Assignment 1

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1 Question1:

1.1 2D case

1.1.1 Velocity Distribution

$$f(\vec{v}) = n \frac{m}{2\pi kT} e^{-\frac{m\vec{v}\cdot\vec{v}}{2kT}} \tag{1}$$

1.1.2 Speed Distribution

$$f(v) = n \frac{m}{2\pi kT} v e^{-\frac{mv^2}{2kT}} \tag{2}$$

1.1.3 Energy Distribution

$$f(\epsilon) = \frac{n}{2\pi kT} e^{-\frac{\epsilon}{kT}} \tag{3}$$

Mean velocity:

$$V_{mean} = \sqrt{\frac{kT}{8\pi m}} \tag{4}$$

Rms velocity:

$$V_{rms} = \frac{kT}{\pi m} \tag{5}$$

1.2 1D case

1.2.1 Speed Distribution:

$$f(v) = n\sqrt{\frac{m}{2\pi kT}}e^{-\frac{mv^2}{2kT}} \tag{6}$$

1.2.2 Energy Distribution

$$f(\epsilon) = n \frac{1}{2\sqrt{\pi kT}} \frac{1}{\sqrt{\epsilon}} e^{-\frac{\epsilon}{kT}} \tag{7}$$

Mean velocity:

$$V_{mean} = \sqrt{\frac{2kT}{\pi m}} \tag{8}$$

RMS velocity:

$$V_{rms} = \frac{kT}{4m} \tag{9}$$

2 Question 2:

(Refer code)

3 Question 3:

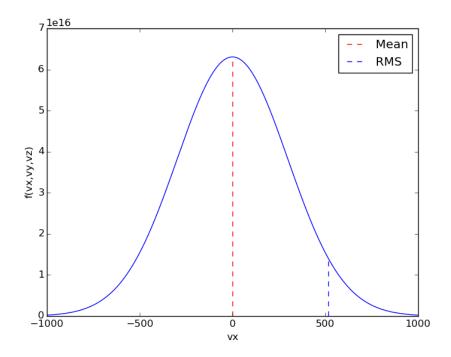


Figure 1: Velocity Distribution Function

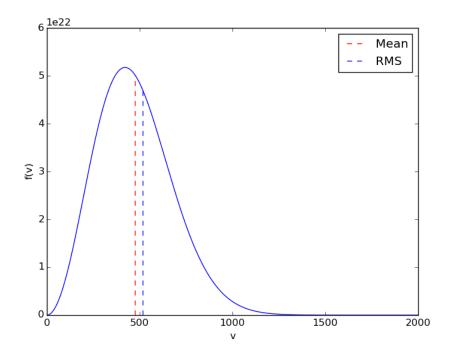


Figure 2: Speed Distribution Function

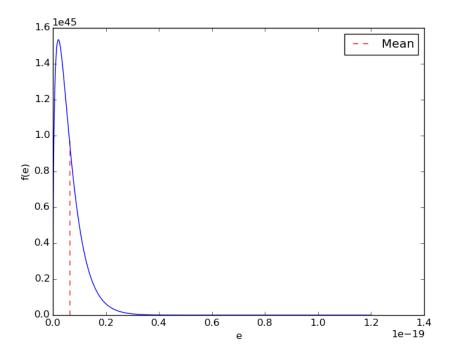


Figure 3: Energy Distribution Function

4 Question 4:

(Done in the above plots)

5 Question 5:

(Refer code) The values given below are for 20000 grids in velocity and energy

Pressure = 108687.853873Pa

Energy = $8.1515 \times 10^4 J$

Vmean = 475.32704008m/s

Entropy = $1.36088286962 \times 10^{2}7$

6 Question 6:

(Refer code) We see that the mean velocity and the pressure are almost independent of grid size and fall down rapidly approaching to zero. The error in Pressure and the Mean velocity was of the order of 10^{-9} and 10^{-11} respectively. So the pattern of the graph isn't of much relevence as the

values of Boltzmann constant and the Avagadro number are taken only till $2\ \mathrm{decimal}\ \mathrm{places}$

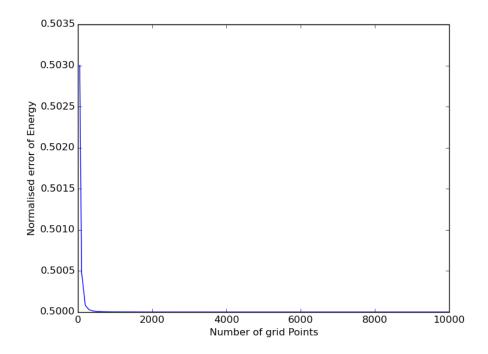


Figure 4: Error in energy calculation

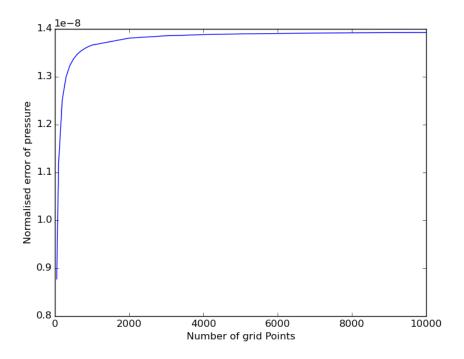


Figure 5: Error in pressure calculation

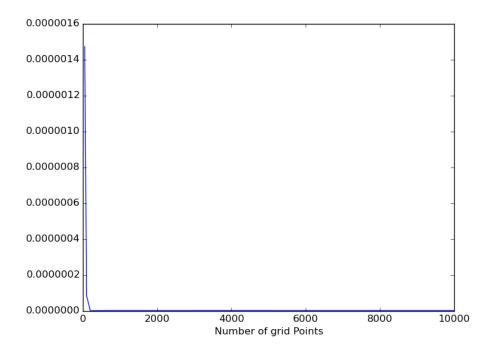


Figure 6: Error in mean velocity

7 Question 7:

Following functions were chosen

• Exponential function Entropy = -0.9991669999117565

$$p(x) = e^{-x} (10)$$

• Exponential Logarithmic function: $p=0.5; \beta=2$ Entropy = $-1.33 \times 10^3 1$

$$p(x, p, \beta) = \frac{-1}{\ln(p)} \frac{\beta(1-p)e^{-\beta x}}{1 - (1-p)e^{-\beta x}}$$
(11)

• Standard Cauchy Distributioon: Entropy = 727.51330695775994

$$p(x) = \frac{1}{\Pi(1+x^2)} \tag{12}$$

The entropy of Maxwell Boltzmann distribution is 2146305679384934.2 and is the highest among all.

8 Question 8:

V has been plotted in logarithmic scale. Blue is for electrons and red for ions

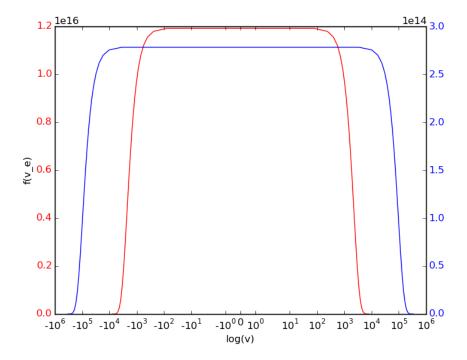


Figure 7: Velocity PDF of electrons and ions

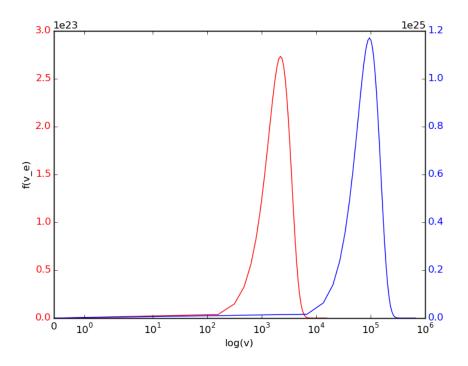


Figure 8: Speed pdf of electrons and ions

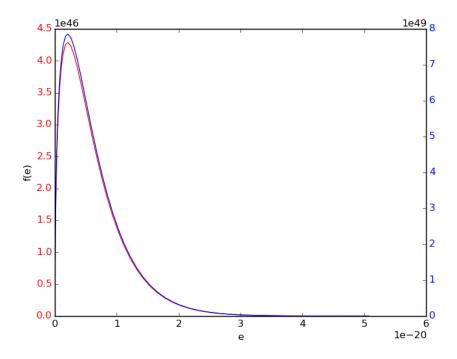


Figure 9: Energy PDF of electrons and ions