

CSIC 5011 Project 2 – How different are the images of digits by Pixel-wise distance and Earth Mover Distance?

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Background and Aim

- Optimal Transport is currently a hot topic in data science or machine learning.
- Earth Mover Distance (EMD) / Wasserstein Distance** is the fundamental measure of optimal transport between probability distribution - 'minimum work done' moving bins among two histogram.
- EMD satisfies properties of metric.
- This study is going to see result of compare images by pixel-wise distance and EMD.

Dataset

- From `sklearn.datasets.load_digits[1]`:
- Classes: 10
- Samples per class: ~180
- Total Samples: 1797
- Dimensionality: 8*8 pixels

Methodology:

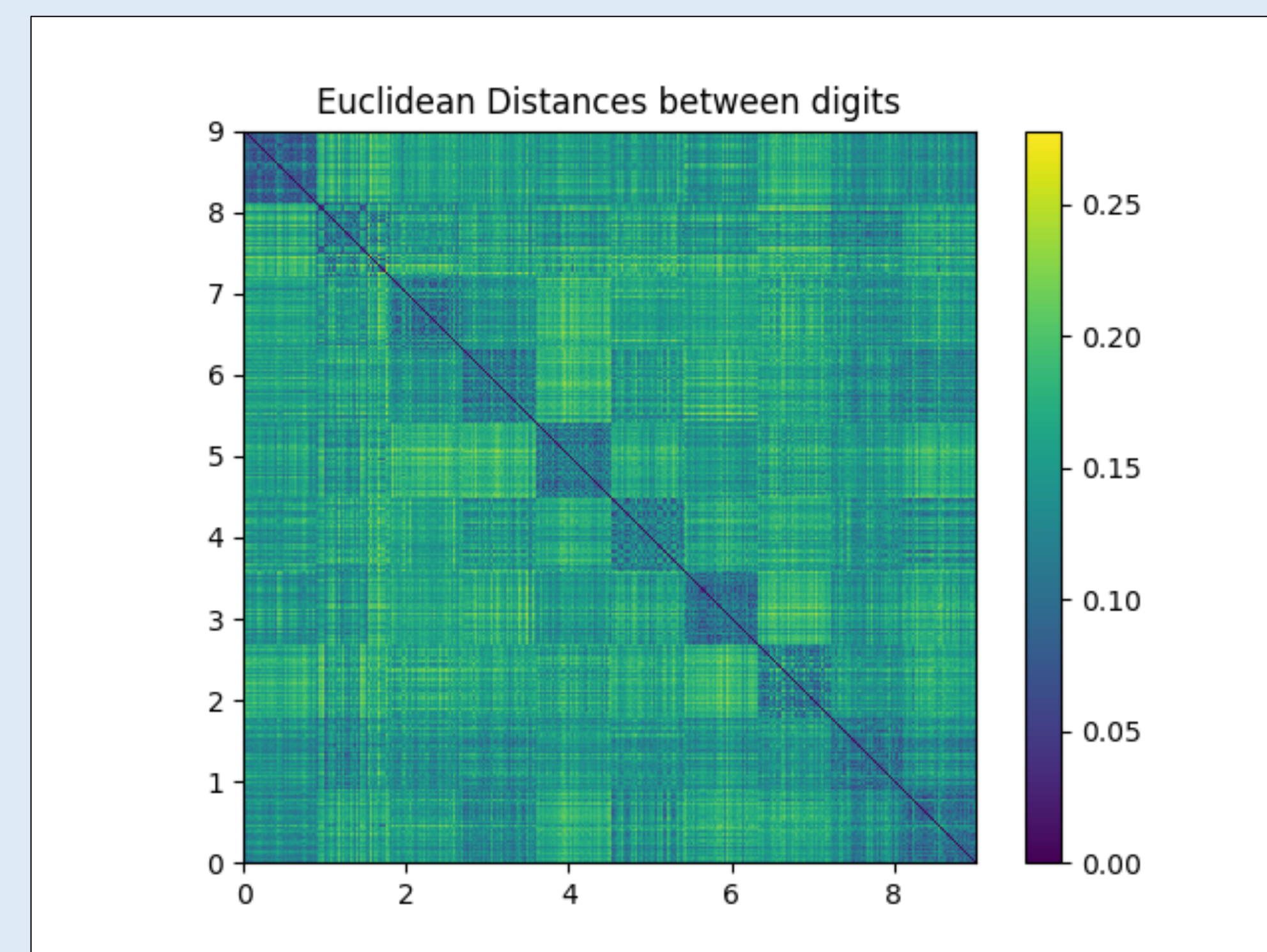
- Construct **EMD matrix** by POT library[2] and **Euclidean distance matrix**
- Use manifold learning techniques: t-SNE with precomputed distance matrix above to compare the results

References

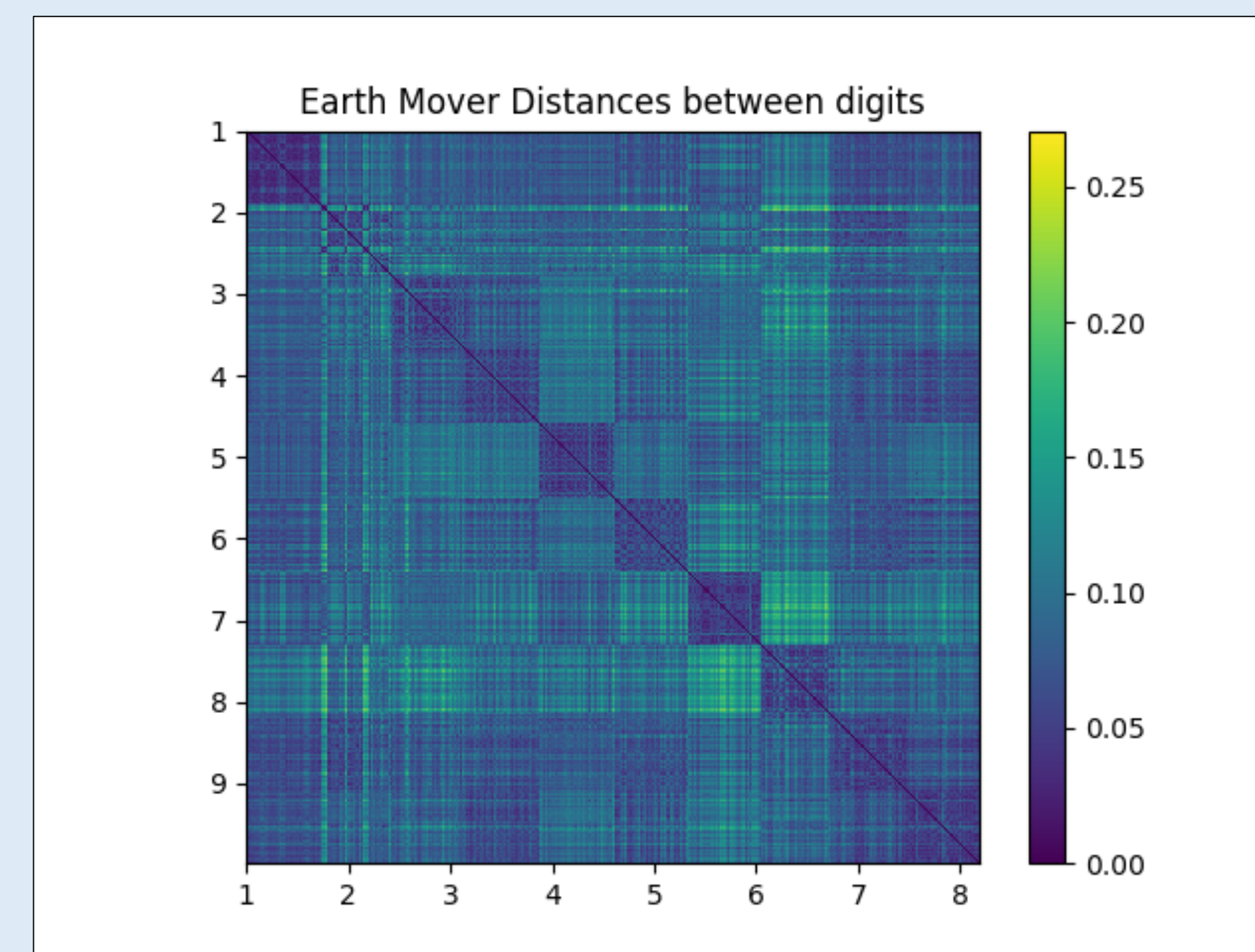
[1] `sklearn.datasets.load_digits` — scikit-learn 0.19.1 documentation
http://scikit-learn.org/stable/modules/generated/sklearn.datasets.load_digits.html

[2] POT: Python Optimal Transport — POT Python Optimal Transport 0.4.0 documentation
<http://pot.readthedocs.io/en/stable/>

Distance Matrices



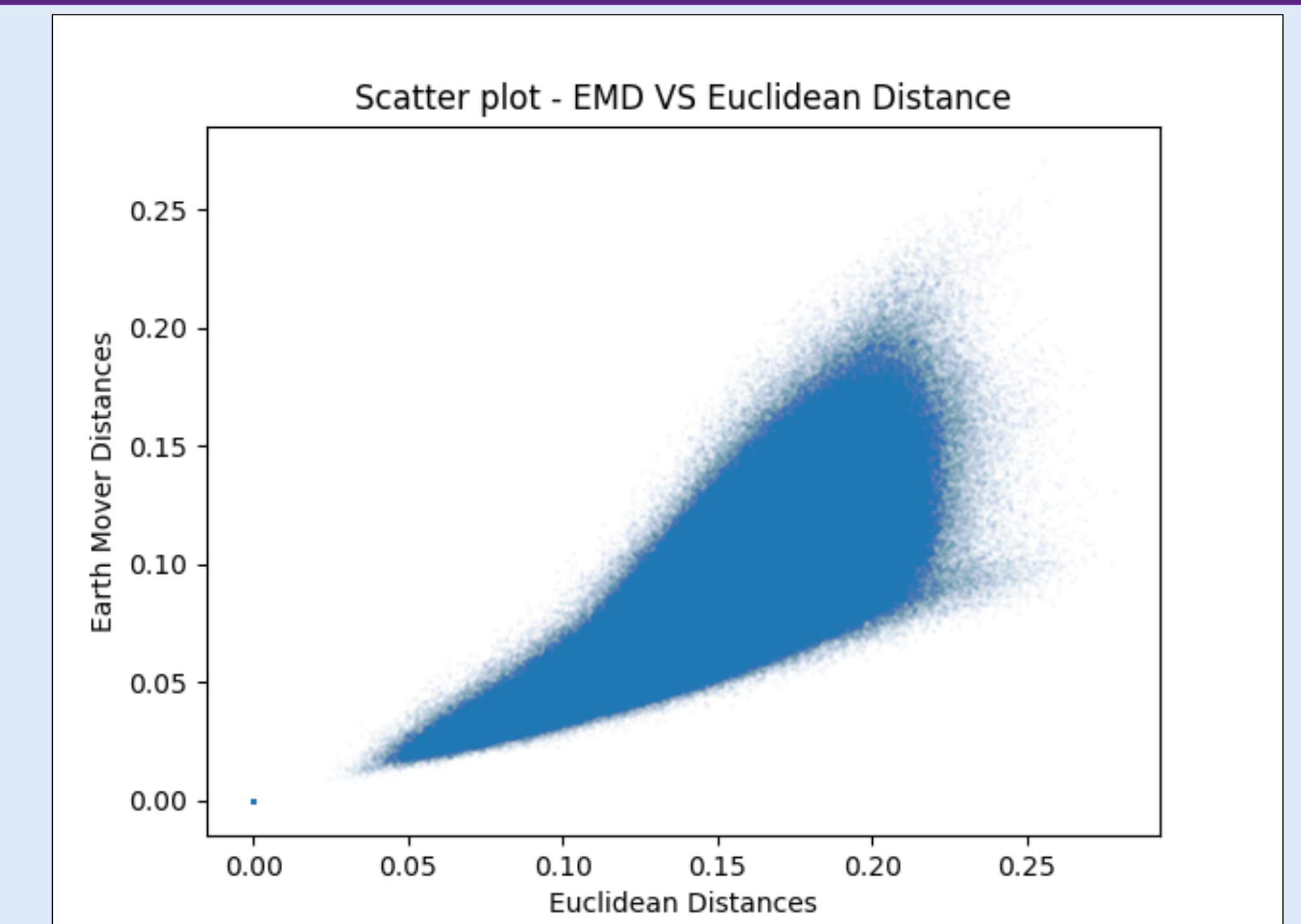
Euclidean Distance Matrix



Earth Mover Distance Matrix

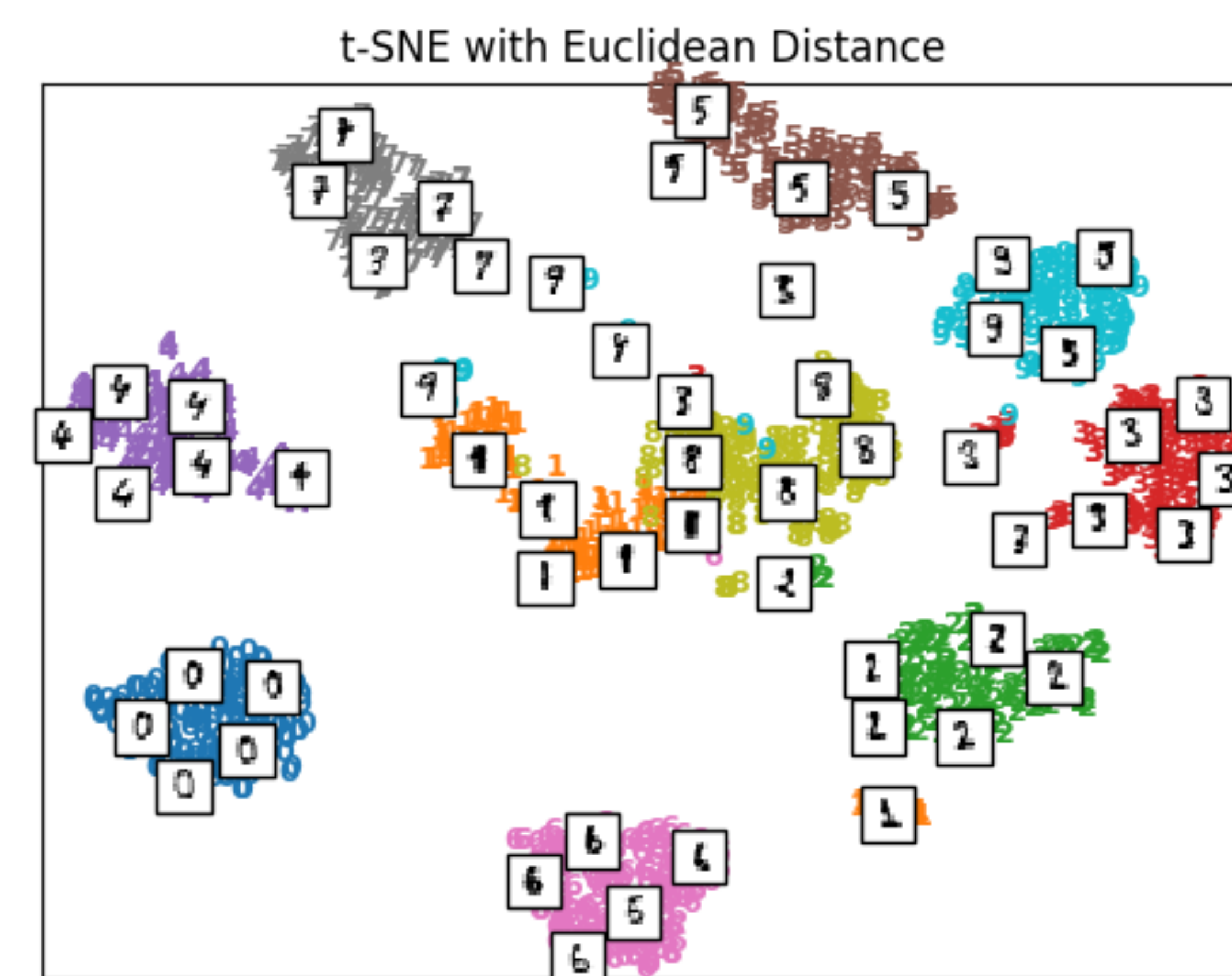
- The distances between classes in **Euclidean Distance Matrix** looks sharper

Scatter plot - Euclidean Distances VS EMD

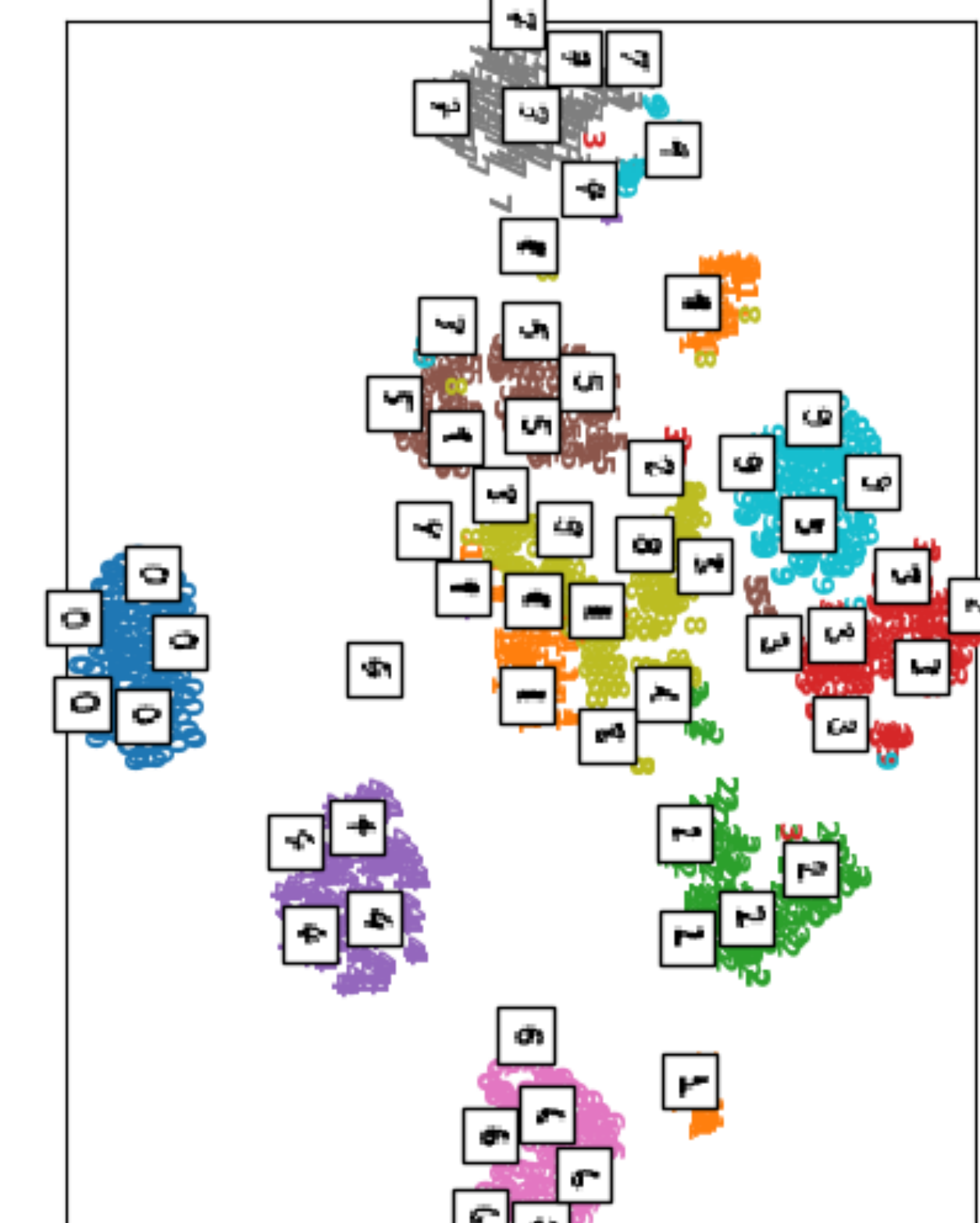


- Positive Correlation between these two distance measures

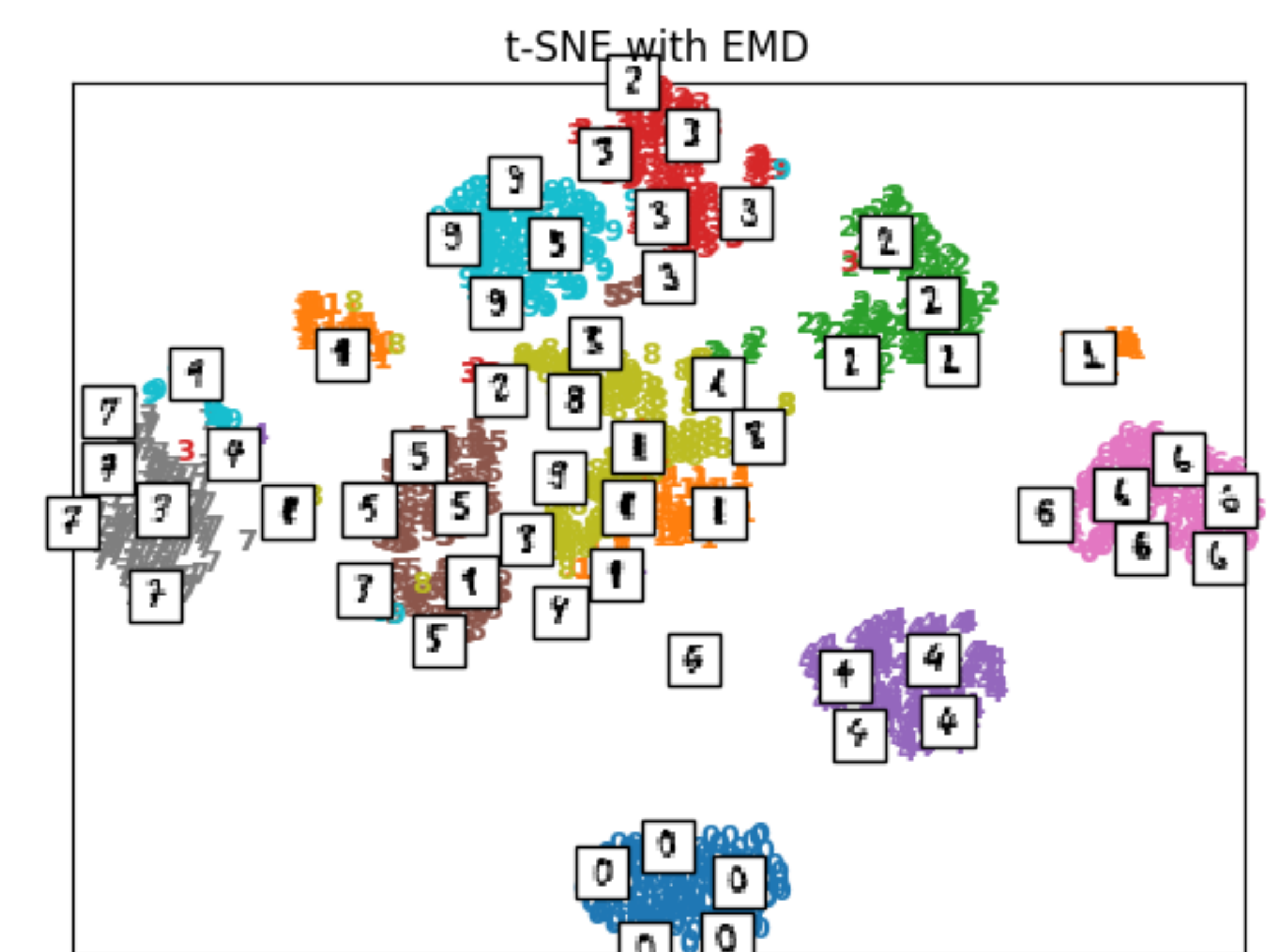
Result - Euclidean Distance t-SNE VS EMD t-SNE



Euclidean Distance t-SNE



Rotated EMD t-SNE



Earth Mover Distance t-SNE

- Roughly say, **EMD t-SNE** is the rotated version of **Euclidean Distance t-SNE**.
- The most significant difference:
 - in **Euclidean Distance t-SNE**, the graph between digits is **4 - 0 - 6**, but in **EMD t-SNE**, the graph between digits is **0 - 4 - 6**.
- In author's view, distance between **0** and **6** is smaller than distance between **0** and **4** and distance between **4** and **6**, as some of the handwritten **4** do not have same topology with **0** and **6**, but EMD t-SNE show the chain **0 - 4 - 6**, somehow show EMD between images do not compare topological structures of digits.