacing control strategy without the use of lead vehicle information, there is an inherent bound on the smallest headway time that can be achieved without saturating the actuators and assuring string stability. Feeding back preceding vehicle acceleration helps reduce the bound for this case, but it cannot reduce the inverse dependence of control gains on the headway time. A lower bound is of practical significance for increasing the traffic lane capacity.

A modified sliding mode headway control strategy based on the assumption that the preceding vehicle acceleration information can be estimated is developed below. Define a sliding surface $S_1$ for the j-th vehicle in the platoon

$$S_1 = c_p \varepsilon_j + c_I \int_0^t \varepsilon_j + (v_{j-1} - v_j) \tag{53}$$

Differentiating the above equation yields:

$$\dot{S}_1 = c_p \dot{\varepsilon}_j + c_I \varepsilon_j + \dot{v}_{j-1} - \dot{v}_j \tag{54}$$

Treating $\ddot{x}_j$ as the control input as in equation 20, choose a desired value of $u_j$ such that

$$S_1 = -k_1 S_1; \quad k_1 > 0 \tag{55}$$

Thus, from the three equations above, the desired acceleration command is:

$$u_j = \frac{1}{1 + c_p h_w} \left[ (c_p + k_1)(v_{j-1} - v_j) + (c_I + k_1 c_p) \varepsilon_j + k_1 c_I \int_0^t \varepsilon_j dt + \dot{v}_{j-1} \right] \tag{56}$$

**Remark 12:**

The proposed control scheme will drive the system trajectories to the sliding surface, i.e

$$c_p \varepsilon_j + c_I \int_0^t \varepsilon_j dt + (v_{j-1} - v_j) = 0.0 \tag{57}$$

**Remark 13:**

For any constant acceleration maneuver of the lead vehicle, the proposed control scheme can guarantee that the spacing error goes to zero by properly choosing parameters $c_p, c_I, k_1$.

In order to overcome the problem of large transient engine and brake torques caused by non-zero initial spacing and velocity errors, the above developed scheme is modified in the following way:

Redefine the first control surface as:

$$S_1' = c_p\hat{\varepsilon}_j + c_I \int_0^t \hat{\varepsilon}_j dt + (\hat{v}_{j-1} - v_j) \tag{58}$$

where

$$\hat{\varepsilon}_j = \varepsilon_j - r(t) \tag{59}$$

$$\dot{\hat{v}}_{j-1} = \alpha_1(v_{j-1}(0))(v_{j-1} - \hat{v}_{j-1});$$

$$\hat{v}_{j-1}(0) = v_j(0)$$

$$r(t) = \left[\varepsilon_j(0) + \left[\alpha_2(\varepsilon_j(0), v_j(0))\varepsilon_j(0) + \frac{c_p + \alpha_1}{c_p}(v_{j-1}(0) - v_j(0))\right]t\right] \tag{60}$$
$$exp[-\alpha_2(\varepsilon_j(0), v_j(0))t]$$

and $\alpha_1(v_{j-1}(0))$, $\alpha_2(\varepsilon_j(0), v_j(0))$ are strictly positive functions. In order to avoid instantaneously high desired engine torque, the controlled vehicle is designed to catch up with the leading vehicle smoothly. This idea motivates us to replace $v_{j-1}$ with $\hat{v}_{j-1}$ and to replace $\varepsilon_j$ with $\hat{\varepsilon}_j$. The choice of $r(t)$ is modified from the technique proposed in [10].

Following the control scheme proposed earlier, $u_j$ is chosen to make

$$\dot{S}_1' = -k_1 S_1'; \quad k_1 > 0 \tag{61}$$

and the value of $u_j$ to achieve this is:

$$u_j = \frac{1}{1 + h_w c_p}\left[(c_I + k_1 c_p)\hat{\varepsilon}_j + k_1 c_I \int_0^t \hat{\varepsilon}_j dt + c_p(v_{j-1} - v_j) - c_p\dot{r}(t) \right. \tag{62}$$
$$\left. + \alpha_1(v_j(0))(v_{j-1} - \hat{v}_{j-1}) + k_1(\hat{v}_{j-1} - \hat{v}_j)\right]$$

**Remark 14:**

It is clear that $S_1'$ converges to $S_1$ with a rate of convergence determined by $\alpha_1$ and $\alpha_2$. Since $u_j$ is chosen to make $S_1'$ go to zero, it follows that $S_1$ goes to zero.

**Remark 15:**

A negative $\varepsilon_j(0)$ or $v_{j-1}(0) - v_j(0)$ corresponds to a braking situation. The more negative they are, the more dangerous the controlled vehicle is. Thus, it is necessary to reflect the dangerous situations to the controller as soon as possible, which enables the controller of the controlled vehicle to be able to respond to it properly avoiding a collision. In this case, the convergence rate for

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$S_1' \rightarrow S_1$ should be faster and hence $\alpha_1(v_j(0))$ and $\alpha_2(\varepsilon_j(0), v_j(0))$ should be large.

**Remark 16:**

For the schemes developed above, for the zero initial conditions, the gain $c_l + k_1 c_p/(1 + c_p h_w)$ is inversely proportional to $h_w^2$ or the gain $c_p/(1 + c_p h_w)$ is inversely proportional to $h_w$. When the preceding vehicle acceleration can be estimated, the constant of proportionality can be made small.

Simulation results with preceding vehicle acceleration feedback are presented for $h_w = 0.1$ in Figure 6 and with no initial condition errors. The ride quality, in terms of acceleration, is better although the error magnitude is high. Similarly, with preceding vehicle acceleration feedback, maximum spacing errors are attenuated. Implementation of this algorithm assumes that preceding vehicle acceleration information is available (for semi-autonomous ICC) or that it can be estimated perfectly (for AICC).

## 6. EVALUATION

In this section, the two platoon control strategies are compared using lane capacity in *vehicles/lane/hour* as a measure. Consider a platoon of N vehicles maintaining a distance $L_p$ from its preceding one. Let $L_v$ be the inter-vehicle spacing in the platoon and $L_c$ be the car length. The lane capacity, [6,11], is given by

$$\phi_{id} = \frac{3600v}{L_v + L_c + \frac{L_p}{N}} veh/lane/hr \tag{63}$$

For the case of the spacing control strategy, $L_v = L_0$, a constant. For the case of the headway control strategy, $L_v = L_0 + h_w v$. The ideal lane capacity given by (63) should be derated by 20% to account for merge and lane changing. $L_p$ is estimated assuming that no collisions are allowed when the platoons are moving at $v_c m/s$ and when the lead platoon decelerates at $d_1 m/s^2$ and the following platoon decelerates at $d_2 m/s^2$, $\Delta t sec$ after the lead platoon has started decelerating.

$$L_p = v_c \Delta t + \frac{v_c^2}{2}\left[\frac{1}{d_1} - \frac{1}{d_2}\right] \tag{64}$$

Typical values of the parameters are: $v_c = 30m/s$; $\Delta t = .3s$; $d_1 = 4m/s^2$ $d_2 = 10m/s^2$; $L_0 = 1m$; $L_v = 5m$; $L_p = 76.5m$.

For spacing control strategy,

$$\phi_{act} = \frac{2880v}{6 + \frac{76.5}{N}} \tag{65}$$

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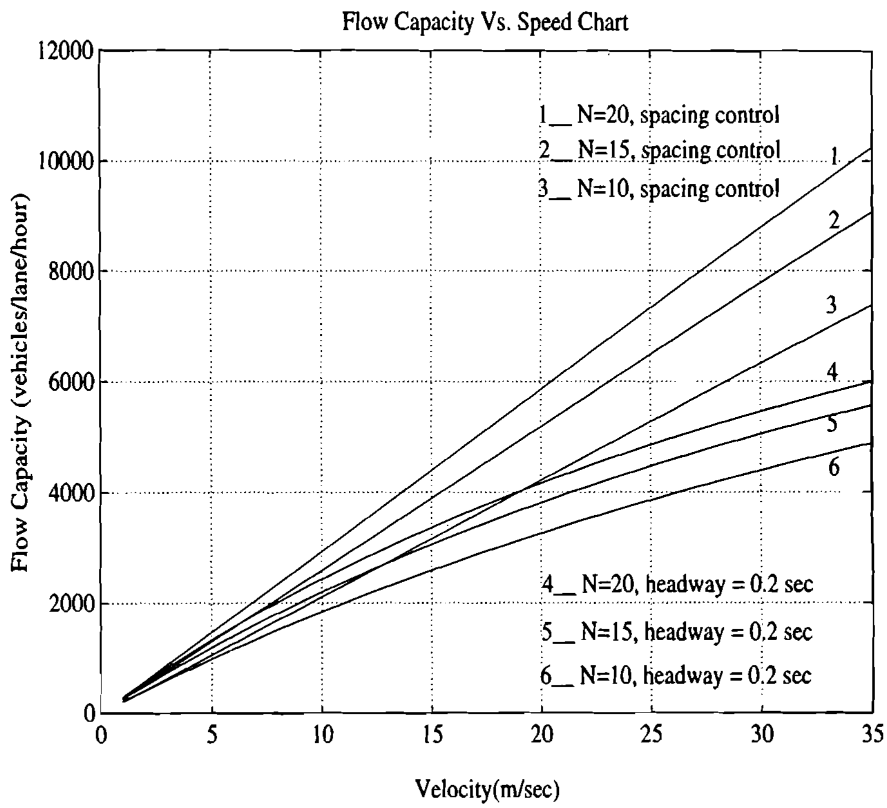


Fig. 7. Flow Capacity Vs. Platoon Speed Chart.

For headway control strategy,

$$\phi_{act} = \frac{2880v}{6 + h_w v + \frac{76.5}{N}} \tag{66}$$

From the above formula, it is clear that lower headway times yield higher lane capacities. The lane capacities of both the schemes, given by the two equations above, are shown in Figure 7 where $N$ in the plot refers to the corresponding platoon sizes. From the previous section, a headway time of 0.20 sec is chosen for comparision. It can be seen that the spacing control has atleast 30% more traffic capacity than the headway control. When the preceding vehicle acceleration is estimated, a headway time of 0.1 sec can be used. Even for this case, the spacing control scheme has 25% more traffic capacity than the headway control.

## 7. CONCLUSIONS

A longitudinal model of an automotive vehicle is establised and used to design vehicle following control laws for automatically operated vehicles. It is shown that two types of stability issues are important, first, individual vehicle stability with respect to a desired following distance, and second the stability (attenuation) of disturbances propogated from vehicle to vehicle within a string of following vehicles. The first type of stability, individual vehicle stability is straightforward to establish, the second type, "string stability" is more involved, and is shown to depend upon the type of information available to each vehicle. Two types of control systems were analyzed, the first was constant spacing control and the second was constant headway control. Conditions required for maintaining string stability during constant spacing control were established under the assumption that ideal input/output linearization could be achieved. Inter-vehicle communication is required to transmit preceding and platoon lead vehicle information to all the vehicles. It is shown that knowledge of the relative position of each vehicle with respect to the lead vehicle can guarantee geometric attenuation of disturbances upstream in the vehicle. Constant headway control has the desirable feature of providing string stability without requiring inter-vehicle communication. This make it a very attractive method for AICC systems.

The use of these two vehicle control concepts for fully automated "platooning" to achieve increased lane capacities was considered. It was shown that the constant headway law requires control torques inversely proportional to the headway which can lead to input saturation for small headways. It is argued that for high capacity, small vehicle to vehicle spacing, that the constant spacing control method is necessary at the price of inter-vehicle communication.

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## APPENDIX

**Proof of FACT 1:**

$$\int_0^\infty |h(t)|dt \geq |\int_0^\infty h(t)dt| = |\hat{h}(0)| \tag{67}$$

Since $h(t)$ is continuous, let $t_1$ be the only time in the interval $[t_0, t_2]$ when it changes sign. Without loss of generality, it is assumed that $h(t) > 0$ for $t\varepsilon[t_0, t_1)$ and $h(t) < 0$ for $t\varepsilon(t_1, t_2]$. Therefore,

$$\left|\int_{t_0}^{t_2} h(t)dt\right| = \left|\int_{t_0}^{t_1} h(t)dt - \int_{t_1}^{t_2}\right|h(t)|dt| < \int_{t_0}^{t_2} |h(t)|dt \tag{68}$$

If

$$\left|\int_0^\infty h(t)dt\right| = \int_0^\infty |h(t)|dt \tag{69}$$

Then

$$\begin{aligned}\left|\int_0^\infty h(t)dt\right| &= \int_0^\infty \left|h(t)\right|dt \leq \left|\int_0^{t_0} h(t)dt\right| + \left|\int_{t_0}^{t_2} h(t)dt\right| + \left|\int_{t_2}^\infty h(t)dt\right| \\ &< \left|\int_0^{t_0} h(t)dt\right| + \int_{t_0}^{t_2} |h(t)|dt + \left|\int_{t_2}^\infty h(t)dt\right| < \int_0^\infty |h(t)|dt\end{aligned} \tag{70}$$

which is a contradiction.

Proof of FACT 1 b) can be found in [7].

**Proof of Remark 1:**

If $k_a > k_p/(k_p + c_p)$, then a sinusoidal input of unit amplitude at a sufficiently high frequency results in a sinusoidal output whose amplitude is greater than $k_p/(k_p + c_p)$. Therefore, $\gamma > k_p/(k_p + c_p)$.

If $k_a = k_p/(k_p + c_p)$, then

$$\hat{h}(s) = \frac{k_p}{k_p + c_p}\left[1 + \frac{(k_v k_p(c_p + k_p) - (k_v + c_v + k_1))s}{s^2 + (k_v + c_v + k_1)s + k_p + c_p}\right] \tag{71}$$

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Unless $c_v + k_1/(k_v) = c_p/(k_p), ||h||_1 > k_p/(k_p + c_p)$. If $c_v + k_1/(k_v) = c_p/(k_p)$, $\hat{h}(s) = k_p/(k_p + c_p)$, i.e a static attenuator is obtained.

if $k_a < k_p/(k_p + c_p), k_a \neq 1$.

Therefore, define

$$\hat{f}(s) := \frac{(k_v + c_v + k_1 - \frac{c_v+k_1}{1-k_a})s + k_p + c_p - \frac{c_p}{1-k_a}}{s^2 + (k_v + c_v)s + k_p + c_p} \tag{72}$$

where

$$\hat{h}(s) = k_a + (1 - k_a)\hat{f}(s) \tag{73}$$

Since $k_a < k_p/(k_p + c_p), 1 - k_a > 0$ and hence,

$$1 - \frac{c_p}{(1 - k_a)(k_p + c_p)} = \frac{\frac{k_p}{k_p+c_p} - k_a}{1 - k_a} > 0.$$

$$||f||_1 \geq |\hat{f}(0)| = \frac{\frac{k_p}{k_p+c_p} - k_a}{1 - k_a} \tag{74}$$

$$\begin{aligned} \hat{h}(s) = k_a + (1 - k_a)\hat{f}(s) \Rightarrow ||h||_1 &\geq |k_a| + |1 - k_a||\hat{f}(0)| \\ &= |k_a| + \left|\frac{k_p}{k_p + c_p} - k_a\right| \end{aligned} \tag{75}$$

If $k_a < 0 \Rightarrow ||h||_1 > k_p/(k_p + c_p)$. If $k_a \varepsilon [0, k_p/(k_p + c_p))$, then $||h|| \geq (k_p/k_p + c_p)$.

If $\phi(s) = s^2 + (k_v + c_v + k_1)s + k_p + c_p$ has complex conjugate poles, then $f(t)$ changes sign, consequently, by equations 28–30, $||f||_1 > k_p/(k_p + c_p) - k_a)/(1 - k_a)$ and $||h||_1 > k_p/(k_p + c_p)$.

Let $\phi(s) = (s + \beta_1)(s + \beta_2)$ with $\beta_2 > \beta_1 > 0$. By partial fraction expansion and inverse Laplace transform,

$$f(t) = \frac{\beta_1^2 - \frac{(c_v+k_1)\beta_1 - c_p}{1-k_a}}{\beta_2 - \beta_1} e^{-\beta_2 t} \left[ -e^{(\beta_2-\beta_1)t} + \frac{\frac{(c_v+k_1)\beta_2 - c_p}{1-k_a} - \beta_2^2}{\frac{(c_v+k_1)\beta_1 - c_p}{1-k_a} - \beta_1^2} \right] \tag{76}$$

If $f(t)$ does not change sign and $k_a \varepsilon [0, (k_p/k_p + c_p))$, then $||h||_1 = (k_p/k_p + c_p)$. For no sign change to occur in $f(t)$ for all $t \geq 0$,

$$\frac{\frac{(c_v+k_1)\beta_2 - c_p}{1-k_a} - \beta_2{}^2}{\frac{(c_v+k_1)\beta_1 - c_p}{1-k_a} - \beta_1{}^2} \leq 1 \tag{77}$$

which implies that

$$\left[\frac{(c_v + k_1)}{1 - k_a} - (\beta_1 + \beta_2)\right]\left[\frac{(c_v + k_1)}{1 - k_a} - (\beta_1 + \frac{c_p}{\beta_1(1 - k_a)})\right] \leq 0 \tag{78}$$

Therefore, $((c_v + k_1)/1 - k_a)$ lies between $\beta_1 + \beta_2$ and $\beta_1 + (c_p/\beta_1(1 - k_a))$. $\beta_2 = \beta_1$ is the limiting case and the result still holds.

**Proof of Remark 2:**

By the Routh-Hurwitz criterion, $k_v + c_v + k_1 > 0$, $k_p + c_p > 0$;
Since $w_l(t) - w_l(t_f) = 0$ for $t \geq t_f$,

$$\hat{w}_l(s) = \frac{w_l(t_f)}{s} + \int_0^{t_f} (w_l(t) - w_l(t_f))e^{-st}dt \tag{79}$$

By the final value theorem, equations 32 and 33,

$$\lim_{t\to\infty} \varepsilon_j(t) = \lim_{s\to 0} s\hat{h}^{j-1}(s)\hat{g}(s)\hat{w}_l(s) = \frac{k_1 k_p^{j-1}}{k_p + c_p^j} w_l(t_f) \tag{80}$$

Therefore, for zero steady state error, $k_1 = 0$.

**Proof of Remark 3 :**

Since no lead vehicle information is available, $c_v = 0; c_p = 0; k_l = 0$;

$$\hat{h}(s) = \frac{k_a s^2 + k_v s + k_p}{s^2 + k_v s + k_p} \tag{81}$$

For $k_a \neq 1$,

$$\hat{h}(s) = k_a + (1 - k_a)\frac{k_v s + k_p}{s^2 + k_v s + k_p} \tag{82}$$

Define

$$f(t) = L^{-1}\frac{k_v s + k_p}{s^2 + k_v s + k_p} \tag{83}$$

From equation 31,

$$||h||_1 = |k_a| + |1 - k_a|||f||_1 \tag{84}$$

If $s^2 + k_v s + k_p$ has complex conjugate roots, $f(t)$ changes sign, $||f|| > 1$ by equations 28–30 and $||h|| > |k_a| + |1 - k_a| > 1$. If

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$s^2 + k_v s + k_p = (s+\beta_1)(s+\beta_2)$ with $\beta_2 \geq \beta_1 > 0$, then $f(t)$ changes sign at $t = 2(ln(\beta_2) - ln(\beta_1)/\beta_2 - \beta_1) > 0$ when $\beta_2 > \beta_1$ and at $(2/\beta_2) > 0$ when $\beta_2 = \beta_1$. Consequently $||h|| > 1$. If $k_a = 1$, it is easy to see that $\hat{h}(s) = 1, \hat{g}(s) = 0$

**Proof of Remark 13:**

From equations 20 and 56,

$$\dot{v}_j = c_p\dot{\varepsilon}_j + (c_I + k_1 c_p)\varepsilon_j + k_1 c_I \int_0^t \varepsilon_j dt + k_1(v_{j-1} - v_j) + \dot{v}_{j-1} \tag{85}$$

From equation 47, it follows that

$$\begin{aligned}\dddot{\varepsilon}_j &= \ddot{v}_{j-1} - \ddot{v}_j - h_w\dddot{v}_j \\ &= \ddot{v}_{j-1} - [c_p\ddot{\varepsilon}_j + (k_1c_p + c_I)\dot{\varepsilon}_j + k_1c_I\varepsilon_j + k_1(\dot{v}_{j-1} - \dot{v}_j) + \ddot{v}_{j-1}] \\ &\quad - h_w[c_p\dddot{\varepsilon}_j + (k_1c_p + c_I)\ddot{\varepsilon}_j + k_1c_I\dot{\varepsilon}_j + k_1(\ddot{v}_{j-1} - \ddot{v}_j) + \dddot{v}_{j-1}]\end{aligned} \tag{86}$$

Therefore,

$$\begin{aligned}(1 + h_wc_p)\dddot{\varepsilon}_j + (h_wc_I + h_wk_1c_p + c_p + k_1)\ddot{\varepsilon}_j + (h_wk_1c_I + c_I + k_1c_p)\dot{\varepsilon}_j + k_1c_I\varepsilon_j \\ = -h_w[\dddot{v}_{j-1} + k_1\ddot{v}_{j-1}]\end{aligned}$$

From the above expression, it follows that

$$\frac{\hat{\varepsilon}_1}{v_l}(s) = \frac{-h_w(s^3 + k_1s^2)}{(1 + h_wc_p)s^3 + (h_wc_I + h_wk_1c_p + c_p + k_1)s^2 + (h_wk_1c_I + c_I + k_1c_p)s + k_1c_I}$$

Therefore, for any constant acceleration maneuver of the lead vehicle, the proposed control scheme guarentees that $\varepsilon_1$ and hence $\varepsilon_j$ to zero.

**Proof of Remark 16:**

Define $z_j$ as:

$$z_j = x_j - \dot{x}_j(0)t - x_j(0) \tag{87}$$

Notice that:

$$\dot{z}_j = \dot{x}_j - \dot{x}_j(0); \quad \ddot{x}_j = \ddot{z}_j; \quad z_j(0) = 0; \quad \dot{z}_j(0) = 0.0 \tag{88}$$

If the initial condition error is zero, (i.e $\dot{x}_j(0) = \dot{x}_{j-1}(0)$ and $\varepsilon_j(0) = 0$ then

$$\varepsilon_j = z_{j-1} - z_j - h_w\dot{z}_j + \varepsilon_j(0) \tag{89}$$

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The control $u_j$ is specified as:

$$u_j = \alpha_1(\dot{z}_{j-1} - \dot{z}_j) + \alpha_2\varepsilon_j + \alpha_3\int_0^t \varepsilon_j dt + \alpha_4\ddot{z}_{j-1} \tag{90}$$

Taking Laplace Transforms,

$$\hat{h}(s) = \frac{\hat{Y}_j}{\hat{Y}_{j-1}}(s) = \frac{\hat{Y}_j - \hat{Y}_{j-1}}{\hat{Y}_{j-1} - \hat{Y}_{j-2}}(s) = \frac{\alpha_4 s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3}{s^3 + (\alpha_1 + h_w\alpha_2)s^2 + (\alpha_2 + h_w\alpha_3)s + \alpha_3}$$

Therefore,

$$\frac{\hat{\varepsilon}_j}{\hat{\varepsilon}_{j-1}}(s) = \hat{h}(s) \tag{91}$$

In order to guarentee string stability,

$$|\hat{h}(jw)| \leq 1 \quad \forall w \geq 0 \tag{92}$$

This implies that

$$(\alpha_3 - h_w\alpha_2 w^2)^2 + (\alpha_2 + h_w\alpha_3 - w^2)^2 w^2 \geq (\alpha_3 - \alpha_1 w^2)^2 + (\alpha_2 - \alpha_4 w^2)^2 w^2 \tag{93}$$

Therefore, $\forall w \geq 0$,

$$(1 - \alpha_4^2)w^4 + w^2(2\alpha_2\alpha_4 + h_w^2\alpha_2^2 + 2h_w\alpha_1\alpha_2 - 2\alpha_2 - 2\alpha_3 h_w) + h_w^2\alpha_3^2 \geq 0 \tag{94}$$

This requires that

$$1 - \alpha_4^2 \geq 0 \tag{95}$$

and that one of the two inequalities below have to be satisfied:

$$2\alpha_2\alpha_4 + h_w^2\alpha_2^2 + 2h_w\alpha_1\alpha_2 - 2\alpha_2 - 2\alpha_3 h_w \geq 0 \tag{96}$$

$$-2\sqrt{1-\alpha_4^2}h_w\alpha_3 \leq 2\alpha_2\alpha_4 + h_w^2\alpha_2^2 + 2h_w\alpha_1\alpha_2 - 2\alpha_2 - 2\alpha_3 h_w \leq 2\sqrt{1-\alpha_4^2}h_w\alpha_3$$

Stability of the closed loop requires that

$$\alpha_1 + h_w\alpha_2 > 0; \alpha_3 > 0; \tag{97}$$

$$(\alpha_1 + \alpha_2 h_w)(\alpha_2 + h_w\alpha_3) > \alpha_3 \tag{98}$$

Since $h_w > 0, \alpha_3 > 0$, this implies that

$$\{2\alpha_2\alpha_4 + h_w^2\alpha_2^2 + 2h_w\alpha_1\alpha_2 - 2\alpha_2 > 0 \tag{99}$$

Observing that if $a + b > c$, then $max(a, b) > c/2$, and from the stability conditions and the above equation, it follows that

$$\alpha_1 > (1 - \alpha_4)/2h_w \tag{100}$$

or

$$\alpha_2 > (1 - \alpha_4)/h_w^2 \tag{101}$$

Hence the proof.

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