

Project Report

1. Task 3 - Maximum Likelihood In this task, we will use probability distribution. we shall assume a Gaussian distribution with a mean equal to the value $y(x, w)$. We can also use maximum likelihood to determine the precision parameter β of the Gaussian conditional distribution. Maximizing above equation with respect to β gives:

2.

$$\ln p(t|x, w, \beta) = \left(\frac{\beta}{2}\right) \sum (y(\mathbf{x}_n) - t_n)^2 + \left(\frac{N}{2}\right) \ln \beta - \left(\frac{N}{2}\right) \ln 2\pi.$$

Differentiating with respect to w , we get

$$\frac{d}{dw} (E(w)) = \left(\frac{1}{2}\right) (2X^T X - 2X^T t)$$

$$\frac{d}{dw} (E(w)) = X^T X w - X^T t$$

equate this to 0:

$$w^* = (X^T X)^{-1} - X^T t$$

If we differentiate above equation with respect to β we will get:

$$\left(\frac{1}{\beta}\right) = \left(\frac{1}{N}\right) \sum (y(\mathbf{x}_n) - t_n)^2.$$

Having determined the parameters w and β , we can now make predictions for new values of x . We will see below the width of shaded region varies for different values of M .

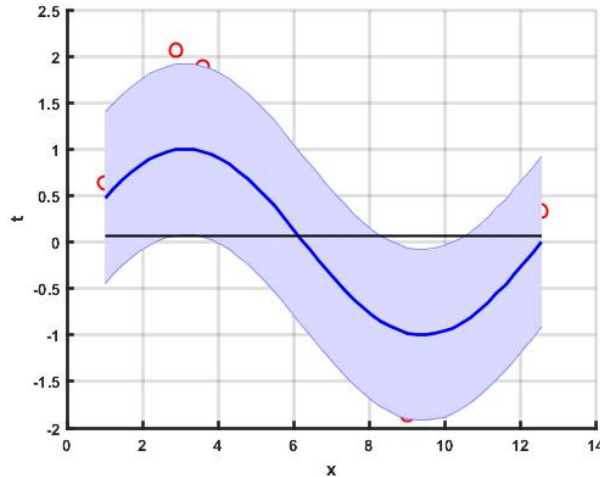


Figure 1: Case 1: ($M = 0$) $w^* = [-0.0501]$

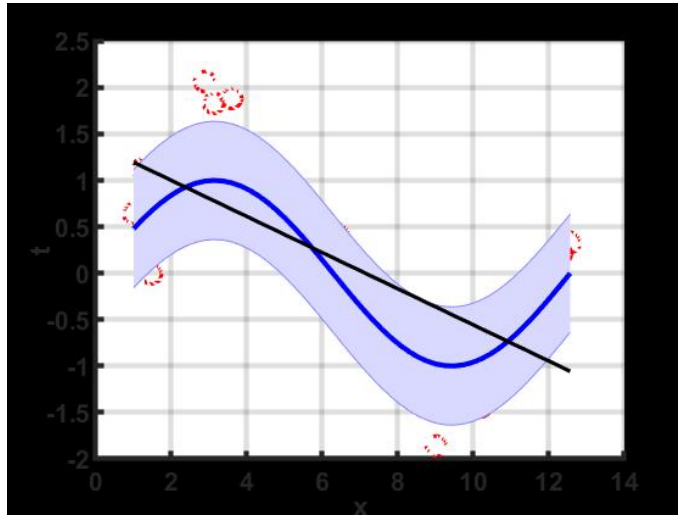


Figure 2: Case 2: ($M = 1$) $w^* = [1.2014, -0.1848]$

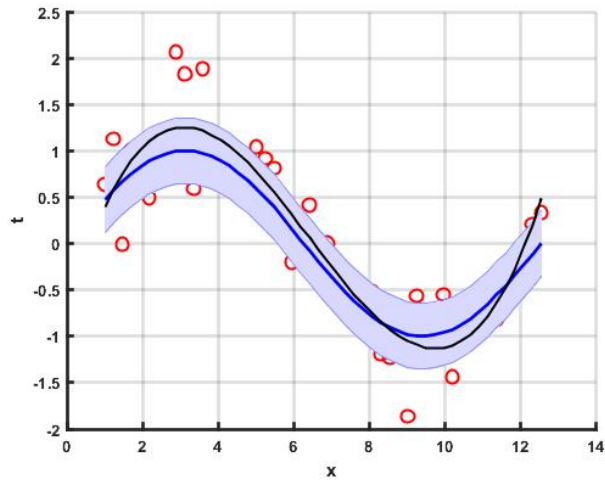


Figure 3: Case 3: ($M = 3$) $w^* = [0.1922, 0.7061, -0.1813, 0.0099]$

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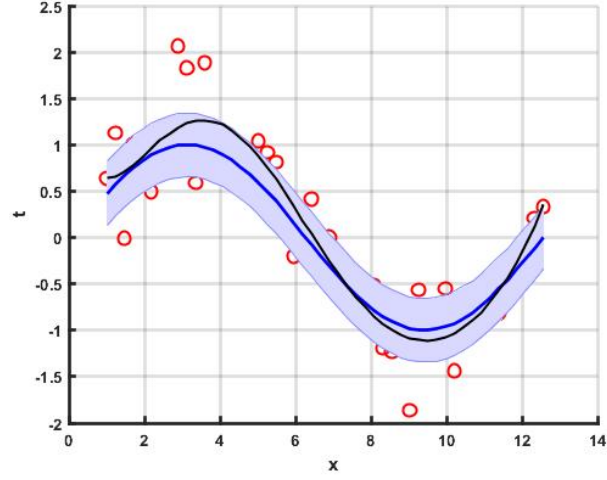


Figure 4: Case 4: ($M = 6$) $w^* = [-0.3129, 1.6050, -0.7302, 0.1619, -0.0209, 0.0014, -0.0000]$

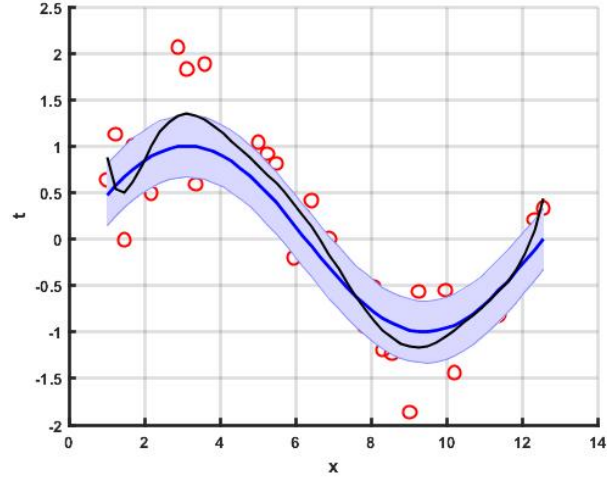


Figure 5: Case 5: ($M = 9$) $w^* = [-9.4270, 25.7526, -25.9357, 13.8811, -4.3654, 0.8423, -0.1007, 0.0073, -0.0003, 0.0000]$

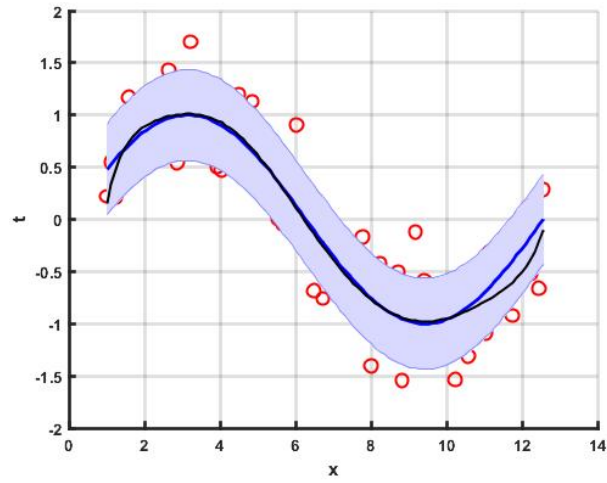


Figure 6: Case 6: ($N=100$)

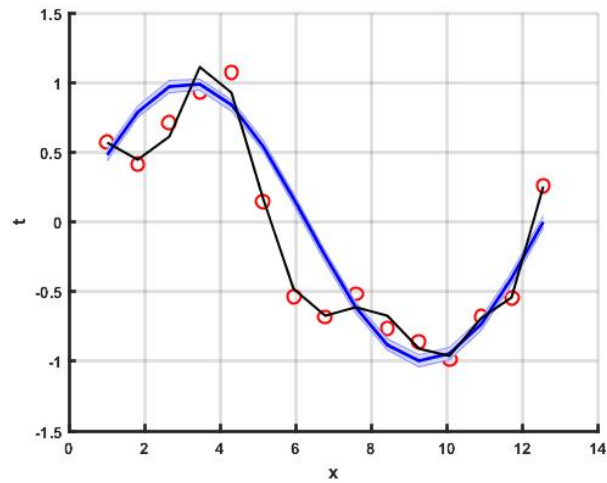


Figure 7: Case 7: ($N=15$, We can see, for $N = 15$ we have very little precision. this is because M and N are almost same so curve is trying to over-fit)