Modulo Arithmetic

$A \% B = \{\text{Remainder}\} \rightarrow 0 \text{ — } (B-1)$

$12 \% 5 = \boxed{2} \rightarrow (0, 1, 2, 3, 4)$

$12 = \text{Divisor} \times Q + \text{Rem}$

Dividend $\quad 5 \times 2 + 2$

$12 \rightarrow 12 - 5 = 7 - 5 = 2$

$a \% M = [0, M-1]$

Modulo Arithmetic

① $(a+b)\%M = (a\%M + b\%M)\%M$

② $(a*b)\%M = (a\%M * b\%M)\%M$

③ $(a-b)\%M$

$$= \left(\overbrace{\underbrace{(a\%M)}_{\downarrow\ 1}}^{(0,M-1)} - \overbrace{\underbrace{(b\%M)}_{\downarrow\ 4}}^{(0,M-1)} + M \right)\%M$$

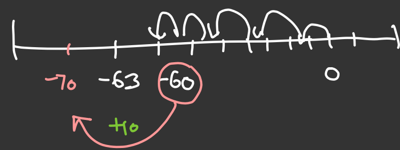
$(1 - 4 + M)\%M$

Correction for a negative no

$(-N0 \% m + m)$

$-63 \% 10$

$12 \% 5$



$-63 - (-70)$

$= 7$ [correct]

$(0, m-1)$

+10

$-63 - (-60)$

$= -3$ [Java] / JS]

$12 - 10$

lower No

Q1. given an array of +ve integers, calculate the no of pairs (i, j) such that $(arr(i) + arr(j)) \% M = 0$ $i \,!= j$

Input   arr = [4, 7, 6, 5, 8, 3] $\boxed{M = 3}$

i=	j=	sum
4	8	12 % 3 = 0
4	5	9 % 3 = 0
7	8	15 % 0
7	5	12 % 3 = 0
6	3	9 % 3 = 0

5 pairs

Brute force

for(i)
  for(j)
    check & count

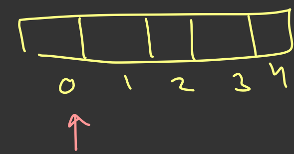
O(N²) time
O(1) space

$a(i) + a(j)$

Significance of $(sum) \% M$

$\underbrace{0 \;----\; M-1}$

$M = 5$



$$(a(i) + a(j)) \% M = 0$$

$$\Rightarrow (a(i)\% M + a(j)\% M)\% M = 0$$

$0 - M-1$ $\quad$ $0 - (M-1)$

$$\Rightarrow (2 + 3) \% 5 = 0$$

$$(x + M-x) \% M = 0$$

store in a hashmap

arr = [4, 7, 6, 5, 8, 3, 10, 15]

5, 10, 15

Any 2 out of 3

5 + 10 = 15 % 5 = 0

10 + 15 = 25 % 5 = 0

5 + 15 = 20 % 5 = 0

Spl case

% 5 → [0, 4]

keys	freq of a[i] % m
0	3
1	1
2	1
3	2
4	1

freq >= 2 → freq C_2

Duplicates

ans = 0 + 3 + 1×1 + 1*2

= 3 + 1 + 3

3

for(i=0; i<=n-1; i++){

hashmap[a[i]%m] += 1,

for(j = ~~0~~ 1 ~~j <~~ (m+1)/2){

ans += hm[j] × hm[m-j],

}

(x + y) % m = 0

1 4

2, 3

hm[1] * hm[4]

hm[2] * hm[3]

arr = [4, 7, 6, 5, 8, 3, 10, 15]

$M = 5$

4 % 5, 7 % 5

a % m	freq
0	3
1	1
2	1
3	2
4	1

M/2

a[i] % m = 0

(a[i] % m + a[j] % m) % m = 0

$^3C_2$ ways = 3 ways + 1 + 2 = $\boxed{5 \text{ ways}}$

4 + 6 = 10

7 + 3 = 10 % M = 0

5 + 10 = 15

5 + 15 = 20

10 + 15 = 25

7 + 8 = 15

(a[i] + a[j]) =

freq(1) 4

x M − x

1 * 1 = 1 way

freq(2) * freq(3) = 2 ways

1 * 2

freq(3) * freq(2)

freq(4) * freq(1)

avoid double counting

One more special case
if m is even

$M = 10$

[- - - 15, 25, 45, 85]
% 10
= 5

$\frac{m}{2}$

0 Spl Case
1 ✓ hm[j] × hm(m−j]
2 ✓
3 ✓
4 ✓
5 — 4 → $^4C_2$ ways
6 ×
7 ×
8 ×
9 ×

x , M−x
[5 , 5]

$= {}^4C_2 = \frac{4\times3}{2}$
= 6

m=20 → 10
m=24 → 12
m

hm[0] C_2
hm[j] × hm[m−j]
hm[m/2] C_2

Special Case

[0 ——— mid−1 , mid   even
[0 ——— < mid   odd

$mid = \frac{M}{2}$

Q **Divisible Subarray**

Len of Array

↑

N = 7

0	1	2	3	4	5	6
3	2	4	$\boxed{7}$	6	1	8

3+4 not a subarray

Print any subarray ~~you~~ have such that

(subarray sum) % N == 0.

7 % 7 = 0

7, 6, 1 = 14 % 7 = 0

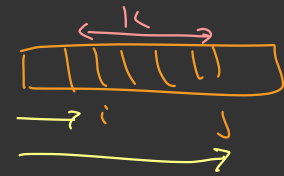
6, 1 = 7 % 7 = 0

2, 4, 7, 6, 1, 8 = 28 % 7 = 0

Brute force

$O(N^3)$

```
for(i)
  for(j)
    for(K)
O(N)  → sum → i ___ j
```



$O(N^3)$

Brute force + Prefix Sum

$O(N^2)$

```
for(i)
  for(j ) {
      sum(i,j) = ps[j] - ps[i-1];
      if (sum % m == 0) cnt++
  }
}
```

$j \geq 1$

Hint

	0	1	2	3	4	5	6
	3	2	4	7	6	1	8
ps →	3	5	9	16	22	23	31
		i	i+1		j		

1

multiple of M

$$(\underbrace{ps[j] - ps[i]}) \;\%\; M = 0$$

$M = 7$

$ps[j] = 15 - 1 = 4$

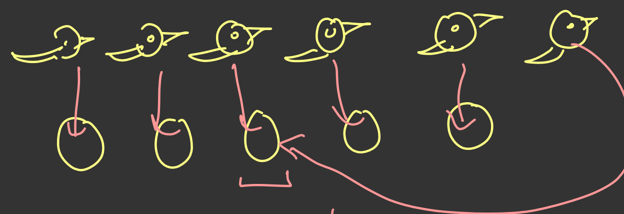
$= 15 - 8 = 7$

$$\Rightarrow (\underbrace{ps[j] \% M}_{(0,\, M-1)} - \underbrace{ps[i] \% M}_{(0,\, M-1)} + M) \;\%\; M = 0$$

$\Rightarrow$

Pigeonhole Principle
(Mathematics)

6 pigeons
5 holes



1 hole
with 2 pigeons

0	1	2	3	4	5	6
3	2	4	7	6	1	8

Subarray → 3 5 9 16 22 23 31

equal in some case

$(ps[j] - ps[i]) \% M = 0$

$\Rightarrow \underbrace{ps[j] \% M} - \underbrace{ps[i] \% M} = 0$ { consider a case }

$\Rightarrow \quad ps[j] \% M = ps[i] \% M$

Claim: Such a case will always exist. WHY?

indices	0	1	2	3	4	5	6
arr	3	2	4	7	6	1	8
prefix	3	5	9	16	22	23	31
prefix % N	3	5	2	2	1	2	3

N = 7

7 Pigeons

Range of

prefix % N

= 0

Mod

	1	3	2		1	
~~0~~	1	2	3	4	5	6

6 Buckets

There will always a case s.t freq of bucket ≥ 2

$ps(j) \Rightarrow 2$

$ps[j] = ps[i]$

$7 \% 7 = 0$

$7, 6, 1 = 14 \% 7 = 0$

$6, 1 = 7 \% 7 = 0$

$2, 4, 7, 6, 7 = 28 \% 7 = 0$

0^{th} $\qquad$ i^{th}

13, 0, 0, 4, 3, 1, 2

13 13 13 17 20 21 23

$\% M$ $\qquad$ $\%$ $\qquad$ $\% M$

1					

0

7 Pigeons

$\cancel{1}$	1	1	2	1	1	1
0	1	2	3	4	5	6

7 holes

$ps(j) \% m = 0$

$\downarrow$

$(0, j)$

Doubts

7 pegeons

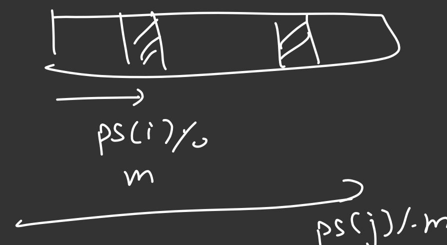
$M = 7$

No pigeon goes here

6 buckets

atleast 1 bucket will contain two pigeons

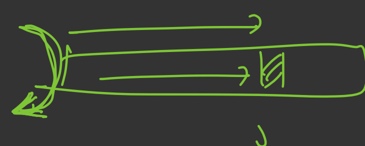
simply means



$\boxed{\text{Case-1}}$

$ps(j)\%m - ps(i)\%m = 0$

Case-II



$(ps(j) - 0)\%m = 0$

$ps(j)\%m = 0$

| 1 | | | | | | |

0

$ps(i) \% m = 0$

Hack

N = 5

M = 7

arr | 5 | 89 | 1 | 3 | 52 | 9 | 3 |

csums (0, 5, 94, 95, 98, 150, 159, 162) % 7

csum % 7 (0, 5, 3, 4, 0, 3, 5, 1)

8 sums (values)

or 8 pigeons

O(N)

O(N)

98%7 == 0

array

~~O(N)~~

O(N) Time

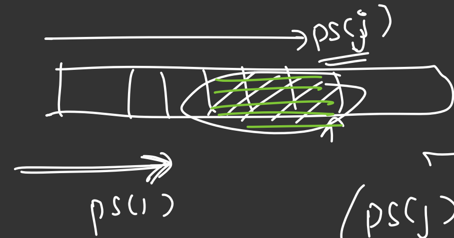
2	1	0	2	1	2	0
0	1	2	3	4	5	6

7 holes

atleast one fre value be 2

i, j → linear

i+1, j → =



$\underbrace{(ps(j) - ps(i))}_{\text{Multiple}} \% \, m = 0$

$\Rightarrow ps(j) \% m = ps(i) \% n$

↑
got a subarray

(5 −

ps%

$ps(j)\%n$ − $ps(i) = 18$

$= 25\%7$ $\quad$ $18\%7$

$= 4$ $\quad$ $= 4$

$= 4 - 4 = 0$

Medium

$\boxed{hm[0] = 1}$ ✓

idx = = 0

| 5 | 7 | 6 | 2 | 3 |

M = 5

5 12 18 20

0 2 3 0

freq

0 → 2 ⇒ stop

2 → 1

3 → 1

sum → ?

↓

Linear Search in ps array

you will notice