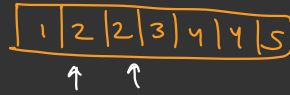




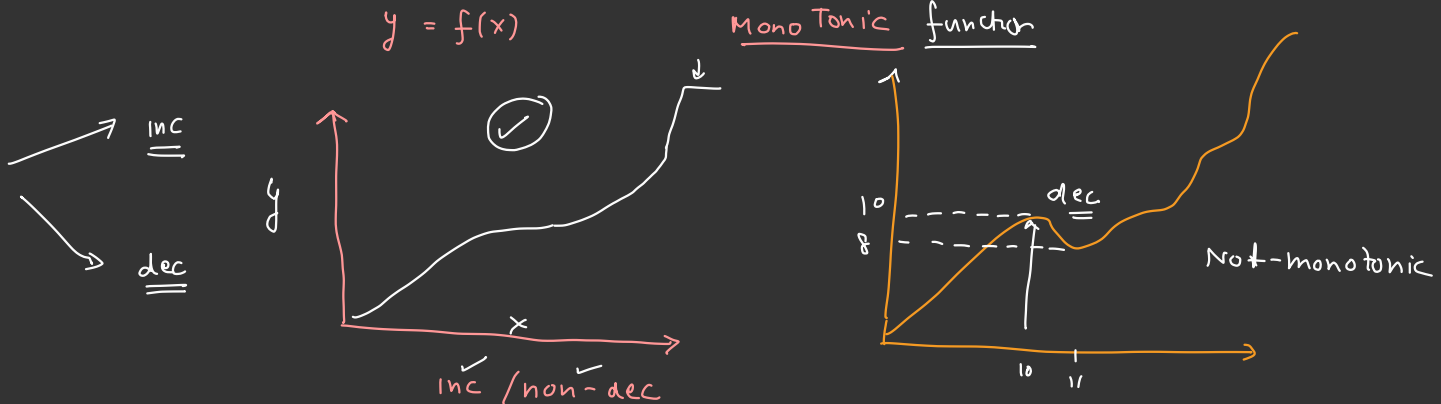
# Binary Search

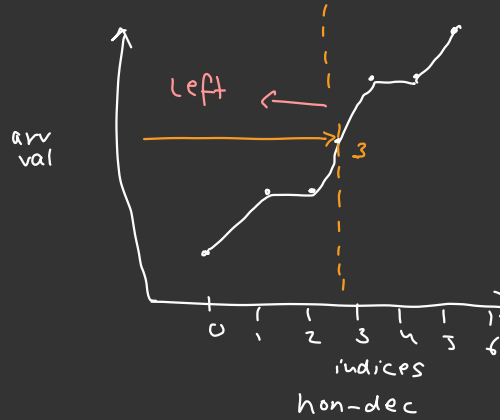
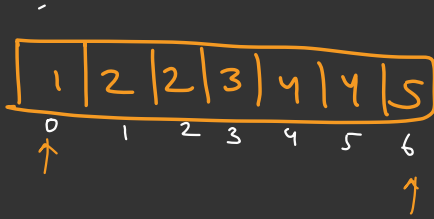
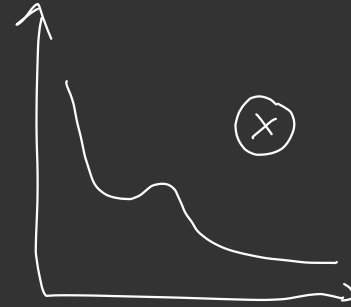
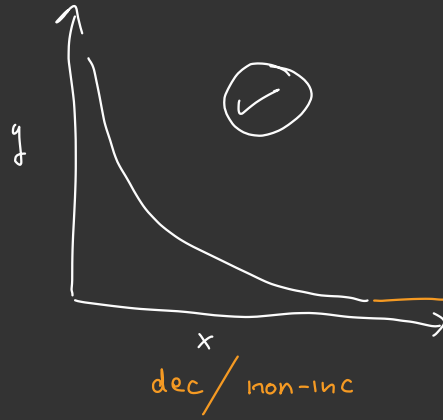
① ✓ Binary Search on Sorted Array



→ Search  
→ first occ  
→ last occ

Today → Monotonic Search Spaces





Target = 2

Square Root

(zomato)

$$N=100 \quad \sqrt{100} = 10$$

$$N=50 \quad \sqrt{50} = 7$$

$$N=150 \quad \sqrt{150} = 12$$

Math.sqrt(100)

Brute force :

(Integer Part)

$$N = (50)$$

$$i = 0$$

while  $i^2 \leq N$  {

$i++$

}

ans = i-1

print(ans)



Time →

$$O(\sqrt{N})$$

Stop

$$i^2 > N$$

$$17 = \sqrt{N}$$

$$N = 10^4$$

$$\sqrt{N} = (100)$$

$$\Rightarrow O(\log N) < O(\sqrt{N}) < O(N)$$

$$\left\{ \begin{array}{l} 0^2 \leq 50 \\ 1^2 \leq 50 \\ 2^2 \leq 50 \\ 3^2 \leq 50 \\ 4^2 \leq 50 \\ 5^2 \leq 50 \\ 6^2 \leq 50 \\ 7^2 \leq 50 \\ i=8 \rightarrow 8^2 \not\leq 50 \end{array} \right.$$

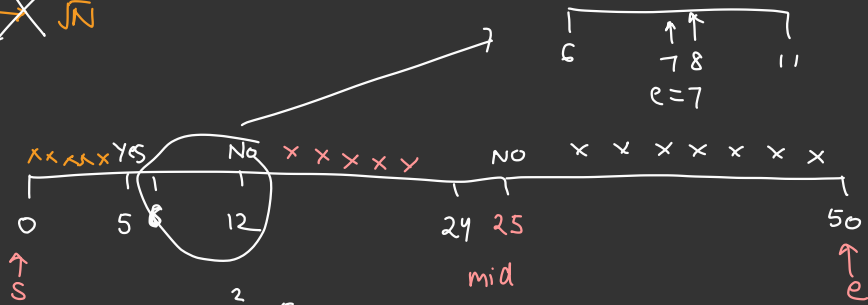
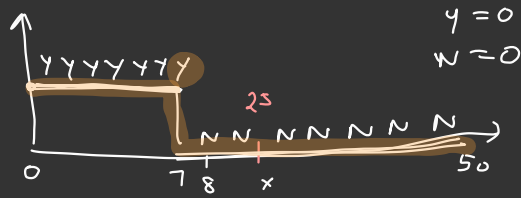
Linear Search

# Binary Search for Square Root

$N = 50$  Search Space (0 — N)

~~$T \propto \sqrt{N}$~~

$x^2$   
 $\text{mid}^2 \leq N$   
 $f(x)$   
Yes No



$2$   
 $\downarrow$   
 $N/2$   
 $\downarrow$   
 $N/4$   
 $\vdots$   
 $1$

$O(\log N)$

$S = 0$	$e = 50$	$\text{mid} = 25$	NO	$25^2 \leq 50$
$S = 0$	$e = 24$	$m = 12$	NO	$e = m - 1$
$S = 0$	$e = 11$	$m = 5$	<u>Yes</u>	$S = m + 1$
$S = 6$	$e = 11$	$m = 8$	NO	$e = m - 1$
$S = 6$	$e = 7$	$m = 6$	<u>Yes</u>	$S = m + 1$
$S = 7$	$e = 7$	$m = 7$	<u>Yes</u>	$S = m + 1$
$S = 8$	$e = 7$		<u>Stop</u>	

$\text{mid}^2 \leq N$

ans

5

6

7

last time you got a Yes will

Return ans;

be the final ans

$$\text{no}^2 \leq N$$

↑

maximise

the no by go Right

$$s = \text{mid} + 1$$

$$N = 50$$



Code

```
int binarySearch (int n) {
```

```
    s=0
```

```
    e=N
```

```
    while (s <= e) {
```

```
        mid = (s+e)/2,
```

```
        if (mid2 > N) {
```

```
            e = mid - 1;
```

```
        }
```

```
        else {
```

```
            ans = mid;
```

```
            s = mid + 1
```

```
        }
```

```
    }
```

```
    return ans,
```

```
}
```

$O(\log N)$

$O(1)$  space

```
    else if (mid2 == ans) {
```

```
        return mid,
```

```
    }
```

(optional)

Decimal Part

$\boxed{N}$   
3

$\boxed{P}$   
3

$\boxed{P \leq 8}$   
↑

$\sqrt{10} = 3.162$

Binary Search  
 $O(\log N)$

Linear Search

10 bits  $O(1)$

$\xrightarrow{P \times 10}$

0-9   0-9   0-9

3. 1 6 2

$O(\log N + P)$  ✓

for loop

while

$3.0^2 \leq 10$

$3.1^2 \leq 10$

$3.2^2 \leq 10$

3.0  
↓ 0.1  
3.1  
↓ 0.1  
3.2

3.10  
3.11  
3.12  
3.13  
3.14  
3.15

3.16  
3.17

$3.16^2 \leq 10$

$3.17^2 \leq 10$

ans = ans - inc

ans =

3.160  
3.161  
3.162  
3.163

$3.16^2 \leq 10$

$3.17^2 \leq 10$

Yes

No

SSS

9.56

Build for Decimal part

float Square Root ( $N$ ,  $p$ )  $\rightarrow$  No of places after decimal

$N=10$

ans = 3.0000

$\log N$  float ans = binSearchForSqRoot( $N$ ),

float inc = 0.1  
 for( $t=1$ ;  $t \leq p$ ,  $t++$ ) {  
     while( $ans \times ans \leq N$ ) {  
         ans = ans + inc  
     }  
     ans = ans - inc,  
     inc = inc / 10;  
 }  
 return ans,

10 times  
 3  
 0.1  
 0.01  
 0.001

places  
 one place  
 3.0<sup>2</sup> < 10  
 3.1<sup>2</sup> < 10  
 → 3.2<sup>2</sup> < 10 ←  
 inc = 0.01  
 ans = 3.1000  
 3.10<sup>2</sup> < 10  
 3.11<sup>2</sup> < 10  
 3.12<sup>2</sup> < 10

ans = 3.1  
 = 3.2  
 = 3.3

3.1619997  
 String format("%3f", ans)



→ 3.1620000 )  
↓  
3162

## Aggressive Cows (Code)

stalls = [1, 2, 8, 4, 9]

C = 3

2 cows

[1, 2, 4, 8, 9]

Sort(stalls)

s = 0

e = stalls[n-1] - stalls[0], ans = 0

while (s <= e) {

mid = (s + e) / 2,

if (CanPlace Cows with mid as Min Sep) {

ans = mid

s = mid + 1



Binary Search

$\log(e - s)$

```

    }
    else {
        e = mid - 1
    }
}

```

print(ans);

// ③  
mid cows stalls

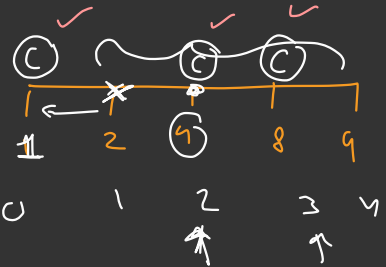
boolean canPlace ( stalls[], sep, C, N ) {

lastCow = stalls[0]

cnt = 1

for ( i = 1; i <= N - 1; i++ ) {

if ( stalls[i] - lastCow > sep ) {  
    cnt ++;



N stalls  
O(N)

```

    {
        last cow = stalls[i],
        if (cnt == c) { return true; }
    }
    return false;
}

```

↑ Place 3 cows  
with min sep = sep

2 - 1 > 3

4 - 1 > 3

8 - 4 > 3

Sorted  
 $2N+1$

# Search

1 1   2 2   3 3   4 4   5 5   6 6   7 7   8 8  
 ↑         ↑         ↑         ↑  
 mid

XOR  $\rightarrow O(n)$

How

## Next Class

Find unique element in an array containing  $2N + 1$  elements, where every element repeats twice. Array is sorted