Warm welcome

Recap

- % operator
- % for a negative no

Rules → MODULO AIRTHMETIC

① $(a + b)\%\, m = ((a\%\, m + b\%\, m))\%\, m$

$(a+b)$ → $[0 - (m-1)]$ ← max val

$(a\%\, m)$ → $[0 \text{ to } m-1]$, $(b\%\, m)$ → $[0 \text{ to } m-1]$

$([0 \text{ to } m-1] \quad [0 \text{ to } m-1])\%\, m$

$[0 - (m-1)]$

② $(a * b)\%\, m = (a\%\, m * b\%\, m)\%\, m$

$(a*b)$ → $[0 \text{ to } m-1]$

$a\%\, m$ → $[0 \text{ to } m-1]$, $b\%\, m$ → $[0 - m-1)$

$([0 \text{ to } m-1] \quad [0 - m-1))\%\, m$

$(m-1)^2$ ∪ $(m-1)^2\%\, m$

$[0 \text{ — to — } m-1]$

Problem Given int a, n, p Calculate $a^n \% p$

Example $a = 3$, $n = 4$, $p = 7$

$(3^4) \% 7$

$= 81 \% 7$

$= 4$ ✓

($a^n \rightarrow$ Bitmasking $\rightarrow$ $\log n$)

```
power ( int a, int n, int p ){
    ans = 1
    for(i=1 , i<=n , i++){
        ans = ans * a
    }
    return ans % p;
}
```

Fail for large inputs because of overflow

Example: $a = 10$, $n = 14$, $p = 25$

$ans = 1$ $a = 3$ $n = 5$

ans	
1	$1 * 5$
2	5×5
3	5^3
4	5^4
5	5^5

10^{14} → int ✗ overflow

→ Take ans as "long"

```
power ( int a, int n, int p ){
    long ans = 1
    for(i=1 , i<=N , i++){
        ans = ans * a
    }
    return (int) (ans % p)
}
```

long → 10^{18} max Range

p is int
↓
0 — p-1
↓
int

Ex- a=10, n=20, p=25

10^{20} % p

↓ long

↓ can't store a value $\geq 10^{18}$

What to do ?

- Build a very large bucket for storage (Big Integer → Java (Later)) — Storage → a^n
- involves the use of %, final ans would be still an int, Exploit the properties of Modulo arithmetic

$a^n \% p$ → (0 to p-1)

Largest int → 10^9

```
power ( int a, int n, int p ){
    long ans = 1
    for(i=1 , i<=N , i++){
        ans = ((ans % p) * (a % p)) % p
    }
    return ans % p;
}
```

$p \le 10^9$

$p^2 \le 10^{18}$

Overflow can be avoided

$10^{20} = 1\times10,\ 10\times10,\ 10^2\times10,\ \ldots,\ 10^{18}\times10,\ 10^{19}\times10 = 10^{20}$

$\le p-1 \le 10^9$, $\le p-1 \le 10^9$

$< p^2 \le 10^{18}$

$((a \times a \times a \ldots\ (n \text{ times}) \ldots) \times a) \% p$

ans

$= ((a \% p) \times (a \% p) \ldots) \% p$

Modulo (Rule)

$(a * b) \% p = ((a \% p) * (b \% p)) \% p$

$a = 5, b = 7$

$(5 \times 7) \% 3 = 2$

$(5 \% 3 * 7 \% 3) \% 3$

$(2 \times 1) \% 3 = 2$

$p = 3$

$(ans * a) \% p = ((ans \% p) * (a \% p)) \% p$

$a, n = 5, p$

	ans = (ans * a) % p	Equivalent ans
1	$(1 \% p * a \% p) \% p$	$a^1 \% p$ → $(0 \to p-1)$
2	$(a \% p * a \% p) \% p$	$a^2 \% p$ $(0 \to p-1)$
3	$(a^2 \% p) \% p * a \% p$	$a^3 \% p$
4	$a^3 \% p * a$ --	$a^4 \% p$
5	$a^4 \% p * a$	$a^5 \% p$

$((res \% p) * (\_\_ \% p)) p$

a^n

$a = 10$

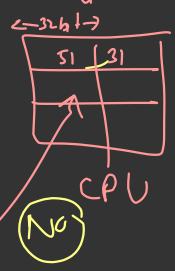
$n = 20$

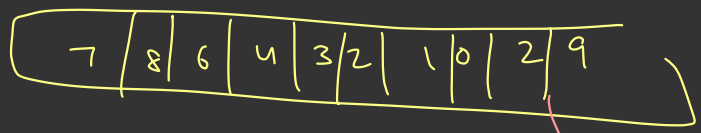
⇒ Big Integer to get the result $-\infty$ to ∞

internally implements Something like an array

52 + 31

int

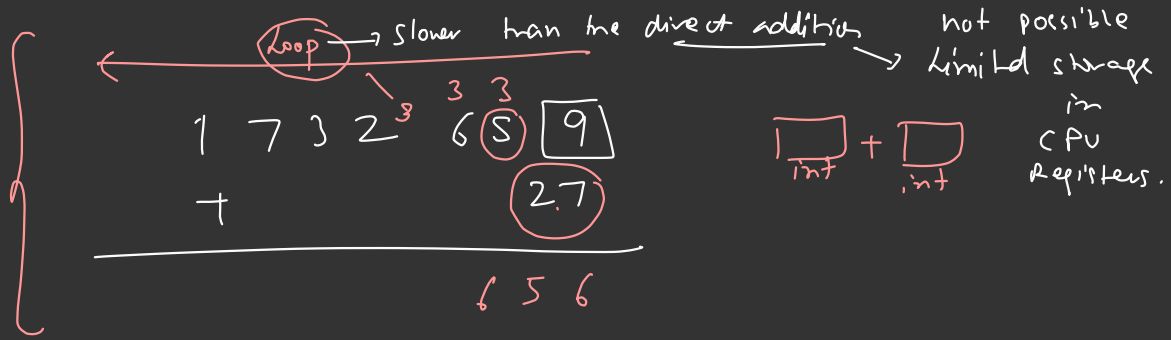




- add
- multiply

↓

Specially Code to work with big in an iterative manner


Loop
slower than the direct addition
not possible
Limited storage
in
CPU
Registers.
1 7 3 2 6 5 9
3 3 3
+
2.7
int
+
int


Passing by
Path
CPU
Heap

Reference Material

Oracle Docs → Big Integer ⇉ list of all methods / Examples.

$(a+b+c+d+\dots)\%m = (a\%m + b\%m + \dots)\%m$

(a) Divisibly Problems

(1) Div by 3 → sum of digit is div by 3

N = 789

$(789)\%3$ ✓ $= 7+8+9 = 24\%3 = 0$

Proof

$(7\times10^2 + 8\times10 + 9\times1)\%3$ (789)

$\Rightarrow ((7\times10^2)\%3 + (8\times10)\%3 + (9\times1)\%3)\%3$

$\Rightarrow (7\%3 \times 1 + 8\%3 \times 1 + 9\%3)\%3$

$\Rightarrow ((7+8+9)\%3)\%3$

$= (7+8+9)\%3$

$= 0$ No is div by 3

$1\%3 = 1$

$10\%3 = 1$

$100\%3 = 1$

$1000\%3 = 1$

$10^x\%3 = 1$

$(a\%p)\%p = a\%p$

263
3 | 789
789
0

$1\%2$ rem
$= 2\times0 + 1$

$3\times0 + 1$ rem
$3\times3 + 1$
$3\times33 + 1$
$3\times333 + 1$

$N =$ 1 2 3 4 5 (10^4 10^3 6^2 10^1 10^0)

$= (1+2+3+4+5) \% m$

$= (15 \% 3) = 0$ ✓

$3 \mid 12345$ → 4115, 12395, 0

Proof

$(1 \times 10^{4} \% 3 + 2 \times 10^{3} \% 3 + 3 \times 10^{2} \% 3 + 4 \times 10^{1} \% 3 + 5 \times 10^{0} \% 3) \% 3$

$(1 + 2 + 3 + 4 + 5) \% 3$

- Divisibility by 9 → Sum of digits by 9

$345 \% 9$ → not div

$648 \% 9$ → Yes

$10 \% 9 = 1$

$100 \% 9 = 1$

$1000 \% 9 = 1$

$6 \times 10^{2} \% 9 + 4 \times 10^{1} \% 9 + 8 \times 1 \% 9$

$\downarrow 1 \quad \downarrow 1 \quad \downarrow 1$

$(6 + 4 + 8) \% 9$

$= 18 \% 9 == 0$

Online Contests

ans % p

Rule — Divisibility by 4 [last 2 digits div by 4]

{ a3 a2 a1 a0 }

2 5 | 1 2 |

$(a_3 \times 10^3 + a_2 \times 10^2 + a_1 \times 10 + a_0) \% 4$

$0 + 0 + 0 + \ldots\ldots + (a_3 \times 10^3 \% 4 + a_2 \times 10^2 \% 4 + (a_1 \times 10 + a_0))$

$a_3 \times 10^3 \% 4 \rightarrow 0$

$a_2 \times 10^2 \% 4 \rightarrow 0$

$100 \% 4 = 0$

$1000 \% 4 = 0$

$10^x \% 4 = 0 \quad x \geq 2$

$(a_1 a_0) \% 4$

2512

$12 \% 4 \rightarrow$

$= 2500 + 12 \% 4 == 0$

15 Min Break :)

10.35

int → 9 digits

10^5 digits

Problem — Given a number → very large no, in the form of arr[].

arr[i] → stores 1 digit of Number

Number arr[5] = | 3 | 2 | 6 | 4 | 9 |

10^4 10^3 10^2 10^1 10^0

(0 + 4)%5

% P → int

40%5

P = 5

$9 \times 10^0 \% P$

$= (9\%5 \times 10^0 \%5)\%5$

$= (4 \times 1)\%5$

$= 4$

$= \left((3\times10^4) + 2\times10^3 + 6\times10^2 + 4\times10^1 + 9\times10^0 \right) \% P$

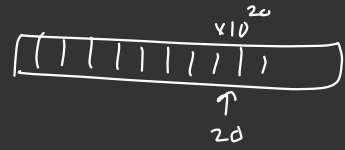
$= \left((3\times10^4)\%P + (2\times10^3)\%P + (6\times10^2)\%P + (4\times10^1)\%P + (9\times10^0)\%P \right) \% P$

$= \left((3\%P \times 10^4\%P)\%P + (2\%P \times 10^3\%P)\%P + (6\%P \times 10^2\%P)\%P + \dots\dots \right) \% P$

$= (x + y + z = \dots\dots)\%P$

Loop

$$[\ arr[0] \times 10^4 + arr[1] \times 10^3 + \cdots\cdots\cdots arr[n-1] \times 10^0\ ] \% p$$



$$(arr[1] \times 10^3) \% p \qquad 10^{large}$$

$$[(arr[1] \% p) * (10^3 \% p)] \% p$$

$$[0 \sim (p-1) * 0 \sim (p-1)] \% p$$

$(0 \sim p-1)$

$N = 10^5$

max val

$$[\ \cdots + (p-1) + (p-1) + (p-1) + (p-1)\ ] \% p$$

$1 \to 10 \to 100 \to 1000 \longrightarrow \cdots \to 10^{20}$

$\Rightarrow \quad a^n \% p$

↑ large value → ~~if~~ already seen

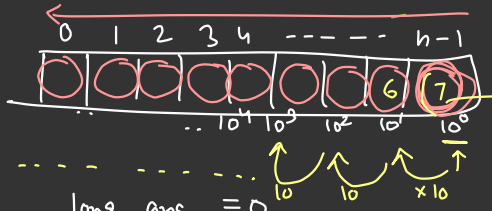
$((10^{18} \times 10) \% p$

no of digit

↑

$n \le 10^5$

10^5



$10000000000 = 10^9$

$100\ldots\ldots\ldots = 10^{105}$

a digit

10^9 int

10^5 digits

10^{10^5} ← Problem

↑ array of size 10^5

7 / 3

long ans = 0

long temp = 10^0 = 1

for (i = N-1 ; i >= 0 ; i--) {

ans = ans + ((arr[i] % p) * (temp % p)) % p ;

ans = ans % p; // to avoid overflow

temp = (10 * temp) % p. // to avoid overflow

}

MUST

Takeaway → Take mod ⟹ every addition & multiplication

int

↓

$p^0 <= 10^9$

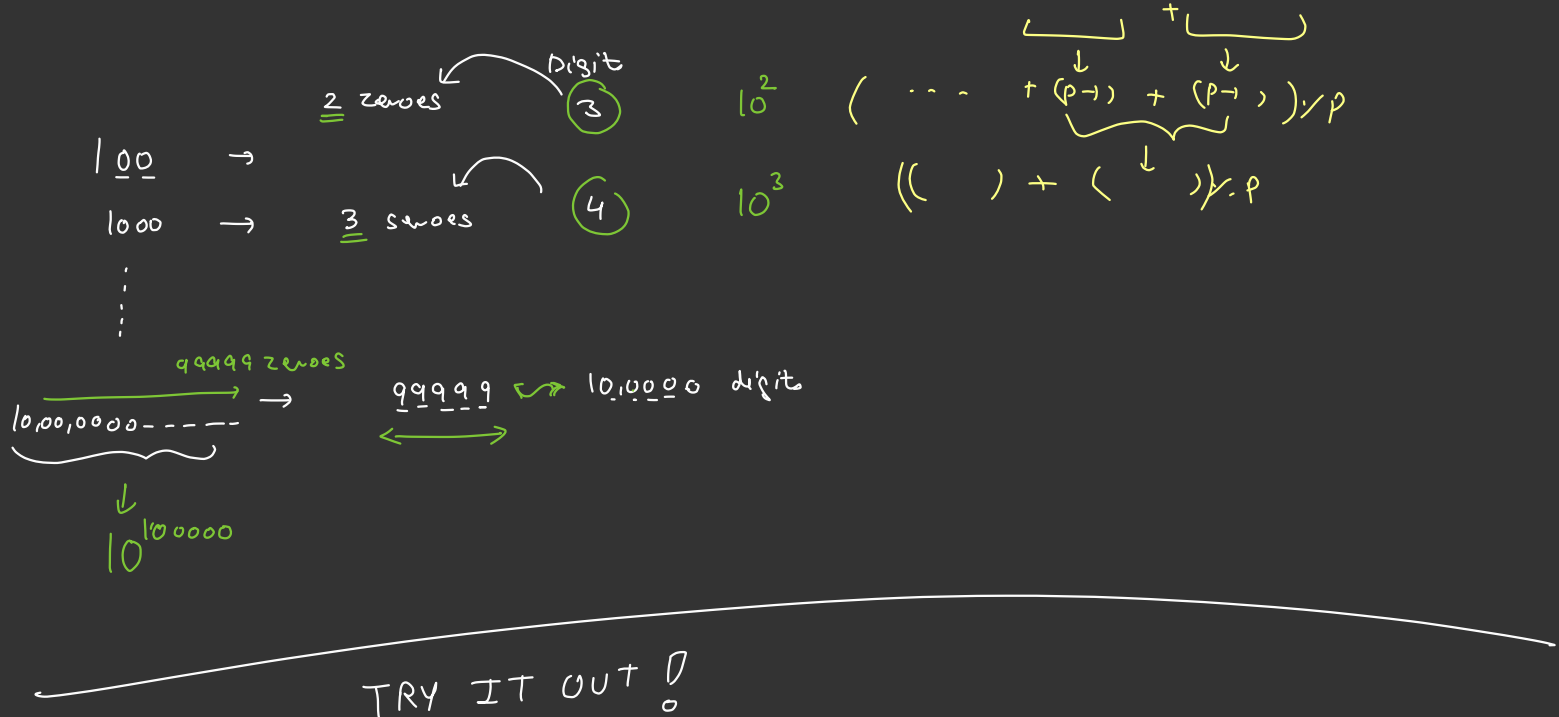
$p^2 <= 10^{18}$

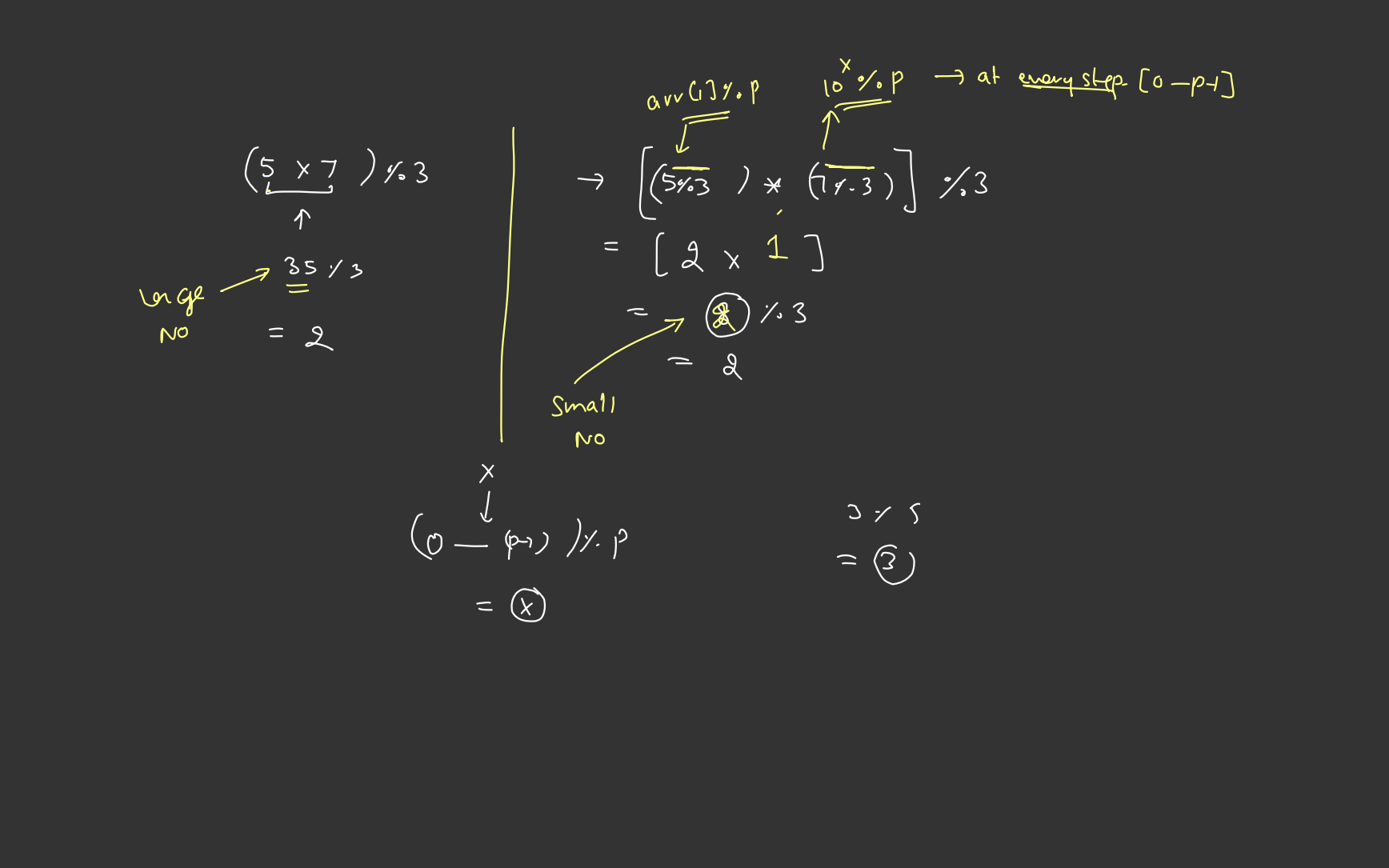
↓

(0 - p-1)

$(arr[i] * 10^x) \% p$

p-1 p-1 p^2 valid


100 →
1000 →
2 zeroes
Digit
3
3 zeroes
4
99999 zeroes
99999 → 10,00,000 digit
10,00,00000-----
10^100000
10^2 (... + (P→) + (P→))%P
10^3 (( ) + ( ))%P
TRY IT OUT !!


arr[i] % p
10^x % p
→ at every step [0 — p-1]
(5 x 7) % 3
35 % 3
= 2
Large No
[(5%3) * (7%3)] %3
= [2 x 1]
= 2 %3
= 2
Small No
x
(0 — (p-1)) % p
= (x)
3 % 5
= 3

→ $(a-b)\%p$

→ $(a/b)\%p$

Adv Batch