# LC Tank with Resistance

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**Abstract**

The problem is to simulate the behaviour of a LC tank with a resistance applied across a voltage in series with given initial conditions of the charge in the capacitor and current in the inductor.

- Public git repo with open source code is available on `https://github.com/vikaskurapatibat/SDES_Project1`
- Ipython 2.3.0 is used to run the IPython notebook. So the version 2.3.0 or higher is preferrable to run the notebook.
- numpy version 1.8.2 is used. So numpy version 1.8.2 or higher is preffered to run the code.
- matplotlib version 1.4.2 is used. So matplotlib version 1.4.2 or higher is preffered to run the code.

# Governing Equation for the problem

The governing equation for this electrical problem is

$$\frac{d^2V_C}{dt^2} + \frac{R}{L}\frac{dV_C}{dt} + \frac{V_C}{LC} = \frac{V_S}{LC} \tag{1}$$

where $V_C$ is the voltage across the capacitor varying with time, R is the resistance of the resistor, L is inductance of the inductor,C is the capacitance of the capacitor and $V_S$ is the voltage of the source voltage under the initial conditions of $V_C(0^+) = V_0$ the initial capacitor voltage and $\frac{dV_C}{dt}(0^+) = \frac{i_0}{C}$, where initial inductor current is $i_0$.

Solving this analytically for two cases of $\Delta = 0$ and $\Delta \neq 0$ where $\Delta = \frac{R^2}{L^2} - \frac{4}{LC}$

## $\Delta = 0$ :

The solution when $\Delta = 0$ under the given initial conditions is:

$$V_C(t) = V_S + e^{\frac{-t}{2RC}}(D_1 t + D_2) \tag{2}$$

where $D_1 = \frac{i_0}{C}e^{\frac{1}{2RC}}, D_2 = (V_0 - V)e^{\frac{1}{2RC}}$

## $\Delta \neq 0$ :

The solution when $\Delta \neq 0$ under the given initial conditions is:

$$V_C(t) = V_0 + A_1 e^{s_1 t} + A_2 e^{s_2 t} \tag{3}$$

where $s_1 = \frac{-R}{2L} + \frac{\sqrt{\Delta}}{2}$, $s_2 = \frac{-R}{2L} - \frac{\sqrt{\Delta}}{2}$
$A_1 = \frac{\frac{i_0}{C} - (V_0 - V)s_2}{s_1 - s_2}$, $A_2 = \frac{\frac{i_0}{C} - (V_0 - V)s_1}{s_2 - s_1}$

The results were plotted showing the voltage accross a capacitor varying with time for three cases of damping.

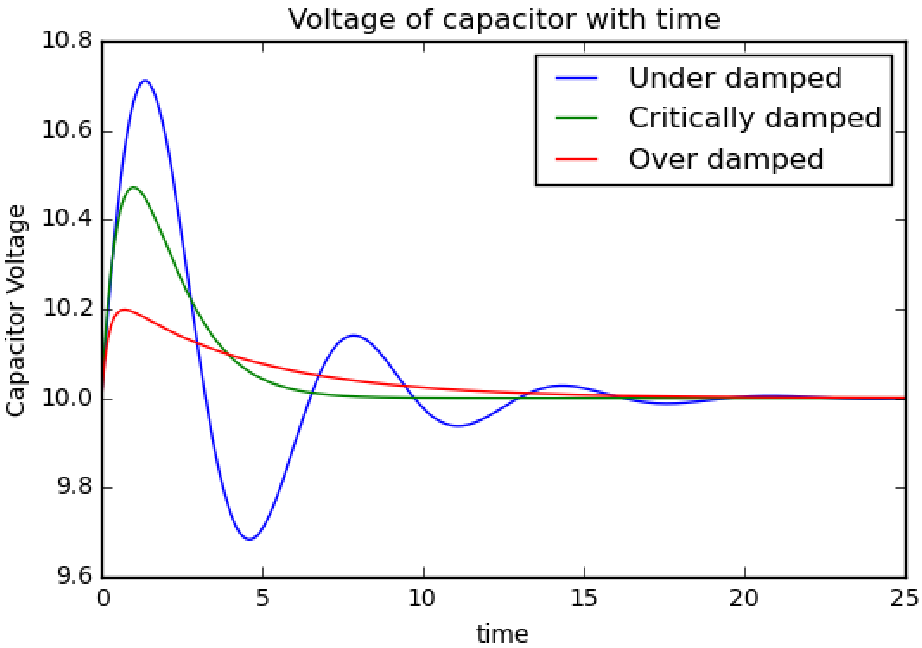


Figure 1: $V_C$ vs time

The results were plotted showing variation of voltage across the capacitor, inductor and resistor for an underdamped case of R = 0.5.

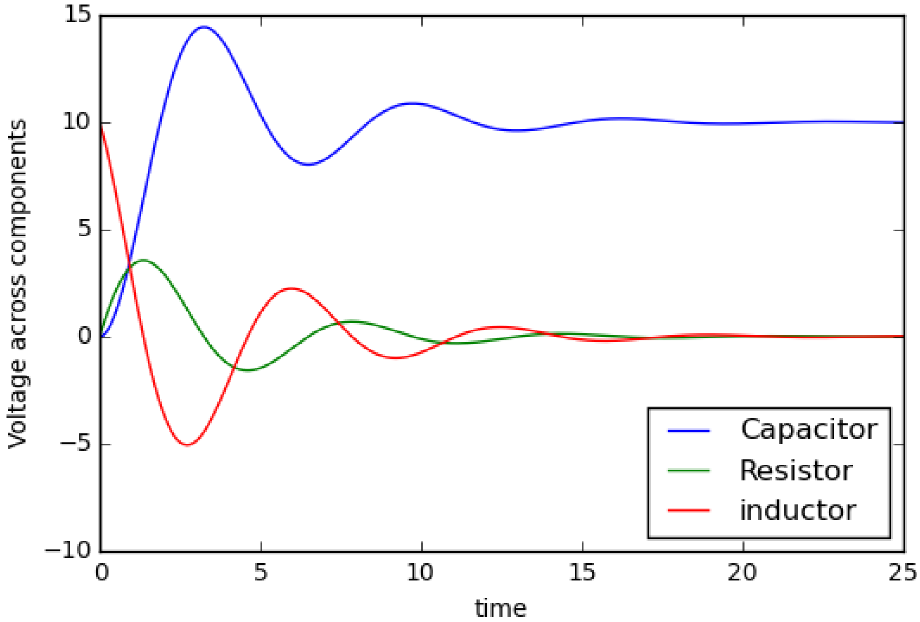


Figure 2: Voltage vs time for different components.