Doubt please explain once again --given q queries find number of primes in range [a,b]

largest N

csum $O(N)$ → 0 1 2 2 3 3 4 . . . .

$O(N \log \log N)$ → | | 2 | 3 | | 5 | | 7 | . . | . . | . . | | . . | . . | . . | | | | | | 9957 | | 1 |

bool sieve ~~array~~ [N+1]

$N = 10^6$

→ Prime sive

prefix[i] = prefix[i−1] + 1 if sieve(i) is True.
+ 0 otherwise

$O(N \log \log N)$

[3 — 7]
||
4 − 1 = 3

$O(1)$ for each query

Combinatorics

→ Counting problems, permutation & combination

Basic Rules

1. Rule of Product

if there are X ways to choose an element from a set A and there Y number of ways to choose an element from set B, then there will be X*Y ways to choose two elements one from set A and one from B.

10 Boys

B1
B2
B3
⋮
B10

7 Girls

G1
G2
⋮
G7

#1 2 Monitors in the class s.t 1 B $\boxed{\text{and}}$ 1 G is selected

10 ways * 7 ways = 70 ways

$^{10}C_1 * {}^{7}C_1 = 70$ ways

#2 Choose 2 Monitors

17 Student

17 ways * 16 ways

Permutation

$^{17}P_2$

X

$\times 2!$

$^{17}C_2 = \frac{17 \times 16}{2}$

Combination

$^{17}C_2$

✓

Same { B3, G7
G7, B3

3

Test 3 Question $\to$ T, F

Answering 3 Questions

T T T
F F F

T F F
F T F
F F T

T T F
F T T
T F T

8 ways

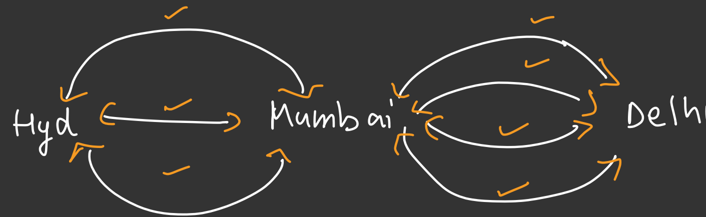
Product Rule

2 2 2
Q1 Q2 Q3

$= 2 \times 2 \times 2$

$= 8$ ways

Flights



Q) ways to Reach Delhi from Hyd via Mumbai

Hyd → Mumbai & Mumbai → Delhi

3 * 4

= 12 ways

Rule of Sum

Rule of sum states that if there are X ways to choose one element from A and Y ways to choose another element from B, then there will be X + Y ways to choose one element that belongs to either A or B.

10 Boys 7 Girls

#1 → Choose a class Monitor → 10 + 7 = 17 ways

#2 → Q1 Q2 Q3

Answer a Question → Q1 or Q2 or Q3

= 3 ways

$^3C_1$

Answer Two Questions $\rightarrow$ Q1, Q2 + Q1, Q3 + Q2, Q3

$^3C_2$

= 3 ways

$^3C_2$ = denotes the no of ways to pick 2 items from a set of 3 items

$$\left[\frac{3!}{(3-2)!\,2!} = \frac{3!}{2!} = 3 \right]$$

"Permutation & Combination"

Combination

Combination of choosing R distinct items of a collection of N objects is given by ${}^{N}C_{R}$.

$$\boxed{{}^{N}C_{R} = \frac{N!}{(N-R)!\,R!}}$$

5 Boys, choose 2 for leading the cricket team

$$ {}^{5}C_{2} = \frac{5!}{(5-2)!\,2!} = \frac{5!}{3!\,2!} = \frac{4 \times 5}{2} = 10 \text{ ways} $$

A, B, C, D, E

$$ \Rightarrow \underbrace{\begin{matrix} AB \\ AC \\ AD \\ AE \end{matrix}}_{4 \text{ ways}} + \underbrace{\begin{matrix} BC \\ BD \\ BE \end{matrix}}_{3 \text{ ways}} + \underbrace{\begin{matrix} CD \\ CE \end{matrix}}_{2 \text{ ways}} + \underbrace{DE}_{1 \text{ way}} $$

$= 4 + 3 + 2 + 1$

$= 10$ ways

5 Boys, choose 2 for leading the cricket team for the post of Captain & vice-captain

A, B, C, D, E

$\Rightarrow$

$\boxed{C}$ $\boxed{VC}$						
AB		BC		CD		DE
AC	+	BD	+	CE	+	1 way
AD		BE		2 ways		
AE		3 ways				
4 ways						

} 10

BA	CB	DC	ED
CA	DB	EC	
DA	EB		
EA			

} 10

Arrange a set of 2 items

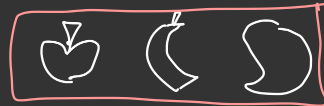
$$ {}^5C_2 \times 2! $$

$$ = \frac{5\times 4}{\cancel{2}} \times \cancel{2} $$

= 20 ways.

~~Choosing~~

"Arrangements"



A , B , M

$3! = 6$

Arrange (in possible orders)

3	2	1

$= 3 \times 2 \times 1$

$= 3!$

$= 6$ ways

$\boxed{A}$ — B, M
M, B

B — A, M
M, A

M — B, A
A, B

$\underbrace{2}$ + $\underbrace{2}$ + $\underbrace{2}$ 6 ways

K distinct items and you want arrange them
n

then it can be K! ways.

N=4

B1 B2 B3 B4

$\boxed{B1}$ — (B2, B3, B4)
6 ways

B2 — $\boxed{B1, B3, B4}$
6 ways

B3 — _ _ _
6 ways

B4 — _ _ _
6 ways

$= 6 \times 4 = 24$ ways

$= 3! \times 4$

$= 4!$

order is important

Permutation (Selecting & Arrangements)

Permuations of choosing R distinct items out of N objects can be calculated using NPR.

$\boxed{{}^{N}P_{R} = {}^{N}C_{R} * R!}$

$\boxed{{}^{N}P_{R} = \frac{N!}{(N-R)!}}$

${}^{N}P_{R}$

Select * Arrange

${}^{N}C_{R} * R! = \frac{N!}{(N-R)!\,\cancel{R!}} \times \cancel{R!}$

$\Rightarrow$ Choose 2 ppl out of 5 ppl

${}^{5}C_{2} = \frac{5 \times 4}{2} = 10$

$\Rightarrow$ Choose 2 ppl out of 5 for PM & President

${}^{5}P_{2} = \frac{5!}{3!} = 5 \times 4 = 20$

$\rightarrow {}^{5}C_{2} \times 2!$

$= 10 \times 2$

$= 20$

Q) Given an array of size N, what is the prob that array contains integer in sorted order (asc & desc)

[1, 3, 5, 2, 7]

$N = 5$

1, 2, 3, 5, 7 ✓

or

7, 5, 3, 2, 1 ✓

$$\text{Prob} = \frac{\text{Favourable}}{\text{Total}} = \frac{2}{5!} = \frac{2}{120} = \frac{1}{60}$$

Q Given a string of len N, find all permutations of string

"count" → all possible arrangements.

$N = 3$

ABC

Arr = 3!

ABC BCA CBA

ACB BAC CAB

$^{N}P_{R} = {}^{3}C_{3} \times 3!$

$= 1 \times 3! = 6$

$^{3}P_{3} = \frac{3!}{(3-3)!} = 3! = 6$

Subsets

$N=4$

01	02	03	04

$$1+1 = 2 \times 2 \quad 2 \quad 2 = 2^4 = \boxed{16} \checkmark$$

ways to choose 0 item $= {}^{N}C_0 = {}^{4}C_0 = \frac{4!}{(4-0)!\,0!} = 1$

$\{\}$

1 item $= {}^{4}C_1 = \frac{4!}{(4-1)!\,1!} = 4$ ways

$\{01\}$ $\{02\}$ $\{03\}$ $\{04\}$

2 item $= {}^{4}C_2 = \frac{4 \times 3}{2} = 6$ ways

3 item $= {}^{4}C_3 = \frac{4 \times \cancel{3} \times \cancel{2}}{1 \cancel{2} \cancel{3}} = 4$ ways

4 items = $^4C_4$ = 1 ways

$$^4C_0 \underset{\text{OR}}{+} {^4C_1} \underset{\text{OR}}{+} {^4C_2} \underset{\text{OR}}{+} {^4C_3} \underset{\text{OR}}{+} {^4C_4}$$

$$= 1 + 4 + 6 + 4 + 1$$

= 16 ways

01	02	03	04

$$\underbrace{(I \text{ or } E)}_{2} \times \underbrace{(I \text{ or } E)}_{2} \times \underbrace{(I \text{ or } E)}_{2} \times \underbrace{(I \text{ or } E)}_{2} = 2^4 = 16 \text{ ways}$$

Prop

$$^{N}C_{R} = {}^{N}C_{N-R}$$

$$\frac{N!}{(N-R)!\,R!} \qquad \frac{N!}{(N-(N-R))!\,(N-R)!}$$

$$= \frac{N!}{R!\,(N-R)!}$$

Hack

$$^{8}C_{3} \rightarrow \frac{8!}{5!\,3!} = \frac{\cancel{5!}\,\cancel{6}\cdot 7\cdot 8}{\cancel{5!}\,1\cdot\cancel{2}\cdot\cancel{3}} = 56$$

Shortcut

Quick calc

Num 3 terms ↓

Dem 3 terms ↑

r terms

$$\frac{8\cdot 7\cdot \cancel{6}}{1\cdot\cancel{2}\cdot\cancel{3}} = 56$$

$$^{10}C_{8} \rightarrow {}^{10}C_{2} \rightarrow \frac{10\cdot 9}{1\cdot 2} = 45$$

Prop

$$^{N}C_{R} = {}^{N-1}C_{R-1} + {}^{N-1}C_{R}$$

[Recursive Rec Relation]

$^{N}C_{R} \to$ Binomial coefficient

Pascal's Triangle

$^{N}C_{R}$

↑

choosing R objects out of N.

lookup Table

N=0	1 ($^{0}C_{0}$)					
N=1	1 ($^{1}C_{0}$)	1 ($^{1}C_{1}$)				
N=2	1 ($^{2}C_{0}$)	2 ($^{2}C_{1}$)	1 ($^{2}C_{2}$)			
N=3	1 ($^{3}C_{0}$)	3 ($^{3}C_{1}$)	3 ($^{3}C_{2}$)	1 ($^{3}C_{3}$)		
N=4	1 ($^{4}C_{0}$)	4 ($^{4}C_{1}$)	6 ($^{4}C_{2}$)	4 ($^{4}C_{3}$)	1 ($^{4}C_{4}$)	
N=5	1 ($^{5}C_{0}$)	5 ($^{5}C_{1}$)	10 ($^{5}C_{2}$)	10 ($^{5}C_{3}$)	5 ($^{5}C_{4}$)	1 ($^{5}C_{5}$)

Constraints: $N \leq 5$

$N(4,2) = \frac{4!}{2!\,2!} = \bigcirc$ overflow

$^{4}C_{3}$, $^{2}C_{2}$, $^{5}C_{3} = \frac{5!}{3!\,2!}$

$^{50}C_{3} = \frac{50!}{47!\,3!}$ → overflow

'long' X

← Not easily overflow

multiple lookups.

$$\underbrace{table[i][j]}_{} = table[i-1][j] + table[i-1][j-1]$$

$$\boxed{{}^{i}C_{j} = {}^{i-1}C_{j} + {}^{i-1}C_{j-1}}$$

$$\boxed{{}^{4}C_{2} = {}^{3}C_{2} + {}^{3}C_{1}}$$

$$= 3 + 3$$

$$= 6$$

$$\frac{4 \cdot 3}{2} = 6$$

Q1 # valentine gift "selection"

$$\underbrace{\left(\overset{5}{\text{Pen}} \ \& \ \overset{3}{\text{Book}}\right)}_{{}^5C_1 \cdot {}^3C_1} \text{ or } \left(\overset{6}{\text{Flowers}} \ \& \ \overset{10}{\text{Chocalte}}\right)_{{}^6C_1 \cdot {}^{10}C_1} \text{ or } \left(\overset{3}{\text{Ring}}\right)_{{}^3C_1}$$

Choose Gift $\Rightarrow 5 \cdot 3 + 6 \cdot 10 + 3$

$= 15 + 60 + 3$

$= 78$

Q2 # Compute $({}^NC_R) \% P$ given N, R, P

$N = 50$

$R = 17$

$P = 11$

Code -

$$^{N}C_{R} = \frac{N!}{(N-R)!\,R!}$$ factorial → overflow

$$\left(^{N}C_{R}\right)\%p = \left(^{N-1}C_{R-1} + {}^{N-1}C_{R}\right)\%p$$

$$= \left(\underbrace{^{N-1}C_{R-1}\%p}_{t1} + \underline{^{N-1}C_{R}\%p}\right)\%p$$

```
int Compute( N, R, P) {
    if (N==0 || R==0){
        return 1;
    }
    t1 = Compute (N-1, R-1, P)
    t2 = compute (N-1, R) P)
    return (t1 + t2) % P
}
```

Q

Input "CAB" find out the rank in all permutation of this string

"order"

position in sorted

N

sorted acc to dic

1 ABC
2 ACB
3 BAC
4 BCA
5 CAB → Position
6 CBA

N! time

$\boxed{\text{5 Mins}}$

output → 5

"Brute" → generate all, sorting & find.

CADB

$N=4$

$4!$

A _ _ _ $3!$

B _ _ _ $3!$

12

~~C~~ A D B

rank of ABD in its permutation

C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
C	D	B	A

13

14 ⟸ 2

$(cnt) * (N-1)!$

$2 \times 3!$

$= 12$ permutation + rank of remaining string after removing first letter

$+ 2$

$= 14$

$$\text{rank}(A) = \underbrace{cnt}_{\text{No of letters having val} < A[0]} * \underbrace{(N-1)!}_{\text{A.length}} + \underbrace{\text{rank}(A')}_{}$$

No of letters having val < A[0]

A.length

String with first char removed.

4!

VIEW

E, I, V, W

3!

E

I 3!

V

W

sub problem

cnt = 2

E

V EIW
V EWI
V IEW
V IWE
V WEI
V WIE

$$ans = 2 \times 3! + \text{rank of } (IEW)$$

$$+ \underbrace{1}_{cnt} * 2! + \text{rank}(EW)$$

$$+ 0 + \text{rank}(W)$$

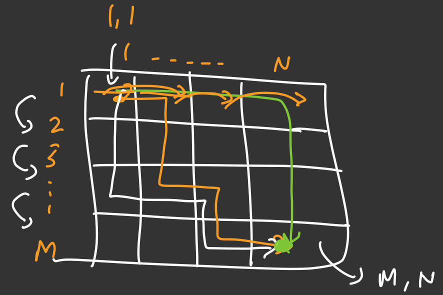
$$= 0$$

$$= 12 + 2 + 0$$

$$= \boxed{14}$$

$$\text{Total} = \frac{(M-1+N-1)!}{(M-1)!\,(N-1)!}$$

Q



4 cols → 3 jumps
4 Rows → 3 jumps

$\boxed{M-1 \text{ Right}}$ → → →

$\boxed{N-1 \text{ Down}}$ ↓ ↓ ↓ ↓

$$\underbrace{\begin{array}{c} RDDDRR \\ DRDRDR \\ RRRDDD \end{array}}$$

in how many ways you can reach M,N.

Concept

5 A, 4 M Arrange them (all apples → identical)
(all mangoes → identical)

$$9 \text{ fruits} = \frac{9!}{5!\,4!}$$

N items
K! are identical

$$\frac{N!}{K!}$$

4 A, 2 Mangoes

identical

A, A, A, A, M $= \dfrac{5!}{4!\,1!}$ = 5 ways

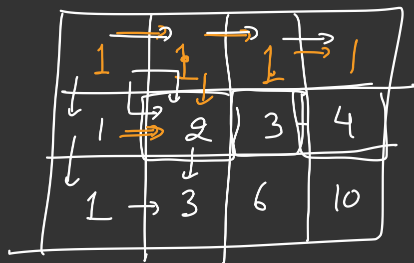
A, A, A, M, A

A, A, M, A, A

A, M, A, A, A

M, A, A, A, A

ways



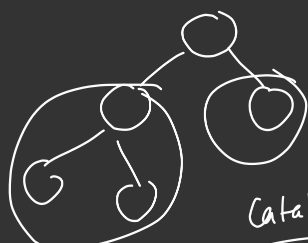
$M=3$, $N=4$

$$ways(i,j) = \underbrace{ways(i-1,j) + ways(i,j-1)}_{time}$$

fast

$$= \frac{(M-1+N-1)!}{(M-1)!\,(N-1)!} = \frac{5!}{2!\,3!} = \frac{4 \times 5}{2} = 10 \text{ ways}$$

correct



Catalan No