Electromagnetic Simulations in FEKO

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Overview

Theory

In performing electromagnetic simulations to be shown in the upcoming slides, FEKO software was used. FEKO software uses different schemes of which Method of Moments(MOM), Multilevel Fast Multipole Method(MLFMM), Physical Optics(PO) schemes were used as and when required according to the frequency of the problem being simulated.

Theory

RCS: RCS(Radar Cross section is defined as a measure of reflective strength of a target defined as 4π times the ratio of the power per unit solid angle scattered in a specified direction to the power per unit area in a plane wave incident on the scatterer from a specified direction. More precisely it is the limit of that ratio as the distance from the scatterer to the point where the scattered power is measure approaches infinity:

$$\sigma = \lim_{r \to \infty} 4\pi r^2 \frac{|\mathbf{E}^{scat}|^2}{|\mathbf{E}^{inc}|^2} \tag{1}$$

Due to large dynamic range of RCS, a logarithmic power scale is most often used with the reference value of $\sigma_{ref} = 1m^2$:

$$\sigma_{dBsm} = \sigma_{dBm^2} = 10log_{10}(\frac{\sigma_{m^2}}{\sigma_{ref}}) = 10log_{10}(\frac{\sigma_{m^2}}{1}) \tag{2}$$

Theory

To find RCS of a particular body under a particular illumination, we need to solve Maxwell's equations and find the scattered portion of the illumination.
The electromagnetic integral equations were obtained by Stratton Chu using the vector Green's Theorem in conjunction with Maxwell's equations. The total electric and magnetic fields are written as the sum of the incident and scattered fields:

$$\mathbf{E}^T = \mathbf{E}^i + \mathbf{E}^s \tag{3}$$

$$\mathbf{H}^T = \mathbf{H}^i + \mathbf{H}^s \tag{4}$$

$$\tag{5}$$

Theory

The scattered **E** and **H** are given by Stratton-Chu integrals:

$$\mathbf{E}^s = \oint_S [-j\omega\mu(\hat{n} \times \mathbf{H})\psi + (\hat{n} \times \mathbf{E}) \times \nabla\psi + (\hat{n}.\mathbf{E})\nabla\psi] dS \tag{6}$$

$$\mathbf{H}^s = -\oint_S [-j\omega\epsilon(\hat{n} \times \mathbf{E})\psi - (\hat{n} \times \mathbf{H}) \times \nabla\psi + (\hat{n}.\mathbf{H})\nabla\psi] dS \tag{7}$$

where ψ is the free space Green's Function, ω is the radian frequency, μ and ϵ are the permeability and permittivity, $\hat{n}$ is the outward normal of the surface.

Theory

The tangential and perpendicular components of the surface fields are interpreted as currents and charges:

$$\mathbf{J} = \hat{n} \times \mathbf{H}^T electriccurrent \tag{8}$$

$$\mathbf{M} = -\hat{n} \times \mathbf{E}^T magneticcurrent \tag{9}$$

$$\rho = \epsilon \hat{n}.\mathbf{E}^T electriccharge \tag{10}$$

$$\rho^* = \mu \hat{n}.\mathbf{H}^T magneticcharge \tag{11}$$

$$\tag{12}$$

The Green's function ψ and its gradient $\nabla\psi$ are the mathematical equivalents of Huygen's wavelets i.e., each elemental surface current or charge is related to the scattered fields by means of the Huygen wavelet and the total field is simply the sum(integral) over all such surface current elements.

Theory

Mathematically, the Green's function relates an elemental source current or charge to the field at the observation point. The three dimensional Green's function in polar coordinates is an outward scalar spherical wave whose intensity falls off as inverse to distance:

$$\psi = \frac{e^{-jkr}}{4\pi R} \tag{13}$$

where an $e^{j\omega t}$ time dependence is assumed and R is the distance from the elemental source to the observer. This gives the gradient to be:

$$\nabla\psi = (1 - jkR)\psi\frac{\hat{R}}{R} \tag{14}$$

Theory

The definition of Green's function is not valid when the source and field points coincide, as $R = 0 \implies \psi = \infty, \nabla\psi = \infty$. Self terms for currents and charges are derived from Maxwell's equations using the integral form of the curl and divergence equations with elemental loops(lines) and pill boxes.

$$(\hat{n} \times \mathbf{H})_{self} = \frac{\mathbf{J}}{2}, (\hat{n} \times \mathbf{E})_{self} = \frac{\mathbf{M}}{2} \tag{15}$$

$$(\hat{n}.\mathbf{E})_{self} = \frac{\rho}{2\epsilon}, (\hat{n}.\mathbf{H})_{self} = \frac{\rho^*}{2\mu} \tag{16}$$

$$\tag{17}$$

Theory

The implementation of Boundary conditions:
The surface charge density is rewritten invoking the conservation of charge using the continuity equation:

$$(\hat{n}.\mathbf{E}) = \frac{\rho}{\epsilon} = -\frac{j}{\omega\epsilon}(\nabla.\mathbf{J}) \tag{18}$$

If the observation point is on the surface, where the field values are known from the boundary conditions, the resulting forms of the EFIE and MFIE are obtained as:

$$\hat{n} \times \mathbf{E}^T = \hat{n} \times (\mathbf{E}^i + \mathbf{E}^s) = 0 \tag{19}$$

$$\hat{n} \times \mathbf{H}^T = \hat{n} \times (\mathbf{H}^i + \mathbf{H}^s) = \mathbf{J} \tag{20}$$

Theory

This leads to:

$\hat{n} \times \mathbf{E}^i = -\hat{n} \times \mathbf{E}^s$

$= -\hat{n} \times \oint_S [-j\omega\mu(\hat{n} \times \mathbf{H})\psi + (\hat{n} \times \mathbf{E}) \times \nabla\psi + (\hat{n}.\mathbf{E})\nabla\psi] dS$

$\hat{n} \times \mathbf{H}^i = \mathbf{J} - \hat{n} \times \mathbf{H}^s$

$= \mathbf{J} + \hat{n} \times \oint_S [-j\omega\epsilon(\hat{n} \times \mathbf{E})\psi - (\hat{n} \times \mathbf{H}) \times \nabla\psi + (\hat{n}.\mathbf{H})\nabla\psi] dS$

Theory

The procedures required to find the unknown current density involve:

- Expressing the unknown terms of a set of basis functions with unknown coefficients.
- Defining weighting or testing functions.
- Explicitly defining the interaction matrix elements
- Inverting the matrix
- Specifying the polarization and direction of the incident field and computing the resultant current density.
- Computing the scattered field radiated by these induced currents.

Theory

The unknown surface currents are typically expanded as:

$$\mathbf{J} = \sum_{i=1}^{N} b_{x,i} f(t) \hat{u_x} + b_{y,i} f(t) \hat{u_y} \tag{21}$$

where $\hat{u_x}$ and $\hat{u_y}$ are the orthogonal unit surface vectors, f(t) is the expansion function, b is the complex unknown current coefficient.
For solving electric and magnetic field integral equations, an electric and a magnetic operator is defined.

$$L_E(\mathbf{J}) = \hat{n} \times \int [-j\omega\nu \mathbf{J}\psi - \frac{1}{j\omega\epsilon}(\nabla.\mathbf{J})\nabla\psi dS] \tag{22}$$

$$L_H(\mathbf{J}) = \frac{\mathbf{J}}{2} - \hat{n} \times \int \mathbf{J} \times \nabla\psi dS \tag{23}$$

The physical interpretation of these operators is that they give the tangential scattered field on the surface due to a surface current J.

- Method of Moments:

With the aid of this operator notation, the solution is obtained by inserting the series expansion of the unknown currents into the MFIE and calculating the constants.

$$L_H(\mathbf{J}) = \sum_{i=1}^{N} b_i L_H(\mathbf{f}_i) = \hat{n} \times \mathbf{H}^i \tag{24}$$

The next is to multiply equation 24 by a vector weighting function $\mathbf{W}_j$ and integrate the result over each surface patch.

$$< \mathbf{W}, L_H(J) >= \int W.L_H(\mathbf{J})dS \tag{25}$$

$$\Longrightarrow \sum_{i=1}^{N} = b_i < \mathbf{W}j, L_H(\mathbf{f}_i >=< \mathbf{W}_j, \hat{n} \times \mathbf{H}^i > \tag{26}$$

for j = 1 to N

Theory

- Method of Moments:

This set of linear equations for the unknown coefficients b_i can be expressed in matrix notation as

$$\overline{\overline{\overline{Z}}}\mathbf{b} = \mathbf{H}^i \tag{27}$$

where the matrix elements are given by

$$Z_{ij} =< \mathbf{W}_j, L_H(\mathbf{f}_i) > \tag{28}$$

And the unknown current coefficients are expressed as a generalized column vector,

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_N \end{bmatrix} \tag{29}$$

Theory

The known incident fields, which represent the forcing function for the solution, are also expressed as a generalized column vector:

$$\mathbf{H}_j^i = < \mathbf{W}_j, \hat{n} \times \mathbf{H}^i > \tag{30}$$

The physical meaning of each term is clear. The matrix elements express the electrical interaction of each part of the scattering surface with every other part. The ijth matrix element is a measure of the fields produced at the ith surface patch created by a unit current located at the jth surface patch.

The solution for the surface currents is formally given by

$$[b] = [Z]^{-1}[H] \tag{31}$$

This solution turns out to be of the order of $O(N^2)$ as the interaction of each element with every other element is calculated.

Theory

Once the currents are known for a given excitation, the scattered fields due to these currents may be computed from EFIE or MFIE expressions.

$$\overline{E^s} = \frac{-j\omega\mu}{4\pi R} e^{-jkR} \int_s [\mathbf{J} - (\mathbf{J}.\hat{R})] e^{+j\mathbf{k.r}} dS \tag{32}$$

$$\overline{H^s} = \frac{+j\omega\epsilon}{4\pi R} e^{-jkR} \int_s \sqrt{\frac{\mu}{\epsilon}} (\mathbf{J} \times \mathbf{R}) e^{+j\mathbf{k.r}} dS \tag{33}$$

Theory

- Fast Multipole Methods:

Fast Multipole methods are developed to improve the complexity of matrix vector products to O(N) in finite arithmetic i.e., for tolerance ϵ.
Dependence on tolerance is $O(\frac{1}{\epsilon})$
Complexity of FMM $= O(\frac{N}{\epsilon})$
This translates to O(N) or O(NlogN)

Theory

- Fast Multipole Methods
  - O(NlogN) scheme

1. Patches are contained within a cube which is subdivided in a recursive manner. In non adaptive case, each has eight children(a tree code is implemented in this scheme), 27 nearest neighbors and dimension of interaction list is 189.
2. Now each cube is a cluster of patches and are treated separately and cluster cluster interactions along with cluster surface interactions and patch patch interactions are done to calculate the values of far field calculations.

Theory

- Fast Multipole Methods
  - O(N) scheme

1. This scheme performs multipole expansions at finest level 1 from source positoins and strengths first.
2. Form multipole expansions at coarser levels by merging by translating local expansions.
3. Account for interactions at each level by converting a multipole expansion into a local expansion.
4. Transmit information to finer levels by translation of a local expansion.

Theory

The theory of physical optics overcomes the catastrophe of the infinities of flat and singly curved surfaces by approximating the induced surface fields and integrating them to obtain the scattered field. This methods is applicable at relatively high frequencies i.e., low electrical circumferences which make the objects look like they are flat.
In this scheme, the gradient of Green's function is approximated by $\nabla\psi \approx ik\hat{s}\psi$ Hence, the Stratton Chu integrals can be written as

$$\overline{E^s} = ik\psi_0 \int_s \hat{s} \times [\hat{n} \times \overline{E} - Z_0 \hat{s} \times (\hat{n} \times \hat{\overline{H}})] e^{ik\overline{r}.(\hat{i}-\hat{s})} dS \tag{34}$$

$$\overline{H^s} = ik\psi_0 \int_s \hat{s} \times [\hat{n} \times \hat{H} + Y_0 \hat{s} \times (\hat{n} \times \overline{E})] e^{ik\overline{r}.(\hat{i}-\hat{s})} dS \tag{35}$$

where $Y_0 = \frac{1}{Z_0}$ is the admittance of free space.

Theory

We can approximate the total fields within the integrals by making the tangent plane approximation. That is, we assign the surface fields the values they would have if the body had been perfectly smooth and flat at the surface patch of integration dS. This approximation can be made for any body material, but we shall assume the body to be perfectly conducting.

This makes the physical optics approximation of RCS to be:

$$\overline{E_s} = -i2kZ_0H_0\psi_0 \int_s \hat{s} \times [\hat{s} \times (\hat{n} \times \overline{h_i})] e^{ik\overline{r}.(\hat{i}-\hat{s})} dS \tag{36}$$

$$\sqrt{\sigma} = \lim_{R\to\infty} 2\sqrt{\pi} R \frac{\overline{E_s}.\hat{e}_r}{E_0} e^{ikR} \tag{37}$$

$$\implies \sqrt{\sigma} = -i\frac{k}{\sqrt{\pi}} \int_s \hat{n}.\hat{e}_r \times \hat{h}_i e^{ik\overline{r}.(\hat{i}-\hat{s})} dS \tag{38}$$

Theory

Upon evaluating the physical optics integral for the sphere, we obtain the result:

$$\sqrt{\sigma} = \sqrt{\pi}a[(1 + \frac{1}{i2ka})e^{-i2ka} - \frac{1}{i2ka}] \tag{39}$$

where a is the radius of the sphere and the phase of the echo has been referenced to the center of the sphere.

Bullet Points

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
- Nam cursus est eget velit posuere pellentesque
- Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block 1

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Block 2

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Block 3

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Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

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Table

Treatments	**Response 1**	**Response 2**
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

The End