① → Bitmasking

→ Bit-manipulation

→ Techniques that build algorithms using the 'bit' of the data

② → Maths

Q Given 3N + 1 Numbers, every number is repeating thrice except one unique number. Find out the unique number

Input = [2, 4, 4, 3, 2, 6, 6, 7, 7, 7, 2, 6, 4]

Output → 3

Approach-1

$O(n^2)$ time

$O(1)$ Space

a[i] → repeating or not

current = ~~2~~ 3, cnt = ~~3~~ 1 ✓ unique no

for(i=0; i<n; i++){

Linear Search for a[i]

at all locations except i

uniq no

Approach-2

Sort → [2, 2, 2, 3, 4, 4, 4, 6, 6, 6, 7, 7, 7]

Count of

no getting visited

$O(N \log N)$ time

$O($ Extra $)$ Space

merge / Quicksort → take extra space

Look for consecutive duplicates (N)

→ $O(N \log N)$ time (fact)

$N \log N + N$

$= O(N \log N)$

Approch-3 Hashmap (Later . . .)

= [2, 4, 4, 3, 2, 6, 6, 7, 7, 7, 2, 6, 4]

Data Structure

No unique entries

extra Space

Key	value
2	3
4	3
3	1
6	3
7	3

$O(N)$ time

Unique No.

$3N + 1$

Insert, Updating → $O(1)$ for each entry

→ Building a hashmap
no → frequency

→ iterating to check what no has freq 1

Time → $O(N)$

Space → $O(N)$ space

Approach-4 Bitmasking

Input = [2, 4, 4, 3, 2, 6, 6, 7, 7, 7, 2, 6, 4]

= ~~2^2^2 ^ 3 ^ 4^4^4 ^ 6^6^6 ^ 7^7^7~~

= ~~2^3 ^ 4^6 ^7~~

Not Helpful

Idea -> we need a way to cancel out bits of the numbers which are repeating trice.

2 -> 0 1 0

4 -> 1 0 0

3 -> 0 1 1

3 times, 3 times

x: 0 1 1

y: 0 0 1

one time

1 ts

	... B3	B2	B1
2 ->	0	1	0
4 ->	1	0	0
4 ->	1	0	0
3 ->	0	1	1
2 ->	0	1	0
6 ->	1	1	0
6 ->	1	1	0
7 ->	1	1	1
7 ->	1	1	1
7 ->	1	1	1
2 ->	0	1	0
6 ->	1	1	0
4 ->	1	0	0
	9,	10,	4

3, 3, 1, 3 -> (3(3) + 1) % 3

0, $\boxed{1}$ <- Unique NO

cancel out bits repeating 3 times

$\boxed{0, 1, 1}$ -> $\boxed{3}$

3N,

3N+1

even

4N + 1 → XOR ⇒ ~~4~~ ^ ~~4~~ ^ ~~4~~ ^ ~~4~~ ^ ~~4~~ ^ 5

5N + 1 → % 5

odd

Bitwise sum % 5

$\cancel{3} \wedge \cancel{3} \wedge \boxed{3}$

i=0

011

3, 7 → 111, - - - - - - - -

0	1	2	3	-	-	-	-	-	31
2	2	1	0	0	0	0	0	0	0

freq array of bits (fixed)

Time → O(N)

space → O(1)

Technique : Finding Power of a Number

Given a, N find a^N. → exponentiation

Google

Math.power

↑

don't know how it implemented.

$a = 5, n = 4$

$a^n = 5^4 = 625$

$a = 2, n = 10$

$a^n = 2^{10} = 1024$

$$\underbrace{a \times a \times a \times \dots a}_{n \text{ times}}$$

```
ans = 1
for (i=1, i<=N; i++) {
    ans = ans*a;
}
```

O(N) time
O(1) Space

Better?

Recursive

$2^{10} \rightarrow (2^5)^2 = 32 \times 32 = 1024$

$2 \cdot (2^2)^2 = 2 \times 16 = \underline{32}$

$= 2 \cdot 2^4$

$= 2^{4+1} = 2^5$

$(2^1)^2 = 4$

$2(2^0)^2 = 1$

10 → 5 → 2 → 1 → 0

n → n/2 → ... → 1 → 0

Log N

exponential decay in power

$a^n = (a^{n/2})^2$ n even

$= a \cdot (a^{n/2})^2$ n odd

(Later)

Bitmasking

$8 + 1 = \boxed{9}$

$$a^{9} = a^{1001}$$

Squaring - - -

$\boxed{a^8}$ a^4 a^2 $\boxed{a}$

1 0 0 1

$$= a^{1001} = a^8 \cdot a^1 = a^9$$

a

$a = a \times a$ // a^2

$a = a \times a$

$a^4 \times a^4 = a^{4+4} = a^8$

$$\boxed{x^{n1} \cdot x^{n2} = x^{n1+n2}}$$

$\boxed{a}$	$a = \boxed{5}$	$a = a \times a$
$\boxed{a^2}$	$a = 25$	$a = a \times a$
$\boxed{a^4}$	$a = 625$	$a = a \times a$
$\boxed{a^8}$	$a = 390625$	

$a^4 \quad a^2$
$\boxed{16} \; \boxed{4} \; a = \boxed{2}$

$2^7 = 2^{111}$

$a^4 \cdot a^2 \cdot a^1$
$= 16 \times 4 \times 2$
$= \underline{128}$

$5^7 = 5^{111}$ ($5^4\, 5^2\, 5$)

$= 5^4 \cdot 5^2 \cdot 5^1$
$= 5^{4+2+1} = 5^7$

$3^8 = 3^{1000}$ ($3^8\; 3^4\; 3^2\; 3^1$)

$= 3^8 \cdot 1$
$= \boxed{3^8}$

Implement

$a^9 = a^{1001}$

n =

$ans = 1 \cdot a \cdot a^8$
$= \boxed{a^9}$

```
a = input
ans = 1
while ( n > 0 ){
    last_bit = n&1
    if ( last_bit == 1){
        ans = ans * a;
    }
    n = n >> 1
    a = a * a
}
```

$\longrightarrow n = n/2$

$n \downarrow n/2 \downarrow \dots 0$ — $\log N$

Time $O(\log N)$
Space $O(1)$

Maths

10:40

↳ Modulo operator
↳ Modulo arithmetic
↳ Problems (next class) +

2^4-1

15 ✓

Recap

int

$\underbrace{-2^{31} \quad \text{to} \quad 2^{31}-1}_{\text{int Range}}$

Ranges

approx $\quad -2\times10^9 \quad$ to $\quad +2\times10^9$

64 bit

long

-2^{63} ——— to ——— $2^{63}-1$

-8×10^{18} to 8×10^{18}

Modulo $\quad a \% b$

$a = K * b + \text{rem}$

$a=10, \; b=4$

$10 = \boxed{2} * 4 + \boxed{2}$

a%b

$a = \underline{60}$, $b = 9$

$a = 9 \times \boxed{6} + \boxed{6}$

$\left(\frac{60}{9}\right) \times 9$

$= 6 \times 9 + 6$

$= \boxed{60}$

$\underline{rem} = a - \boxed{k}\, b$

$= a - \left[\frac{a}{b}\right] b$

$= 60 - \left[\frac{60}{9}\right] \times 9$

$60 - 6 \times 9$

$= \boxed{6}$

Mod fn

$7 \% 5 \rightarrow 2$

$8 \% 5 \rightarrow 3$

$80 \% 5 \rightarrow 0$

$24 \% 5 \rightarrow 4$

$\left\{ \begin{matrix} -\infty \\ \vdots \\ \infty \end{matrix} \right\} \underset{\text{limit the range}}{\% M} \rightarrow \left\{ \begin{matrix} 0 \\ \vdots \\ M-1 \end{matrix} \right\} \begin{matrix} \leftarrow \text{Min} \\ \\ \leftarrow \text{max} \end{matrix}$

Later

- ↳ Consistent Hashing
- ↳ Hashmaps
- ↳ Encryption/ Cryptography.

+ve $\quad 10 \% 7 \rightarrow 3$

-ve NO $\quad -10 \% 7 \rightarrow$

Conceptually

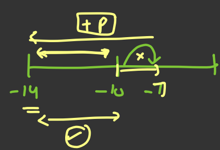
0 — $(M-1)$

$-10-(-9)$
$= -10+9$
$= -1$

$-10-(-14)$
$\Rightarrow -10+14$
$\Rightarrow 4$

```
if ( x < 0 ){
    { x%p + p }
}
```

+ve



$-3\%7 + 7$

$= -3+7$

$= 4$

- if mod is -ve (Java), $\quad x\%p + p$

Modulo Airthmetic

Thm 1
Addition

$$\boxed{\begin{aligned}&(a+b)\ \%\ M\\ &= (\underline{a\ \%\ M} + b\ \%\ M)\ \%\ M\end{aligned}}$$ ✓

$a=6$ $b=\underline{13}$ $m=7$ $6=0\times 7+\underline{6}$	$(a+b)\ \%\ M$ $=19\ \%\ 7$ $=\boxed{5}$	$(6+6)\ \%\ M$ $=12\ \%\ 7$ $=\boxed{5}$

$a \rightsquigarrow 10^9$ $\quad$ $b \rightsquigarrow 10^9$

int a
int b

$(a+b) \to$ overflow

To prevent overflow in certain scenarios

$$(\underline{a+b})\ \%\ M \longrightarrow \left(\underbrace{(a\ \%\ M)}_{\downarrow\ 0\ --\ M-1} + \underbrace{b\ \%\ M}_{\downarrow\ 0\ --\ M-1}\right)\ \%\ M$$

$\uparrow$
overflow

$= \underline{\text{Result}}$

$a = 10^5$ $b = 10^5$

$\underline{(a * b)} \% M$
↓
overflow

<u>Thm-2</u>

$\underbrace{(a * b)}\ \% M = ((a \% M) * (b \% M)) \% M$

$a = 6$
$b = 7$
$M = 4$

$(6 \times 7) \% 4$
$= 42 \% 4$
$= 2$

$(2 \times 3) \% 4$
$= 6 \% 4$
$= 2$

<u>Thm</u>

$(a-b) \% M$
$(a/b) \% M$

] Later in adv batch.

Next class → Problems on Mod Arithmetic

Find

$a^n \% p$

$\boxed{a, n, p}$

[HoH]

$a = 100$, $n = 50$

a^n $\% p$
↓
overflow

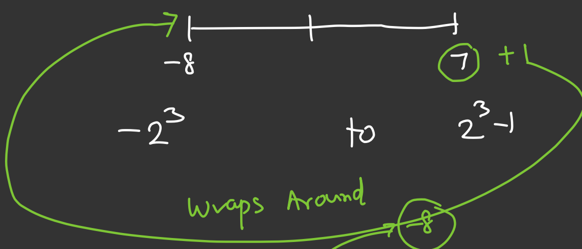
Overflow

sign bit

$\boxed{0 \mid 1\ 1\ 1}$

int

4 bit signed int



-2^3 to 2^3-1

→ $\boxed{1 \mid 0\ 0\ 0}$ → -ve No

Interpret

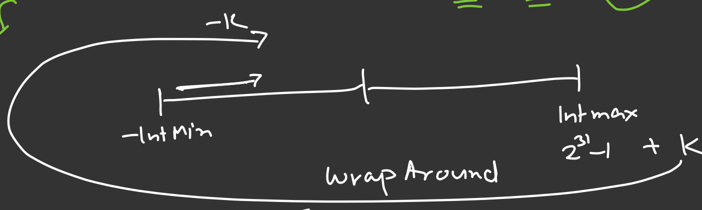
4 bit signed int

7 + 1

$$\begin{array}{r} = \ 0\ 1\ 1\ 1 \\ +\ 0\ 0\ 0\ 1 \\ \hline \boxed{1\ 0\ 0\ 0} \end{array}$$

$7 + 1 = -8$

$+7 + 2 = -7$



Circular (Wrap Around)

7 0111

+ 2 + 0010

1001 → -7

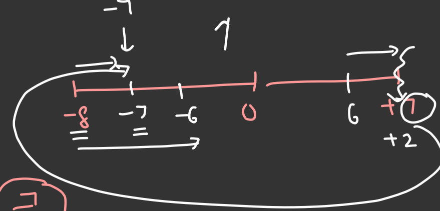
-ve

Signed int

0110

+ 1

0111 → 7



6 + 4 = -6

Invalid value (overflow)

BigInteger → Large No's in Java

% Mod → Mod Thm's