

arr = { 1, 6, 4, 12, 3, 8 }

output = { -1, 1, 1, 4, 1, 3 }

In prev class -

① Find smaller element to the left for every array element

$O(N)$  {

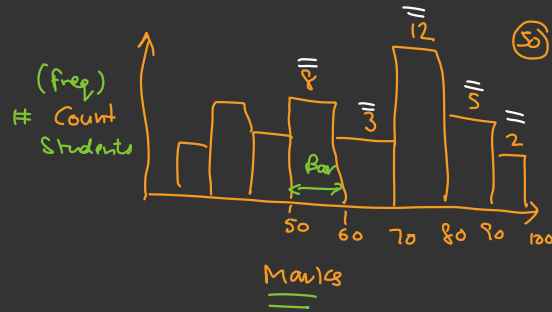
$O(N)$  also using stack.

- smaller on the right
- find big element on the left
- " " " " right (for  $i = n-1 \rightarrow 0$ )

$\Rightarrow$  Loop & comparison condition

Classical Problem : Largest Area under Histogram [Data Visualisation]

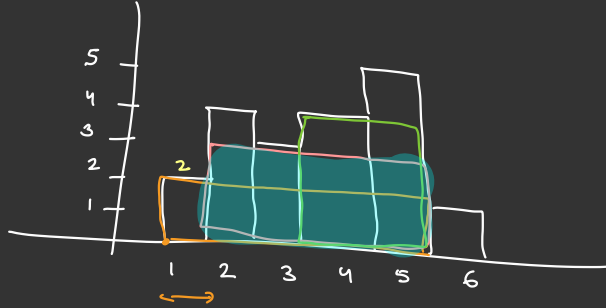
↓  
graph to represent data / bar graph



Challenge

Rect  
Largest Area under Histogram (no re-ordering)

⇒ Area under largest rect block that goes ~~goe~~ lies inside histogram bars.



Input

1D  
array

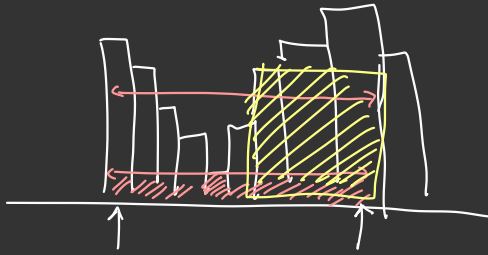
arr = [2, 4, 3, 4, 5, 1]  
B1 B2 B3 B4 B5 B6

Output

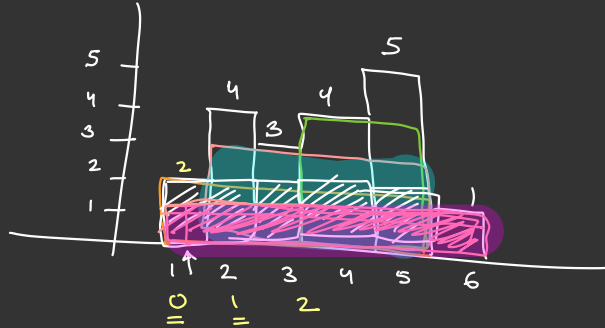
$$2 \times 5 = 10$$

$$3 \times 4 = 12 \leftarrow \text{find out}$$

$$4 \times 2 = 8$$



Brute Force Logic -



largest = 0

for (s → every bar) {  
  for (e → every bar > s) {

    w = e - s + 1  
    H = min (A[s], ..., A[e])  
    area = w \* H

$O(N^3)$

H, W Area

s=1 e=1 2 1 2

s=1 e=2 2 2 4

s=1 e=3 2 3 6

s=1 e=4 2 4 8

s=1 e=5 2 5 10

s=1 e=6 1 6 6

s=2 e=2

e=3

e=4

⋮

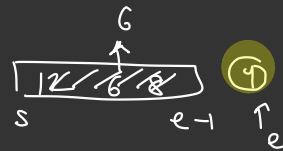
Loop  $O(N)$

→ Carry Forward  $O(1)$  at each position

largest = max (largest, area)

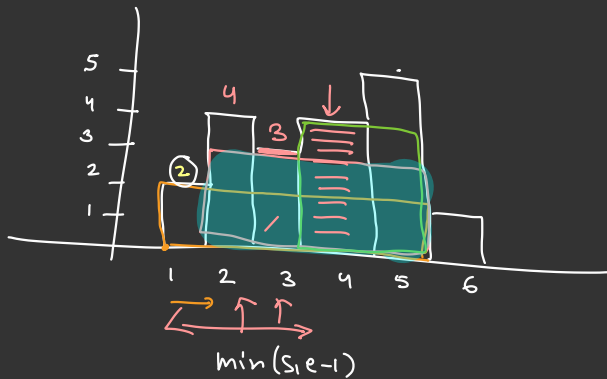
}

3  
print(largest)



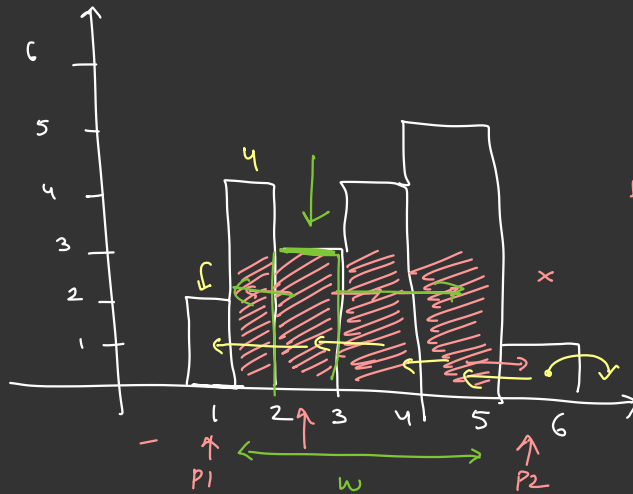
$$\min[s, e] = \min(a[e], \min[s, e-1])$$

$O(N^2)$   
using  
carry  
forward  
technique  
to  
find  
the  
minimum



$$\min[s, e] = \min(\underline{a[e]}, \underline{\min[s, e-1]})$$

		$\min$	$\times$	A
S=1	e=1	2	1	1
	e=2	(2, 2)	2	4
		2		
	e=3	(3, 2)	3	6
	e=4	(4, 2)	<del>4</del>	8



bar[3] is included in the ans.

precompute

~~break at every i~~  
~~O(N)~~

(P1) → idx of bar on the left  
which is less than  
b[i]

(P2) → idx of bar on  
right which is less  
than  
current bar

input ( 2, 4, 3, 4, 5, 1 )

[ P1 = [ (-1), 0, 0, 2, 3, -1 ] O(N)  
idx

P2 = [ , , , , , ] O(N)  
idx

w = P2 - P1 - 1 = 6 - 1 - 1

H = arr[i] = 3 = 3

Area = 3 × 4

$P1 =$   
 $P2 =$  } Compute using Stack Algo

$O(N + N + N)$

$= O(N)$

for (every bar  $b[i]$ ) {

Stack Algo

$P1 \rightarrow$  (get Left)  $P1[i] \Rightarrow O(1)$

$P2 \rightarrow$  (get Right)  $P2[i] \Rightarrow O(1)$

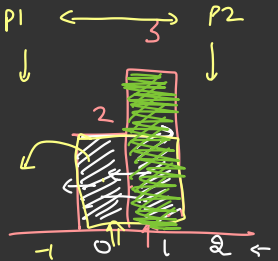
$W = (P2 - P1 - 1)$

$H = b[i]$

$A = W \times H$

Largest = max (A, Largest)

$O(N)$



$P1$   
 $P2$   
 $=$

(put N if

Right small doesn't exist]

Bar 0

$$\begin{aligned}
 W &= P2 - P1 - 1 \\
 &= 2 - (-1) - 1 \\
 &= 2 + 1 - 1 \\
 &= 2
 \end{aligned}$$

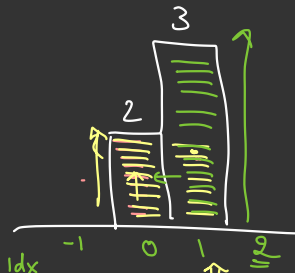
$H = 2$

$A = 4$

Bar 1

$$\begin{aligned}
 &(2 - 0 - 1) \times 3 \\
 &= 1 \times 3 \\
 &= 3
 \end{aligned}$$

everything upto  $N-1$  can be included  
 with current bar



$N=2$

idx -1 0 1 2  
 $P1[] = -1$  0  
 $P2[] =$  2 2  
 N N

$2-0-1$   
 $= (1) \times 3$   
 $= 3$

for ( begin ) {  
 w1 =

$2+1-1$   
 $= 2 \times 2$   
 $P2-P1+1$   
 $=$

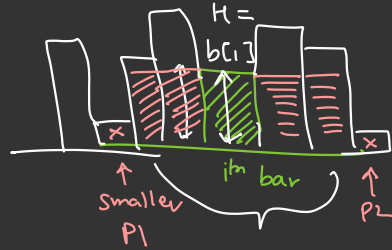
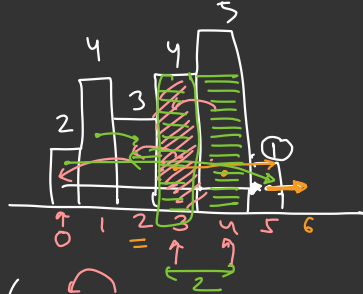
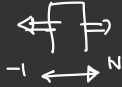
$A1=4$   
 $A2=3$

5



Q

1035



input ( 2, 4, 3, 4, 5, 1 )

Left P1 = ( -1, 0, 0, 2, 3, -1 )  
 Right P2 = ( 5, 2, 5, 5, 5, 6 ) ]  $O(N)$  stack Algo.

w = ( 5, 1, 4, 2, 1, 6 )

H = ( 2, 4, 3, 4, 5, 1 )

A = ( 10, 4, 12, 8, 5, 6 )  
 ↑ ↑

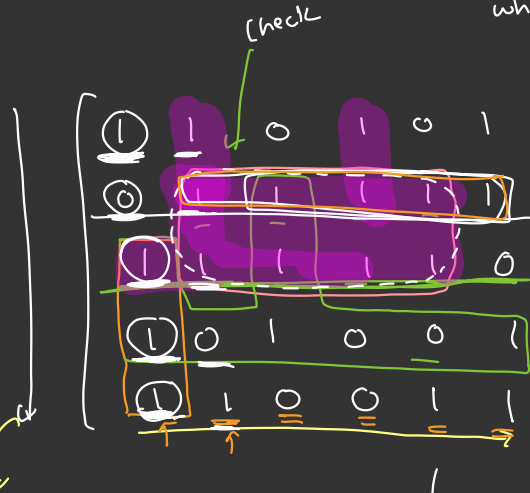
$$w = p2 - p1 - 1$$

$$H = b[i]$$

Q2 2D Matrix of 0's & 1's find out max Rect area,

which contains all 1's.

Prefix Sum col wise except if you get 0, reset sum 0.



①  $1, 0, 1, 0, 1 = 2$

②  $2, 1, 2, 1, 2 = 5$

③  $3, 2, 3, 2, 0 = 8$

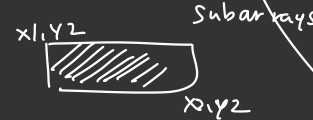
④  $0, 3, 0, 0, 1 = 3$

⑤  $3, 1, 0, 0, 1, 2 = 3$

max = 8



Brute force



$O(N^6)$  Generate all rect  $(x1, y1, x2, y2)$   $O(N^4)$   
 ↳ Checking  $O(N^2)$

⇒ Six nested loops

Matrix

① Prefix Matrix  $O(N^2)$

② For every Row of PM compute histogram  $O(N)$

③ Track the max for every Row

$$O(N^2)$$

## final Question.

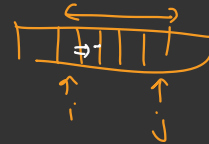
Given N Array elements, find **sum of max** of every subarray

[1, 4, 3]

$$\begin{aligned} \text{max} &= \left\{ \begin{array}{l} [1] = 1 \\ [4] = 4 \\ [3] = 3 \\ [1, 4] = 4 \\ [4, 3] = 4 \\ \cancel{[1, 3]} \\ [1, 4, 3] = 4 \end{array} \right\} \end{aligned}$$

$$16 + 3 + 1 \\ = 20$$

Brute force



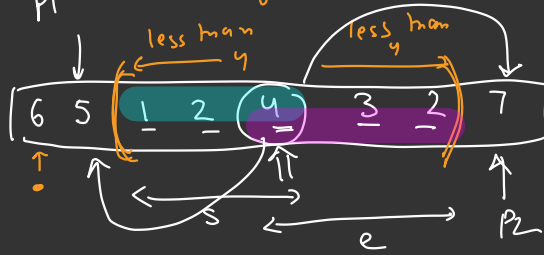
$\left\{ \begin{array}{l} \text{for}(i) \\ \text{for}(j) \\ \text{max} \Rightarrow \text{loop } O(N) \\ \text{or} \\ \text{convy forward } O(1) \end{array} \right\}$

$$O(N^3) \\ = \\ \text{or} \\ O(N^2)$$



Contribution of  
4 in final sum

any subarray start  $1 \leq i^{\text{th}}$  index



4 will be max in  
what subarrays

Range Range  
[s, e]  
||

$$3 \times 3 \text{ Contribution} \\ = \boxed{9} \times 4 \\ = \boxed{36}$$

$$\left. \begin{matrix} (4) \\ (4, 3) \\ (4, 3, 2) \end{matrix} \right\} = 3$$

$$\left. \begin{matrix} (2, 4) \\ (2, 4, 3) \\ (2, 4, 3, 2) \end{matrix} \right\} = 3$$

$$\left. \begin{matrix} [1, 2, 4] = 4 \\ [1, 2, 4, 3] = 4 \\ [1, 2, 4, 3, 2] = 4 \end{matrix} \right\} = 3 \\ = \boxed{9}$$

