- Solutions / Extra class for Test Problems (Prateek)

- Intermediate Batch → wednesday (Problem solving Introduction)
  Monday (OFF)

Maths

- ✓ Prime numbers
- ✓ Divisors (print) i, n/i ⇒ Divisors
- GCD (optimised approach) Euclid's Algo ✓
- AP
- GP
- P & C
- Fibonacci Series.
- Log Function
- Math class (inbuit methods)

Prime Number

12 → 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

More than 1 divisor → Not Prime

11 → [2, 3, 4, 5, ---------- 10]

No Divisor → Prime

Worst case → Prime No ⇒ N-1 times

Time ∝ Loop Runs ∝ N times

Optimisations

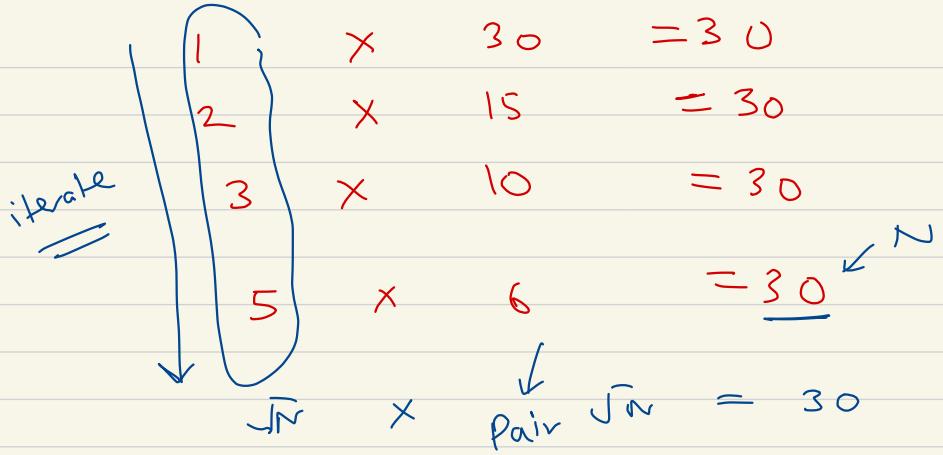
Occ in Pairs

30 → 1, 2, 3, 5, 6, 10, 15, 30

Small | Big

√N

5.xx



if there is no small div ($\leq \sqrt{N}$) → there is no big div

if there is a big div → small div is must

must be less than Root N

a × b = N

$\sqrt{N}$ × $\sqrt{N}$ = N

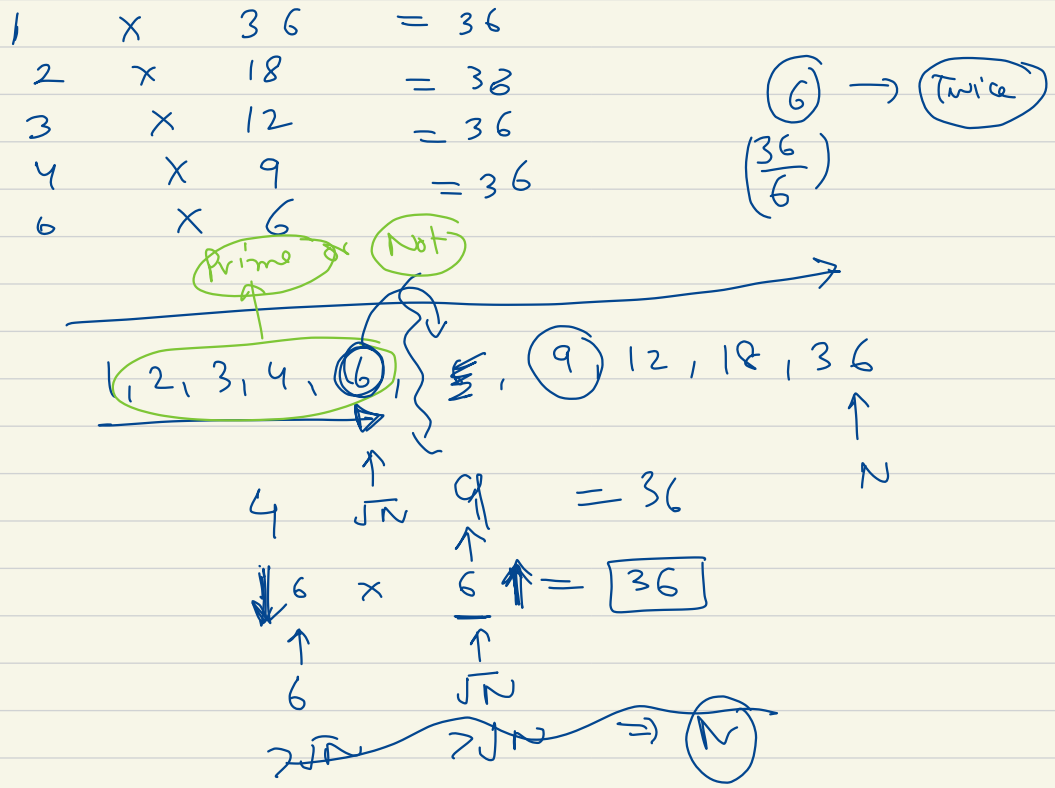
```
for ( i=2; i <= N-1 ; i++ ){
```

i <= √N

i*i <= N

$i^2 <= N$ / $i <= \sqrt{N}$

36


1 x 36 = 36
2 x 18 = 36
3 x 12 = 36
4 x 9 = 36
6 x 6
6 → Twice
(36/6)
Prime or
Not
1, 2, 3, 4, 6, 9, 12, 18, 36
N
4 √N 9 = 36
6 x 6 = 36
6
√N
√N
N

$N = 1000$

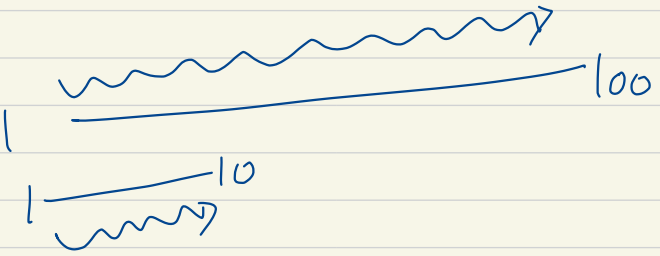
Algo $T \propto N \longrightarrow 1,000,000$

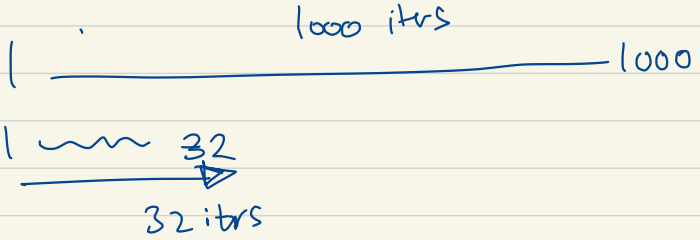
$T \propto \text{iter} \propto \sqrt{N} \longrightarrow 1000$ iterations

$\sqrt{N}$ → Loop

Example: T → 100 s; 1 Million T; $\sqrt{T}$ → 10 s

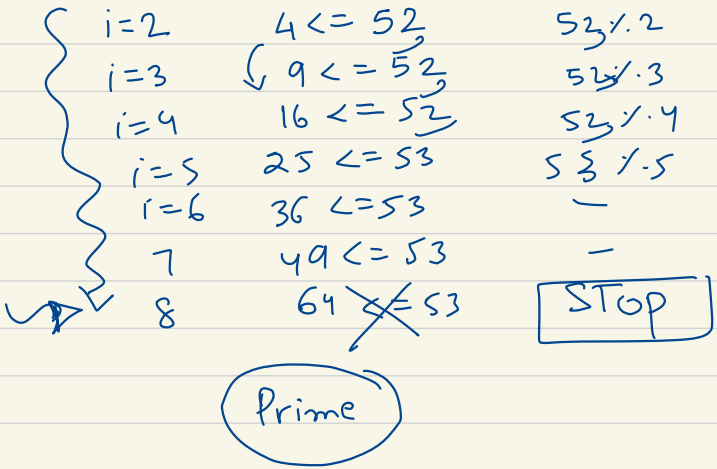
$N = 100$

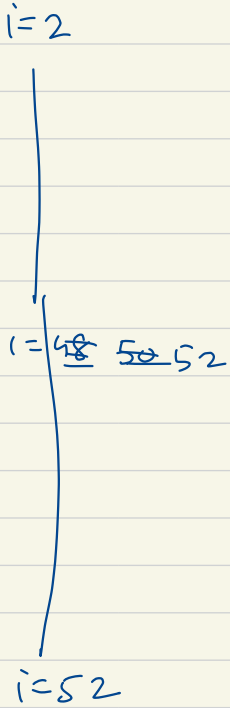




very fast

32
×32
64
96 X
1024


i=2
i=3
i=4
i=5
i=6
7
8
4 <= 52
9 <= 52
16 <= 52
25 <= 53
36 <= 53
49 <= 53
64 <= 53
52 % 2
52 % 3
52 % 4
52 % 5
—
—
STop
Prime


i=2
i=52

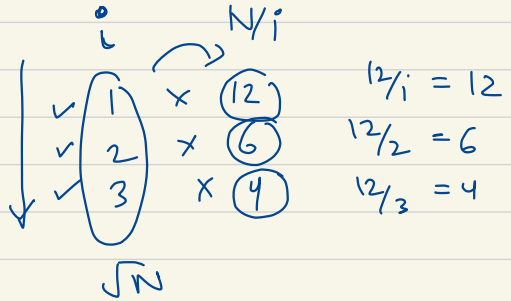
12 → 1, 2, 3, 4, ~~5~~, 6, 7, 8, 9, 10, 11, 12

N iterations

If i divides N

```
sum = 0
for(int = 1 ;  i<= N ; i++) {
    if( N%.i == 0){
        sum = sum + i
    }
}
```

3



0 + 1 + 12 + 2 + 6 + 3 + 4

3 → Save a lot of time

Optimise

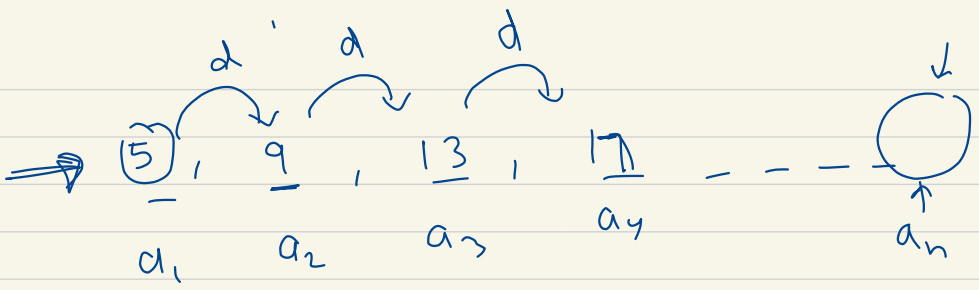
```
for( i=1 ; i<= √N; i++){
    if( N%.i ) {
        sum = sum + i + N/i ;
    }
}
```

3

Perfect square $i == \sqrt{N}$ → Add it once

36

Intuitive Proof



$$a_2 = a + 1d = 5 + 4 = 9$$

$$a_3 = a + 2d = 5 + 2 \cdot 4 = 13$$

$$a_4 = a + 3d = 5 + 3 \cdot 4 = 17$$

$$\vdots$$

$$a_n = a + (n-1)d$$

$a = 5, 9, 13, 17 \cdots$

$5 + 9 + 13 + 17 + 21 + 25$

90

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_6 = \frac{6}{2}\left(2\times 5 + (5)4\right)$$

$$= 3(10 + 20) = 90 \checkmark$$

First Term

$20, \_, \_, \_, \_, 80$

a_1 $\quad$ a_4

$n = 4$

One way

$$a_4 = a_1 + (4-1)d$$

$$\Rightarrow 80 = 20 + 3d$$

$$\Rightarrow 60 = 3d$$

$$\Rightarrow \boxed{d = 20}$$

$$S_4 = \frac{4}{2}\left(2a + (n-1)d\right)$$

$$= \frac{4}{2}(2 \times 20 + 3 \times 20)$$

$$= 2(40 + 60) = 200$$

Trick

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[a + a + (n-1)d\right]$$

$$= \frac{n}{2}\left[a + a_n\right]$$

20 ------ 80

$n = 4$

$$S_4 = \frac{4}{2}(20 + 80)$$

$$= 200$$

Time Complexity →

String Builder ← String Concatenation

"A" — 1
"AB" — 2
"ABC" — 3
⋮
n

$$S_n = \frac{n}{2}(1+n)$$

$$= \frac{n(n+1)}{2}$$

$$\propto n^2$$

GP

Geometric Progression

$a = 5$, $r = 2$

⇒ 5, 10, 20, 40, 80, 160 ...

(×2, ×2, ×2)

a, ar, ar^2, ar^3, ar^4, ar^5

$a_n = ar^{n-1}$

$\frac{10}{5} = \frac{20}{10} = \frac{40}{20}$

$= 2 \quad = 2 \quad = 2$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

$$S_5 = \frac{5(1-2^5)}{(1-2)}$$

$$= 5\left(\frac{-31}{-1}\right)$$

$$= 155$$

$r = 5$

if $r = 1$ ——— an

$$a\left(\frac{1-1^n}{1-1}\right)$$

5, 5, 5, 5, 5

$$S_5 = a \times n = 5 \times 5 = 25$$

Infinite GP

$a = 2$, $r = 1/2$

$r < 1$

$2, 1, 1/2, 1/4, 1/8, \ldots\ldots\ldots$

$$S_\infty = \frac{a}{1-r}$$

$= \frac{2}{1-1/2} = 4$ Converge to 4

$r > 1$

$a = 1$, $r = 5$, $n = \infty$

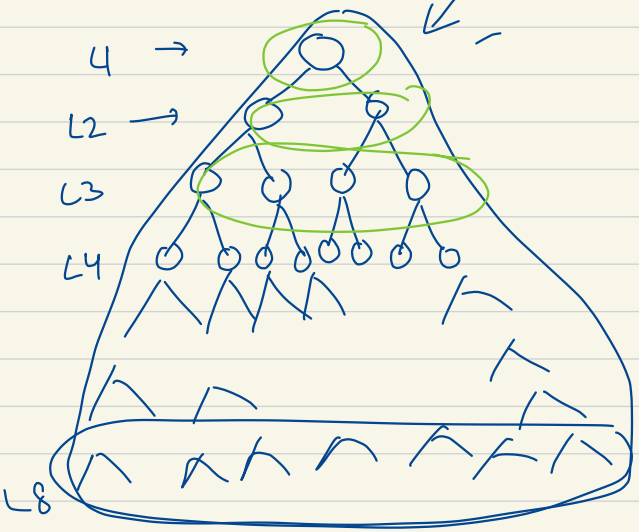
$1, 5, 25, 125, \ldots\ldots\ldots$

$S_\infty \Rightarrow$ can't compute (divergent)

0.000000 ∞

Break

10:35



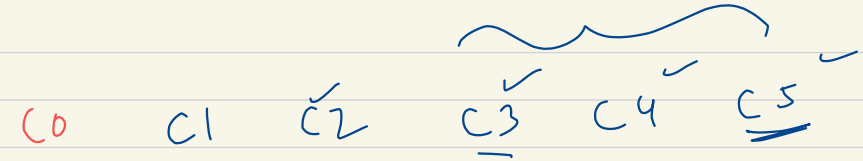
$1 + 2 + 4 + \cdots + 2^{?} = 128$

$n = 8$

$$S_8 = \frac{a(1-r^n)}{(1-r)}$$

$$= \frac{1(1-2^8)}{-1}$$

$$= 255$$



R = 2 cats out 5 = N

(0, 1) (0, 2) (0, 3) (0, 4) (0, 5)

C1, C2 C1, C3 C1, C4 C1, C5

C2, C3 C2, C4 C2, C5

C3, C4 C3, C5

C4, C5

5 ways + 4 ways + 3 ways + 2 ways + 1 ways = 15 ways

$$\binom{N}{R} = {}_5C_2$$

$${}^NC_R = \frac{N!}{(N-R)!\,R!}$$

$${}^5C_2 = \frac{5!}{3!\,2!} = \frac{3! \times 4 \times 5}{3! \times 2!} = 10 \text{ ways}$$

Shortcut

$${}^5C_2 = \frac{5 \times 4}{1 \times 2} = \frac{20}{2} = 10 \text{ ways}$$

2 Terms

easy

$${}^6C_2 = \frac{6 \times 5}{1 \times 2} = 15 \text{ ways}$$

$${}^{10}C_8 = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8} = 45$$

$${}^{10}C_8 \rightarrow {}^{10}C_2 = \frac{10 \times 9}{1 \times 2} = 45$$

fast

$$^{N}C_{R} = {}^{N}C_{N-R}$$

$$^{10}C_{8} = {}^{10}C_{10-8} = {}^{10}C_{2}$$

$$\frac{10!}{2!8!} = \frac{9 \times 10}{2} 5 = \boxed{45}$$

$$\frac{N!}{(N-R)!R!}$$

$$\frac{N!}{(N-(N-R))!\,(N-R)!}$$

Same

$$= \frac{N!}{R!(N-R)!}$$

11 players

↓

Choose 2 players

55

↓

A, B

$^{11}C_2 = \frac{11 \times 10}{1 \times 2} = 55$

&

Captain, vice captain

⇒

A , B

B , A

$55 \times 2 = 110$

A B C D E

Choose C VC

BA

C VC

A, B

A, C

A, D

A, E

B, C

B, D

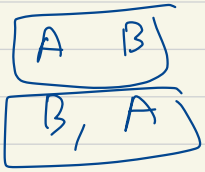
B, E

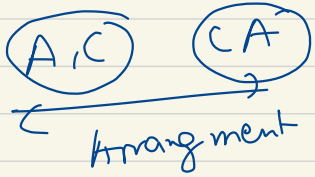
C, D

C, E

D, E

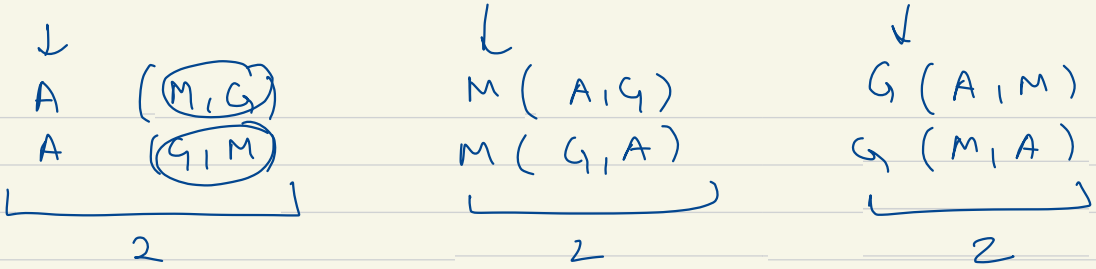
4 + 3 + 2 + 1 = 10 ways



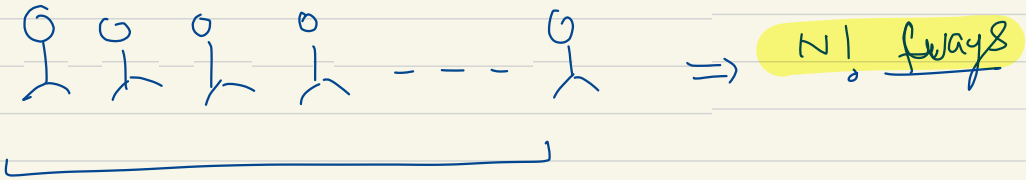


Arrangement

Fruits → A, M, G ⇒ 3! ways.



$3 \times 2! = 6$ ways



Cap V.C

Choose 2 out 5 × arrange (2) them

$$= {}^{5}C_{2} \times 2!$$

$$= \frac{5 \times 4}{1 \times 2} \times 2$$

$$= 20 \text{ ways}$$

${}^{N}P_{R} \Rightarrow$ denotes no of ways of picking R objects out of N object and arranging them

$$= {}^{N}C_{R} * R!$$ ✓

$$= \frac{N!}{(N-R)!} \times \frac{R!}{R!}$$

Permutation

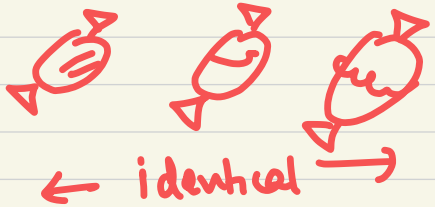
$${}^{N}P_{R} = \frac{N!}{(N-R)!}$$

2 out 5 RR arrange

$${}^{5}P_{2} = \frac{5!}{3!} = 4 \times 5 = 20$$

5 cats

3 candies



← identical →

uniq → ${}^5P_3 = \frac{5!}{(5-3)!} = 3 \times 4 \times 5 = 60$

${}^5C_3$

Logarithm (log spl fn)

$\log_{10} 100 = 2$

$10^2 = 100$

$\log_{10} 1000 = 3$

$\log_5 625 = 4$

$5 \times 5 \times 5 \times 5$

$\log_2 100 = 6.xx$

$2^7 \rightarrow 128$

$2^6 \rightarrow 64$

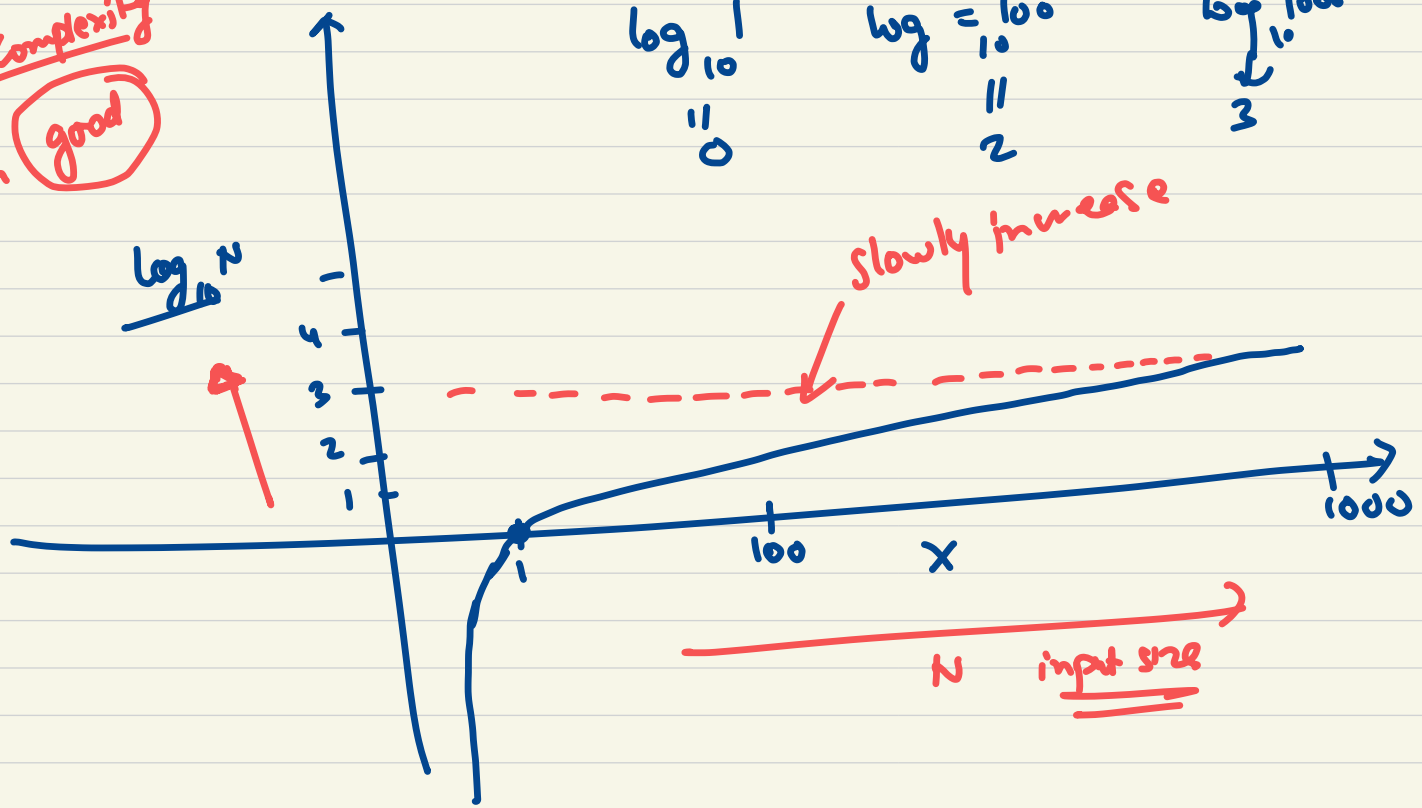
$\log_{20} 1 = 0$ $\quad$ $20^0 = 1$

Time Complexity

↓

Algorithm (good)

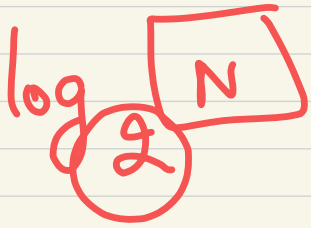
$\log_{10} 1 = 0$ $\quad$ $\log_{10} 100 = 2$ $\quad$ $\log_{10} 1000 = 3$



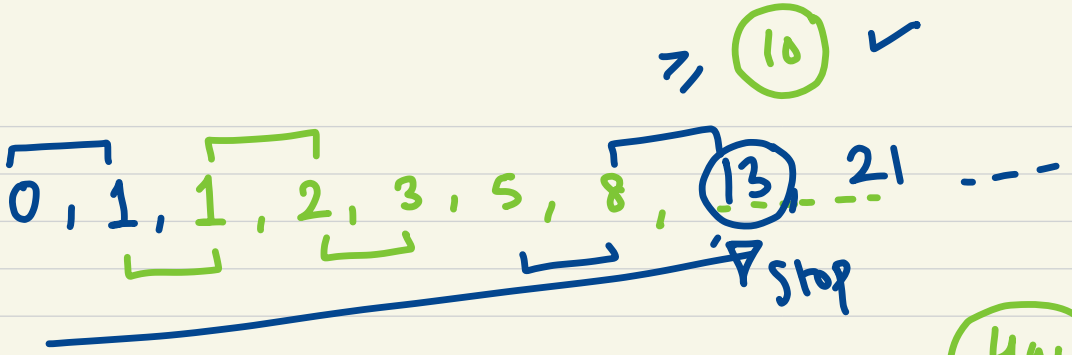
$\log_e N$

$e \simeq 2.78$

NB



Common Base in CS



HW

loop

$a_n = a_{n-1} + a_{n-2}$

a_{n-1} ↑ save

a_{n-2} ↑ save

→ (1) Generate fib series upto N terms

→ (2) N is part of series or not ?

2, 5, 7, 8, 11, 16, 20, 12, 3

$L = \infty$

minOdd → minOdd (main 5 / 3)

maxEven [$L = 0$ 2 8, 16 20]