

LC Tank with Resistance

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Abstract

The problem is to simulate the behaviour of a LC tank with a resistance applied across a voltage in series with given initial conditions of the charge in the capacitor and current in the inductor.

- The IPython notebook used can be found at <https://gist.github.com/vikaskurapatibat/a457aa26bd2928eacbb1077261eb3905.js>
- Public git repo with open source code is available on https://github.com/vikaskurapatibat/SDES_Project1
- Ipython 5.1.0 is used to run the IPython notebook. So the version 5.1.0 or higher is preferable to run the notebook.
- numpy version 1.8.2 is used. So numpy version 1.8.2 or higher is preferred to run the code.
- matplotlib version 1.4.2 is used. So matplotlib version 1.4.2 or higher is preferred to run the code. [1]
- ffmpeg is required for the animation to be saved in mp4 format. ffmpeg 2.7.6 was used for it. So, ffmpeg with version 2.7.6 or higher is preferable for running of the python script.

Governing Equation for the problem

The governing equation for this electrical problem is [2]

$$\frac{d^2V_C}{dt^2} + \frac{R}{L} \frac{dV_C}{dt} + \frac{V_C}{LC} = \frac{V_S}{LC} \quad (1)$$

where V_C is the voltage across the capacitor varying with time, R is the resistance of the resistor, L is inductance of the inductor, C is the capacitance of the capacitor and V_S is the voltage of the source voltage under the initial conditions of $V_C(0^+) = V_0$ the initial capacitor voltage and $\frac{dV_C}{dt}(0^+) = \frac{i_0}{C}$, where initial inductor current is i_0 .

Solving this analytically for two cases of $\Delta = 0$ and $\Delta \neq 0$ where $\Delta = \frac{R^2}{L^2} - \frac{4}{LC}$

$\Delta = 0$:

The solution when $\Delta = 0$ under the given initial conditions is:

$$V_C(t) = V_S + e^{\frac{-t}{2RC}}(D_1t + D_2) \quad (2)$$

where $D_1 = \frac{i_0}{C}e^{\frac{1}{2RC}}$, $D_2 = (V_0 - V_S)e^{\frac{1}{2RC}}$

$\Delta \neq 0$:

The solution when $\Delta \neq 0$ under the given initial conditions is:

$$V_C(t) = V_0 + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (3)$$

where $s_1 = \frac{-R}{2L} + \frac{\sqrt{\Delta}}{2}$, $s_2 = \frac{-R}{2L} - \frac{\sqrt{\Delta}}{2}$
 $A_1 = \frac{\frac{i_0}{C} - (V_0 - V)s_2}{s_1 - s_2}$, $A_2 = \frac{\frac{i_0}{C} - (V_0 - V)s_1}{s_2 - s_1}$

The results were plotted showing the voltage accross a capacitor varying with time for three cases of damping.

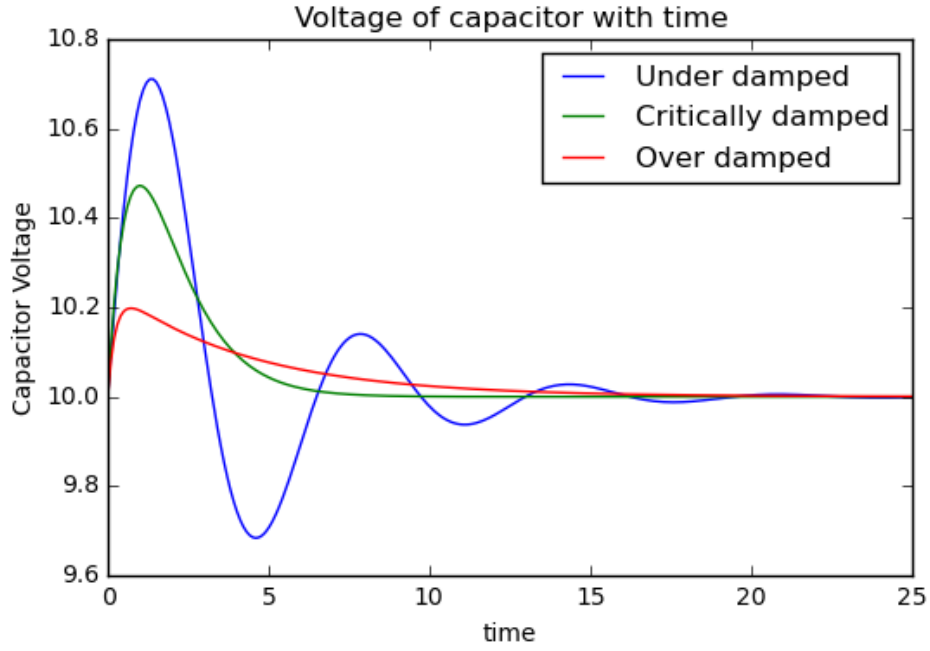


Figure 1: V_C vs time

The results were plotted showing variation of voltage across the capacitor, inductor and resistor for an underdamped case of $R = 0.5$.

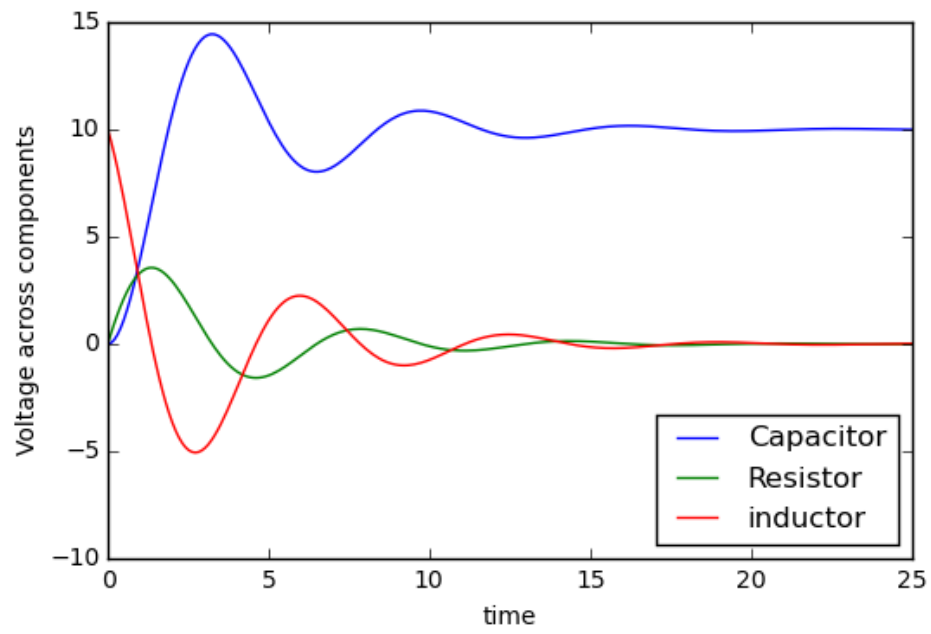


Figure 2: Voltage vs time

Bibliography

- [1] Animations using matplotlib. *<http://matplotlib.org/1.4.1/examples/animation/index.html>*.
- [2] National Tsing Hua University. Chapter 8 natural and step responses of rlc circuits. pages 46–49.