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# Optimizing Non-decomposable Performance Measures

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## Abstract

In a general classification setting, our notions of performance need to be quantified using multiple measures. In some cases like classification problems with label imbalance, it proves to be useful to optimize over non-decomposable measures such as F-measure, G-mean, and H-mean. In our work, we consider certain classes of performance measures and aim to study existing methods to optimize these measures and propose novel methods in the explored settings. We outline the main theoretical results in this topic as well as our objectives for the project.

## 1 Introduction

Every learning problems boils down to optimizing a performance measure. Common performance measures include Accuracy, Precision, Specificity, squared error loss etc. These common additive performance measure are not well-behaved in some particular learning settings like classification with label imbalance. Therefore, non-additive performance measures like  $\text{prec@k}$ , F-measure, G-mean etc. find their application in realizing more practical learning setting. We first present the existing work already done in optimizing non-decomposable performance measure. Secondly, we propose the possible extensions for the existing work in the subsequent section.

## 2 Existing work

The general work of [1] overviews the framework of online regret and provides algorithms to optimally learn under a varying set of notions of losses and penalties. Their generalization capability comes at the cost of trying to solve an NP hard problem at each step of the learning procedure. When we restrict our attention to certain specific loss functions or certain classes of loss functions, we can analyze them a bit better and provide more optimistic bounds for faster algorithms.

For objectives like pAUR and precision@K, [6] provides regret bounds in the non-stochastic adversarial setting. In the work of [2], the authors provide sublinear regret bounds for the stochastic setting for a family of loss functions that can be expressed as a concave or pseudo linear combination of the true positive rate / false positive rate.

We are interested in these settings, where the existence of faster and better algorithms can be hoped for. Specifically, it is not clear whether we can find algorithms that work tractably in the adversarial setting, either with an adaptive adversary or with an oblivious one.

Some extensions that can be studied under the general ambit of online learning include the setting of learning under delay. In [5], the authors show that standard online gradient descent and follow the perturbed leader models achieve  $O(\sqrt{D})$  in the delayed setting, where they denote with  $D$  the sum

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of delays of each round’s feedback. While this is interesting on its own, we would like to analyze specific models that deal with our choice of performance measures. In [7] they provide a black box style meta algorithms that can take algorithms that work traditionally in the non-delayed case and make them work in the case of delays.

### 3 Project Aims

For our project, we propose three directions of research. Our aim would be to tackle these related problems and hope to obtain tangible progress on these in terms of theoretical results.

In order of how we would like to tackle them, they are given below.

#### 3.1 Online Adversarial methods for concave and pseudo linear performance measures

The work of [2] provides us with a stochastic update scheme that let us optimize on concave combinations of the TPR/TNRs, as well as on pseudo-linear combinations. We would like to extend this to a setting where our learning takes place in an online fashion.

**STAMP** technique can be understood as doing a saddle point optimization. Our idea would be to use the analysis in [8] for saddle point optimization using mirror descent to provide a way to understand regret bounds in the online setting.

In addition, in [9], the authors provide an algorithm **NEMESIS** which is a simpler version of the technique used in [2]. While they achieve logarithmic regret bounds in their dual steps, it would be interesting to see if we can obtain similar bounds for the primal steps as well. In particular, since [2] uses an actual gradient descent style method whereas [9] only uses a simple update step, the logarithmic bound for the dual is obtained.

#### 3.2 Learning to optimize concave performance measures with delayed feedback

While the above settings all deal with the generic online learning model, one interesting extension is learning with delayed feedback. The work of [5] provide a starting point for learning with delays. It would be one of our goals to study the algorithms of [2] when faced with delays. If possible, we could then try to do an online to batch conversion on the methods we propose.

#### 3.3 Casting multiclass performance measures as a bandit learning problem

In [4], the authors provide optimization techniques for the multiclass setting with complex measures. They consider settings where performance measures which are linear functions of the confusion matrix, as well as ones like the multi-class extensions of G-mean and micro  $F_1$ -measures. We would try to analyze this in a bandit learning setting, where our algorithm does not receive complete information about the correctness of its predictions.

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