Electromagnetic Simulations in FEKO

Vikas Kurapati

Department of Aerospace Engineering

IIT Bombay

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Overview

1. Theory
   - Definition of RCS
   - Method Of Moments(MOM)
2. Second Section

Theory

In performing electromagnetic simulations to be shown in the upcoming slides, FEKO software was used. FEKO software uses different schemes of which Method of Moments(MOM), Multilevel Fast Multipole Method(MLFMM), Physical Optics(PO) schemes were used as and when required according to the frequency of the problem being simulated.

Theory

RCS: RCS(Radar Cross section is defined as a measure of reflective strength of a target defined as 4π times the ratio of the power per unit solid angle scattered in a specified direction to the power per unit area in a plane wave incident on the scatterer from a specified direction. More precisely it is the limit of that ratio as the distance from the scatterer to the point where the scattered power is measure approaches infinity:

$$\sigma = \lim_{r \to \infty} 4\pi r^2 \frac{|\mathbf{E}^{scat}|^2}{|\mathbf{E}^{inc}|^2} \tag{1}$$

Due to large dynamic range of RCS, a logarithmic power scale is most often used with the reference value of $\sigma_{ref} = 1m^2$:

$$\sigma_{dBsm} = \sigma_{dBm^2} = 10log_{10}(\frac{\sigma_{m^2}}{\sigma_{ref}}) = 10log_{10}(\frac{\sigma_{m^2}}{1}) \tag{2}$$

Theory

To find RCS of a particular body under a particular illumination, we need to solve Maxwell's equations and find the scattered portion of the illumination.
The electromagnetic integral equations were obtained by Stratton Chu using the vector Green's Theorem in conjunction with Maxwell's equations. The total electric and magnetic fields are written as the sum of the incident and scattered fields:

$$\mathbf{E}^T = \mathbf{E}^i + \mathbf{E}^s \tag{3}$$

$$\mathbf{H}^T = \mathbf{H}^i + \mathbf{H}^s \tag{4}$$

$$\tag{5}$$

Theory

The scattered **E** and **H** are given by Stratton-Chu integrals:

$$\mathbf{E}^s = \oint_S [-j\omega\mu(\hat{n} \times \mathbf{H})\psi + (\hat{n} \times \mathbf{E}) \times \nabla\psi + (\hat{n}.\mathbf{E})\nabla\psi] dS \quad (6)$$

$$\mathbf{H}^s = -\oint_S [-j\omega\epsilon(\hat{n} \times \mathbf{E})\psi - (\hat{n} \times \mathbf{H}) \times \nabla\psi + (\hat{n}.\mathbf{H})\nabla\psi] dS \quad (7)$$

where ψ is the free space Green's Function, ω is the radian frequency, μ and ϵ are the permeability and permittivity, $\hat{n}$ is the outward normal of the surface.

Theory

The tangential and perpendicular components of the surface fields are interpreted as currents and charges:

$$\mathbf{J} = \hat{n} \times \mathbf{H}^T electriccurrent \tag{8}$$
$$\mathbf{M} = -\hat{n} \times \mathbf{E}^T magneticcurrent \tag{9}$$
$$\rho = \epsilon \hat{n}.\mathbf{E}^T electriccharge \tag{10}$$
$$\rho^* = \mu \hat{n}.\mathbf{H}^T magneticcharge \tag{11}$$
$$\tag{12}$$

The Green's function ψ and its gradient $\nabla\psi$ are the mathematical equivalents of Huygen's wavelets i.e., each elemental surface current or charge is related to the scattered fields by means of the Huygen wavelet and the total field is simply the sum(integral) over all such surface current elements.

Theory

Mathematically, the Green's function relates an elemental source current or charge to the field at the observation point. The three dimensional Green's function in polar coordinates is an outward scalar spherical wave whose intensity falls off as inverse to distance:

$$\psi = \frac{e^{-jkr}}{4\pi R} \tag{13}$$

where an $e^{j\omega t}$ time dependence is assumed and R is the distance from the elemental source to the observer. This gives the gradient to be:

$$\nabla\psi = (1 - jkR)\psi\frac{\hat{R}}{R} \tag{14}$$

Theory

The definition of Green's function is not valid when the source and field points coincide, as $R = 0 \implies \psi = \infty, \nabla\psi = \infty$. Self terms for currents and charges are derived from Maxwell's equations using the integral form of the curl and divergence equations with elemental loops(lines) and pill boxes.

$$(\hat{n} \times \mathbf{H})_{self} = \frac{\mathbf{J}}{2}, (\hat{n} \times \mathbf{E})_{self} = \frac{\mathbf{M}}{2} \tag{15}$$

$$(\hat{n}.\mathbf{E})_{self} = \frac{\rho}{2\epsilon}, (\hat{n}.\mathbf{H})_{self} = \frac{\rho^*}{2\mu} \tag{16}$$

$$\tag{17}$$

Theory

The implementation of Boundary conditions:
The surface charge density is rewritten invoking the conservation of charge using the continuity equation:

$$(\hat{n}.\mathbf{E}) = \frac{\rho}{\epsilon} = -\frac{j}{\omega\epsilon}(\nabla.\mathbf{J}) \tag{18}$$

If the observation point is on the surface, where the field values are known from the boundary conditions, the resulting forms of the EFIE and MFIE are obtained as:

$$\hat{n} \times \mathbf{E}^T = \hat{n} \times (\mathbf{E}^i + \mathbf{E}^s) = 0 \tag{19}$$

$$\hat{n} \times \mathbf{H}^T = \hat{n} \times (\mathbf{H}^i + \mathbf{H}^s) = \mathbf{J} \tag{20}$$

Theory

This leads to:

$$\hat{n} \times \mathbf{E}^i = -\hat{n} \times \mathbf{E}^s$$
$$= -\hat{n} \times \oint_S [-j\omega\mu(\hat{n} \times \mathbf{H})\psi + (\hat{n} \times \mathbf{E}) \times \nabla\psi + (\hat{n}.\mathbf{E})\nabla\psi] dS$$
$$\hat{n} \times \mathbf{H}^i = \mathbf{J} - \hat{n} \times \mathbf{H}^s$$
$$= \mathbf{J} + \hat{n} \times \oint_S [-j\omega\epsilon(\hat{n} \times \mathbf{E})\psi - (\hat{n} \times \mathbf{H}) \times \nabla\psi + (\hat{n}.\mathbf{H})\nabla\psi] dS$$

Theory

The procedures required to find the unknown current density involve:

- Expressing the unknown terms of a set of basis functions with unknown coefficients.
- Defining weighting or testing functions.
- Explicitly defining the interaction matrix elements
- Inverting the matrix
- Specifying the polarization and direction of the incident field and computing the resultant current density.
- Computing the scattered field radiated by these induced currents.

Theory

The unknown surface currents are typically expanded as:

$$\mathbf{J} = \sum_{i=1}^{N} b_{x,i} f(t) \hat{u_x} + b_{y,i} f(t) \hat{u_y} \tag{21}$$

where $\hat{u_x}$ and $\hat{u_y}$ are the orthogonal unit surface vectors, f(t) is the expansion function, b is the complex unknown current coefficient.
For solving electric and magnetic field integral equations, an electric and a magnetic operator is defined.

$$L_E(\mathbf{J}) = \hat{n} \times \int [-j\omega\nu \mathbf{J}\psi - \frac{1}{j\omega\epsilon}(\nabla.\mathbf{J})\nabla\psi dS] \tag{22}$$

$$L_H(\mathbf{J}) = \frac{\mathbf{J}}{2} - \hat{n} \times \int \mathbf{J} \times \nabla\psi dS \tag{23}$$

The physical interpretation of these operators is that they give the tangential scattered field on the surface due to a surface current J.

Theory

- Method of Moments:

With the aid of this operator notation, the solution is obtained by inserting the series expansion of the unknown currents into the MFIE and calculating the constants.

$$L_H(\mathbf{J}) = \sum_{i=1}^{N} b_i L_H(\mathbf{f}_i) = \hat{n} \times \mathbf{H}^i \tag{24}$$

The next is to multiply equation 24 by a vector weighting function $\mathbf{W}_j$ and integrate the result over each surface patch.

$$< \mathbf{W}, L_H(J) >= \int W.L_H(\mathbf{J})dS \tag{25}$$

$$\Longrightarrow \tag{26}$$

Bullet Points

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
- Nam cursus est eget velit posuere pellentesque
- Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block 1

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Block 3

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Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

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Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

The End