BT6270 Computational Neuroscience Assignment 3

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A Pair of Hopf Oscillators Coupled Through Real Coupling

When two Hopf oscillators with equal natural frequencies are coupled through real coupling coefficients, the system is described by the following equations:

$$\dot{z}_1 = z_1\left(\mu + i\omega_1 - |z_1|^2\right) + W_{12}\text{real}(z_2) \quad \text{(3a)}$$
$$\dot{z}_2 = z_2\left(\mu + i\omega_2 - |z_2|^2\right) + W_{21}\text{real}(z_1) \quad \text{(3b)}$$

In these equations, z_1 and z_2 are complex variables representing the states of the oscillators. W_{12} and W_{21} are real coupling coefficients from the second oscillator to the first and vice versa. The phase difference $(\phi_1 - \phi_2)$ between the oscillators is constrained to either phase-locking at 0 or $(2n + 1)\pi$ depending on the polarity of the coupling and the initial conditions.

A Pair of Hopf Oscillators With Complex Coupling

When two Hopf oscillators with identical natural frequencies are coupled bilaterally through complex coefficients with Hermitian symmetry, the system is described by the following equations:

$$\dot{z}_1 = z_1\left(\mu + i\omega - |z_1|^2\right) + Wz_2 \quad \text{(4a)}$$
$$\dot{z}_2 = z_2\left(\mu + i\omega - |z_2|^2\right) + W^*z_1 \quad \text{(4b)}$$

Here, z_1 and z_2 are complex variables in polar coordinate system representation ($z = re^{i\phi}$). W is the complex coupling coefficient with magnitude A and angle θ. The phase difference $(\phi_1 - \phi_2)$ reaches a steady state at $2n\pi + \theta$ depending on the initial conditions.

COMPLEX COUPLING

A. $\omega_1 = \omega_2 = 5$, phase difference to be achieved = 47°

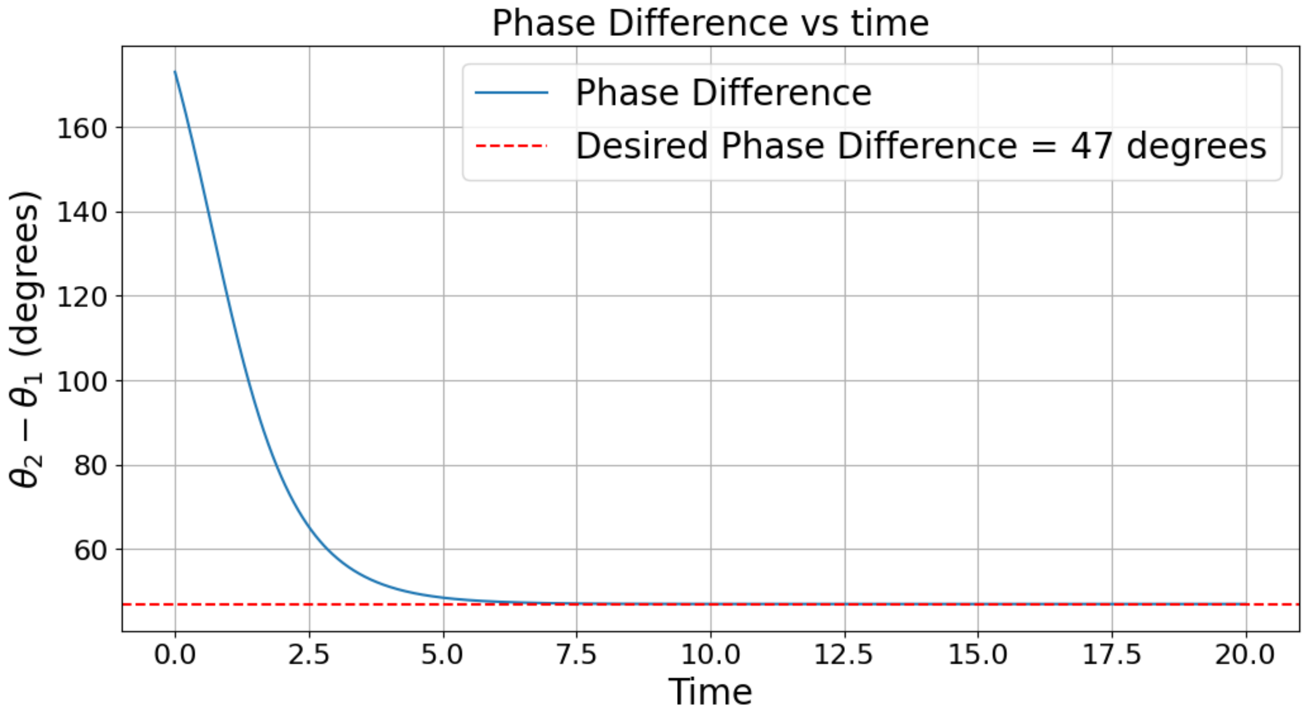

Figure 1: phase difference almost converges to 47° after time t=5 for A=0.5
coupling coefficients $W12 = 0.5e^{i47^\circ}$ = 0.341+0.366i and $W21 = 0.5e^{-i47^\circ}$=0.341-0.366i

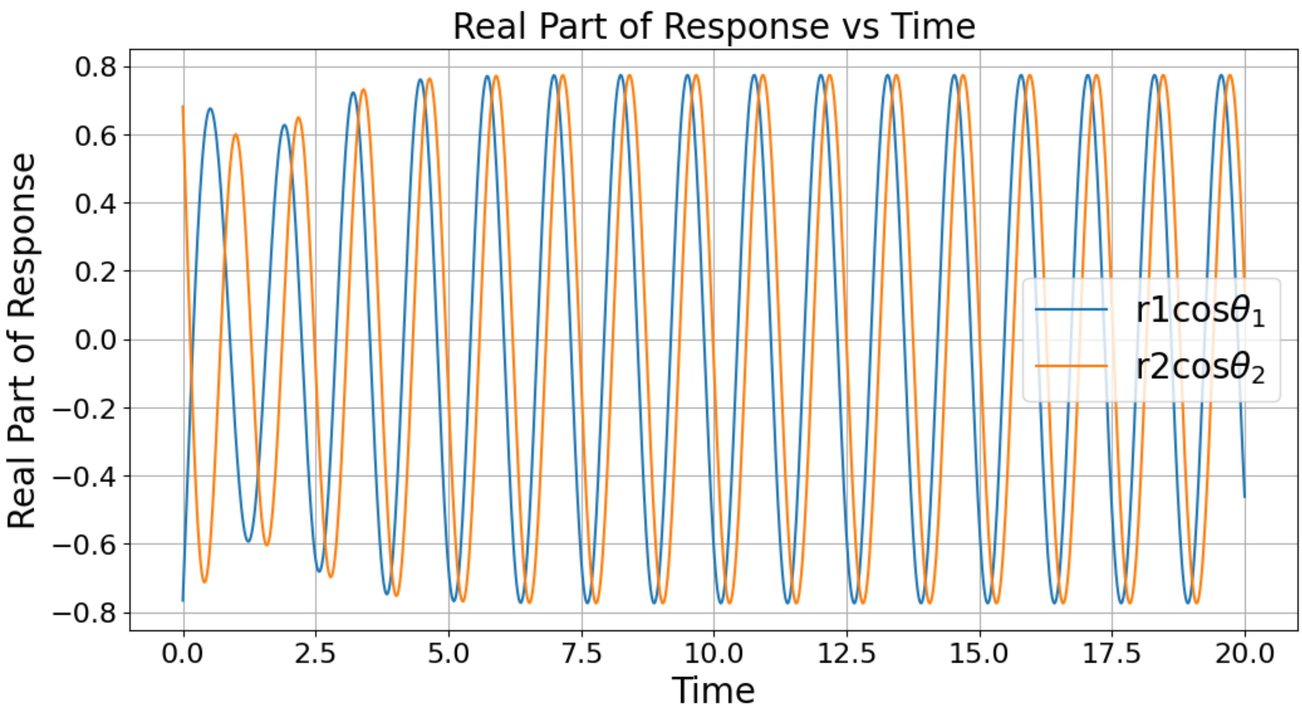

Figure 2: real part response of two hopf oscillators with complex coupling of phase difference=47°

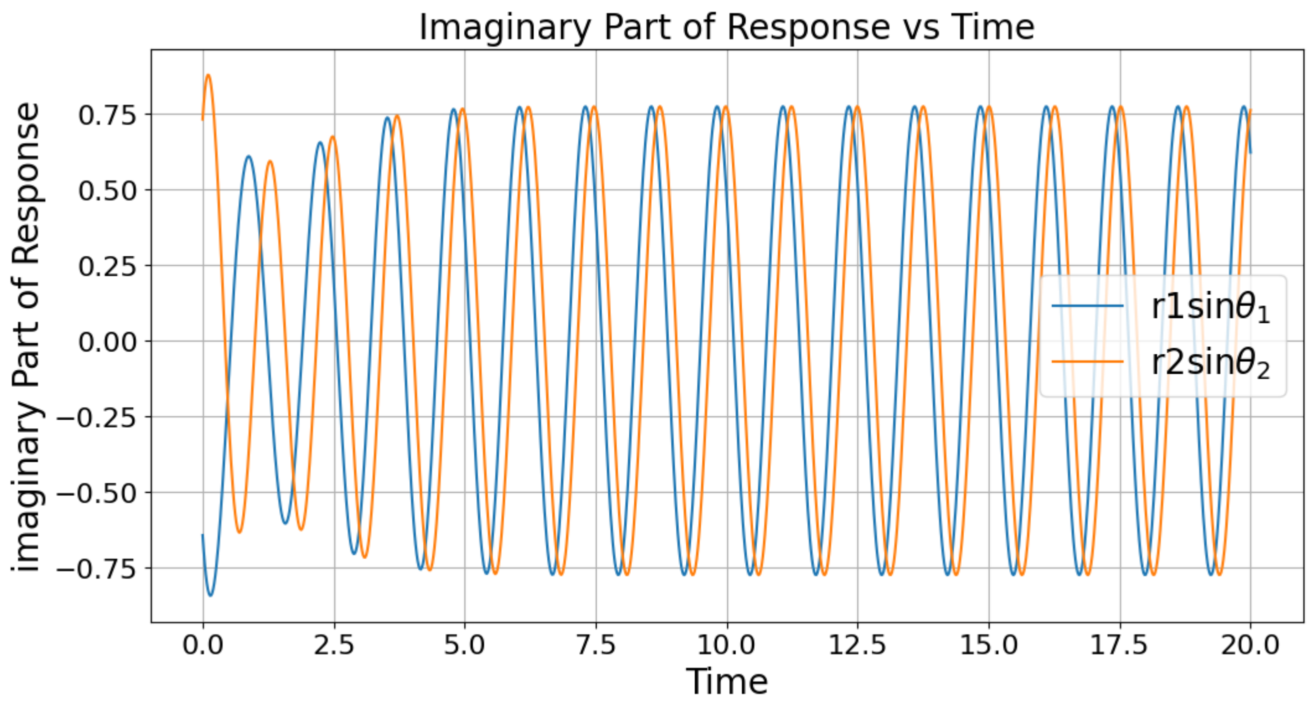

Figure 3: imaginary part response of two hopf oscillators with complex coupling of phase difference=47°

B. $\omega_1 = \omega_2 = 5$, phase difference to be achieved = 98°

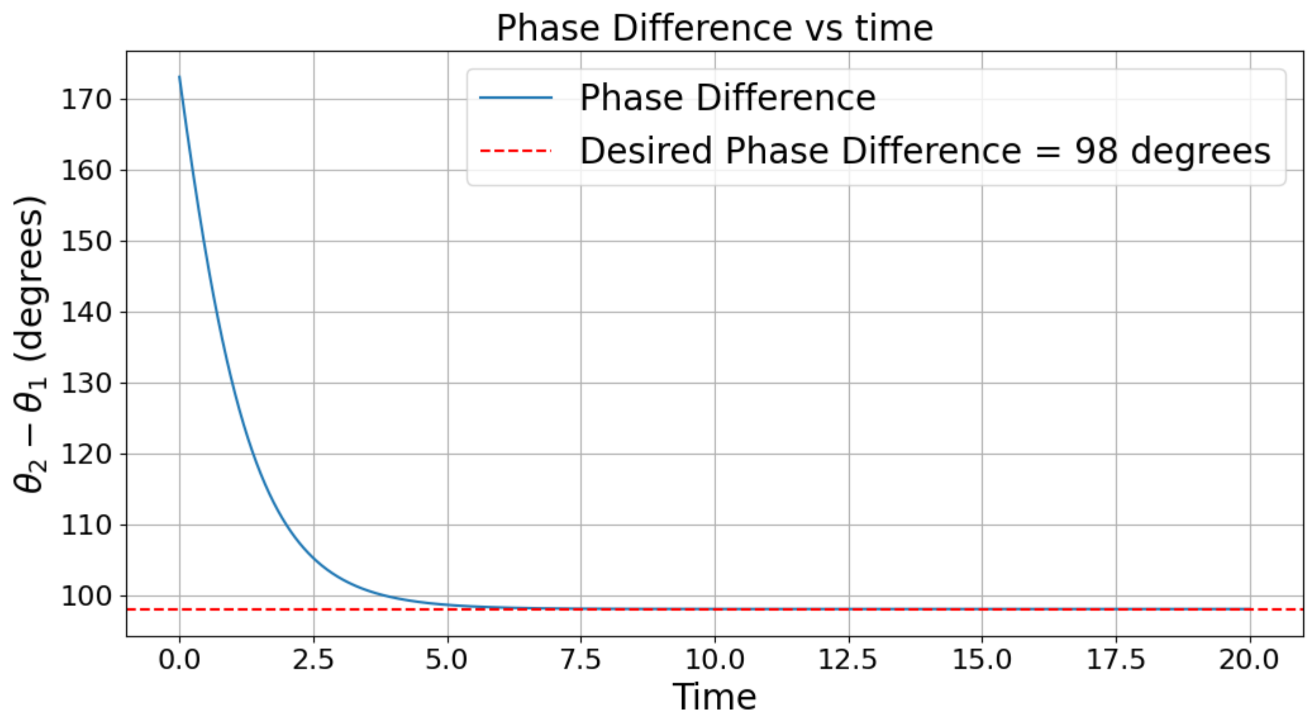

Figure 4: phase difference almost converges to 98° after time t=5 for A=0.5. coupling coefficients $W12 = 0.5e^{i98^\circ}$ = 0.070+0.495i. and $W21 = 0.5e^{-i98^\circ}$=0.070-0.495i

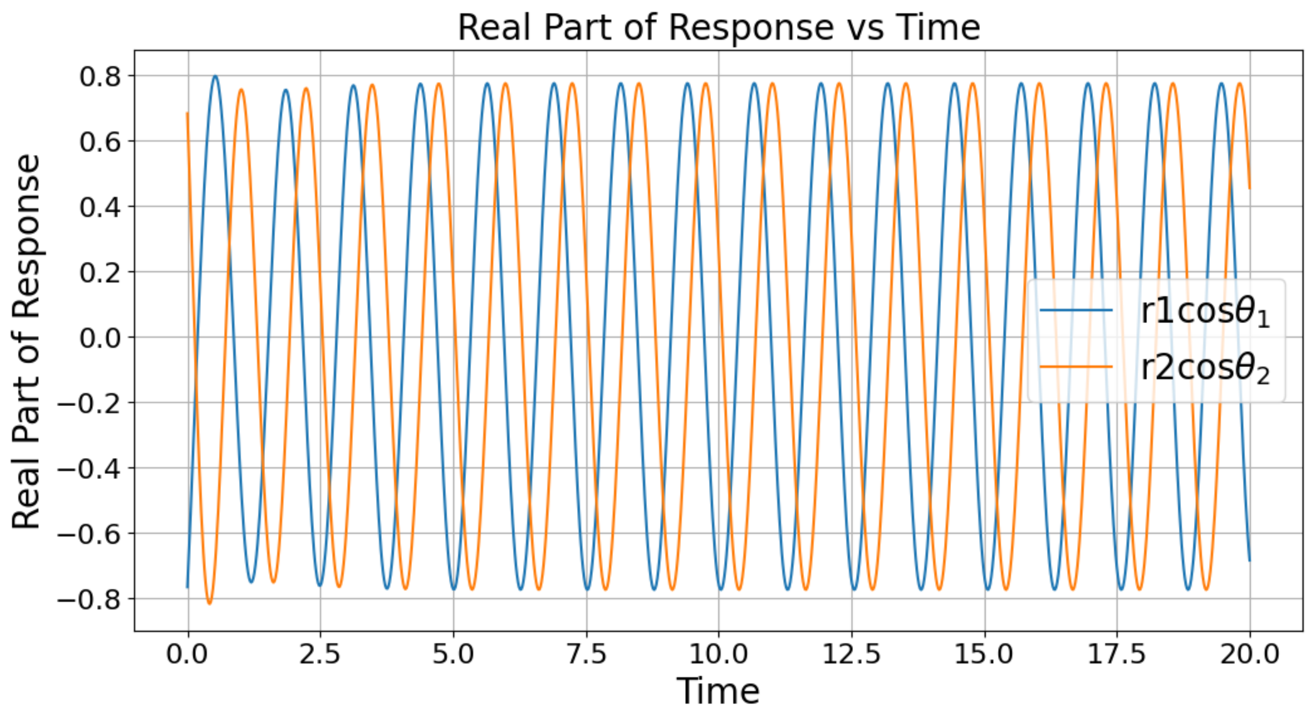

Figure 5: real part response of two hopf oscillators with complex coupling of phase difference=98°

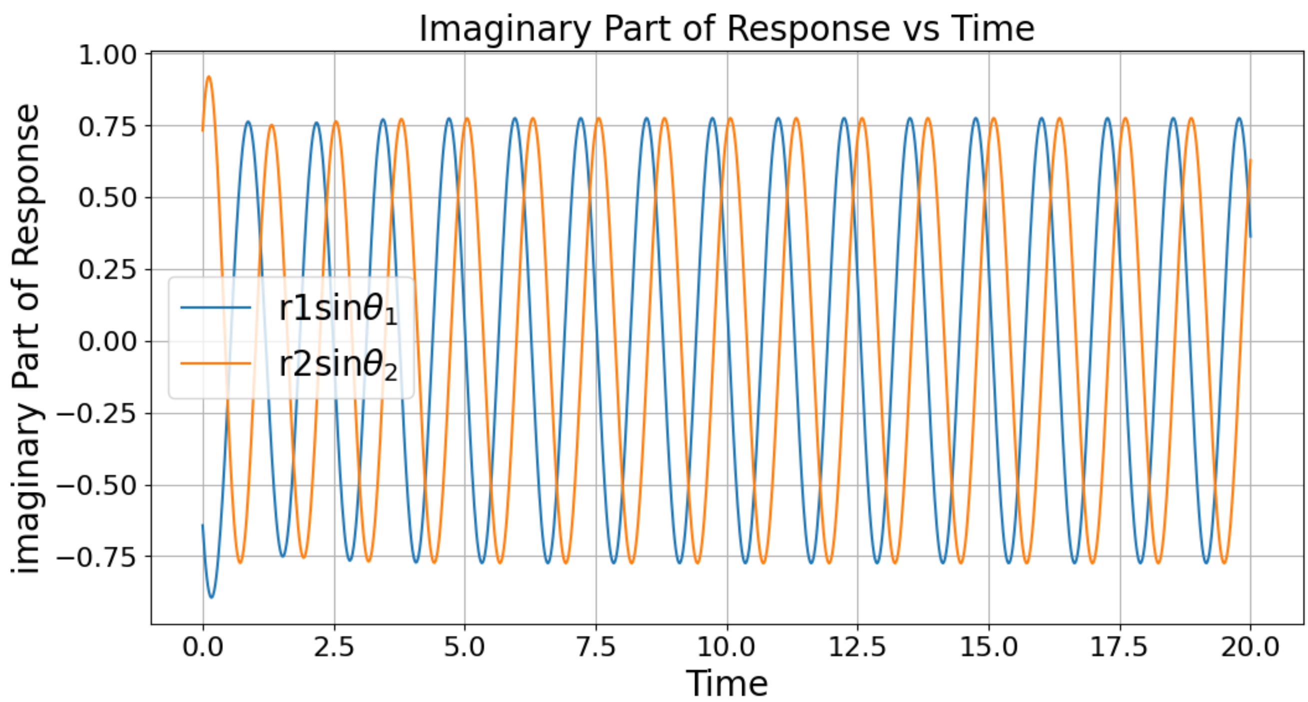

Figure 6: imaginary part response of two hopf oscillators with complex coupling of phase difference=98°

Power Coupling

In the context of coupling sinusoidal oscillators, specifically Hopf or Kuramoto oscillators, the concept of "power coupling" introduces a complex coupling coefficient that facilitates entrainment at a specific normalized phase difference. The dynamics of such coupled oscillators are governed by the following equations:

$$\dot{z}_1 = z_1(\mu + i\omega_1 - |z_1|^2) + Ae^{i(\theta/\omega_2)} z_2^{(\omega_1/\omega_2)}$$
$$\dot{z}_2 = z_2(\mu + i\omega_2 - |z_2|^2) + Ae^{-i(\theta/\omega_1)} z_1^{(\omega_2/\omega_1)}$$

Here, z_1 and z_2 are complex variables representing the amplitudes of the oscillators, μ is a constant determining the fixed point of the uncoupled system, ω_1 and ω_2 are the natural frequencies of the oscillators, A is the amplitude of the power coupling, and θ is the angle of the complex power coupling coefficient. The weights $Ae^{i(\theta/\omega_2)}$ and $Ae^{-i(\theta/\omega_1)}$

signify the strength and phase of the power coupling from oscillator 2 to oscillator 1 and from oscillator 1 to oscillator 2, respectively. These equations capture the interdependence between the oscillators, their intrinsic properties, and the influence of the complex power coupling. The behavior of the coupled system, including the emergence of synchronized states, can be explored through the dynamics of these equations.

C. $\omega_1 = 5, \omega_2 = 15$, *phase difference to be achieved = 47°*

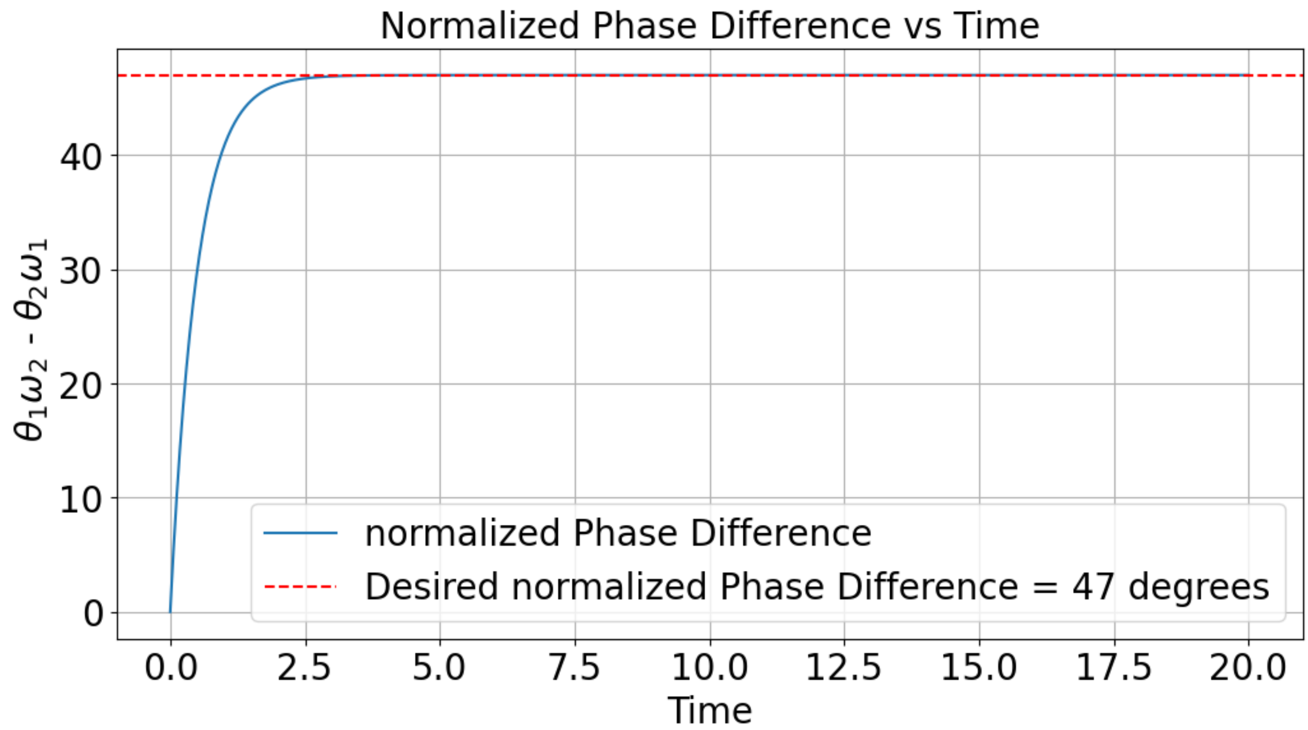

Figure 7: phase difference almost converges to 47° after time t=2.5 for A=1
coupling coefficients $W12 = e^{i47^{\circ}/15}$ =0.9985+0.0547i, and $W21 = e^{-i47^{\circ}/5}$=0.9870.163i

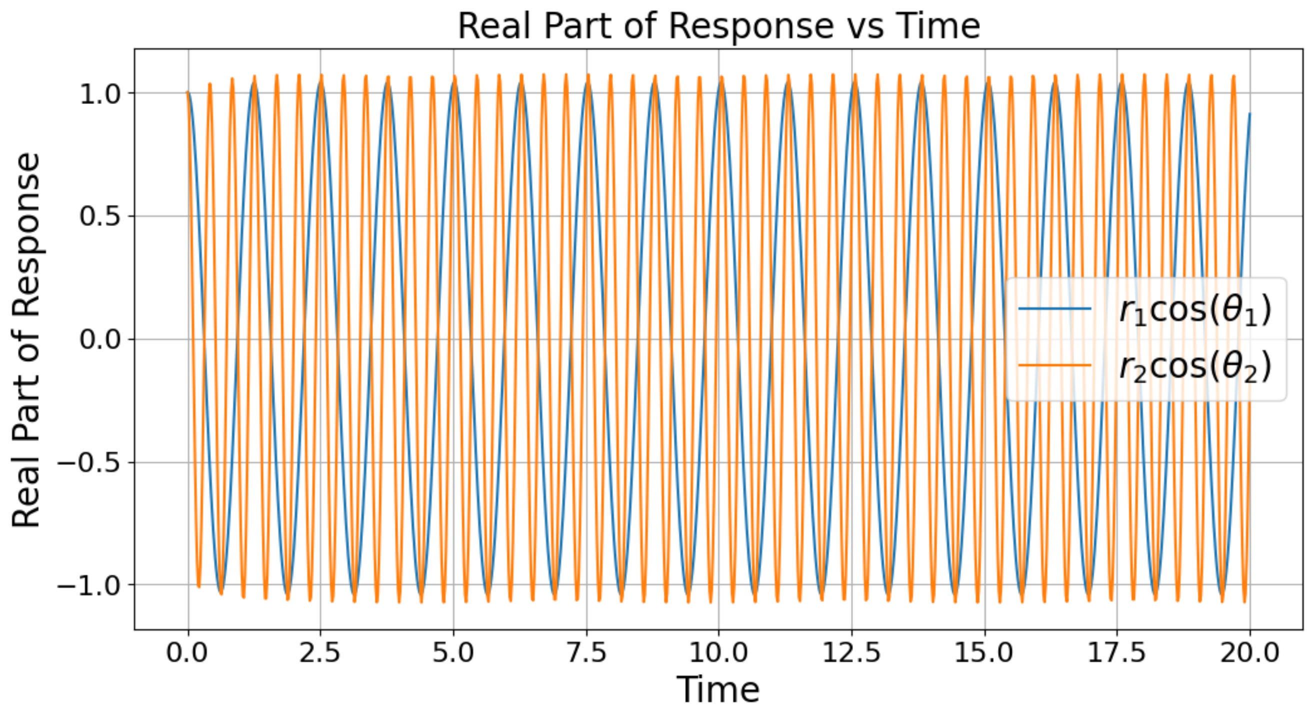

Figure 8: real part response of two hopf oscillators with power coupling of phase difference=47°

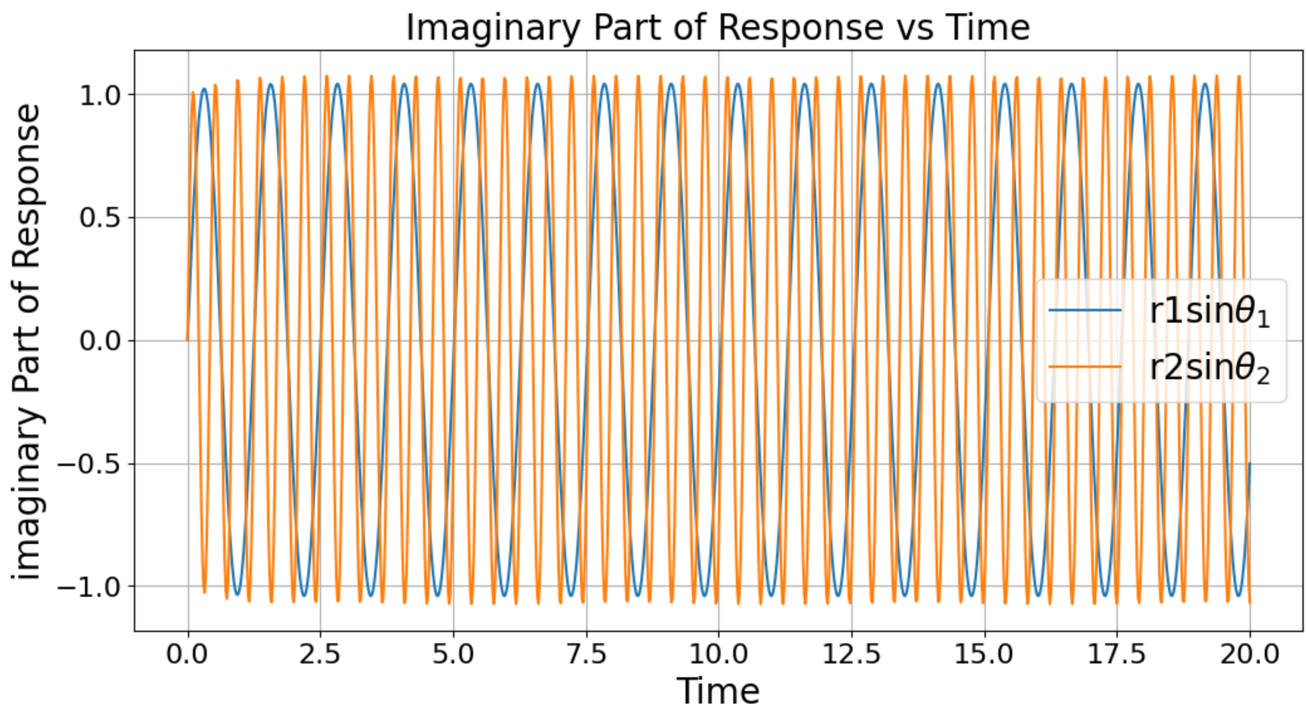

Figure 9: Imaginary part response of two hopf oscillators with power coupling of phase difference=47°

D. $\omega_1 = 5, \omega_2 = 15$*, phase difference to be achieved = 98°*

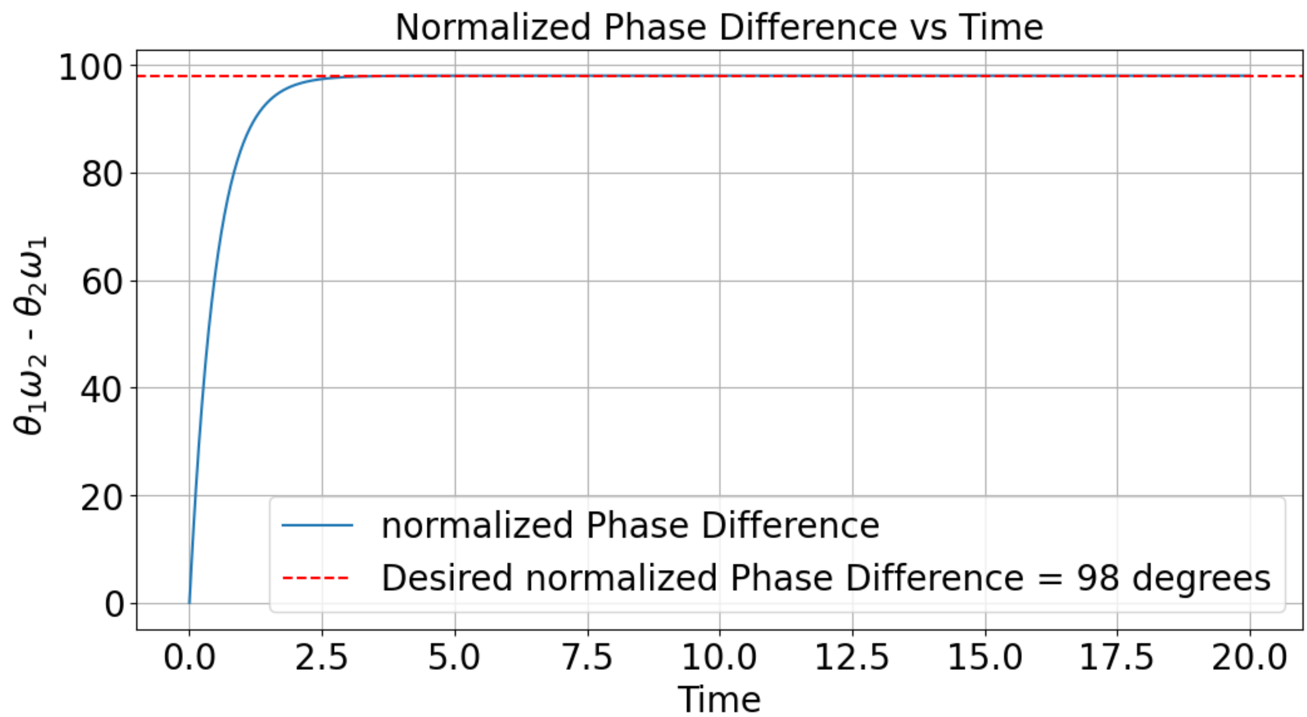

Figure 10: phase difference almost converges to 47° after time t=2.5 for A=1
coupling coefficients $W12 = e^{i98^\circ/15}$ =0.994+0.114i, and $W21 = e^{-i98^\circ/5}$=0.9420.335i

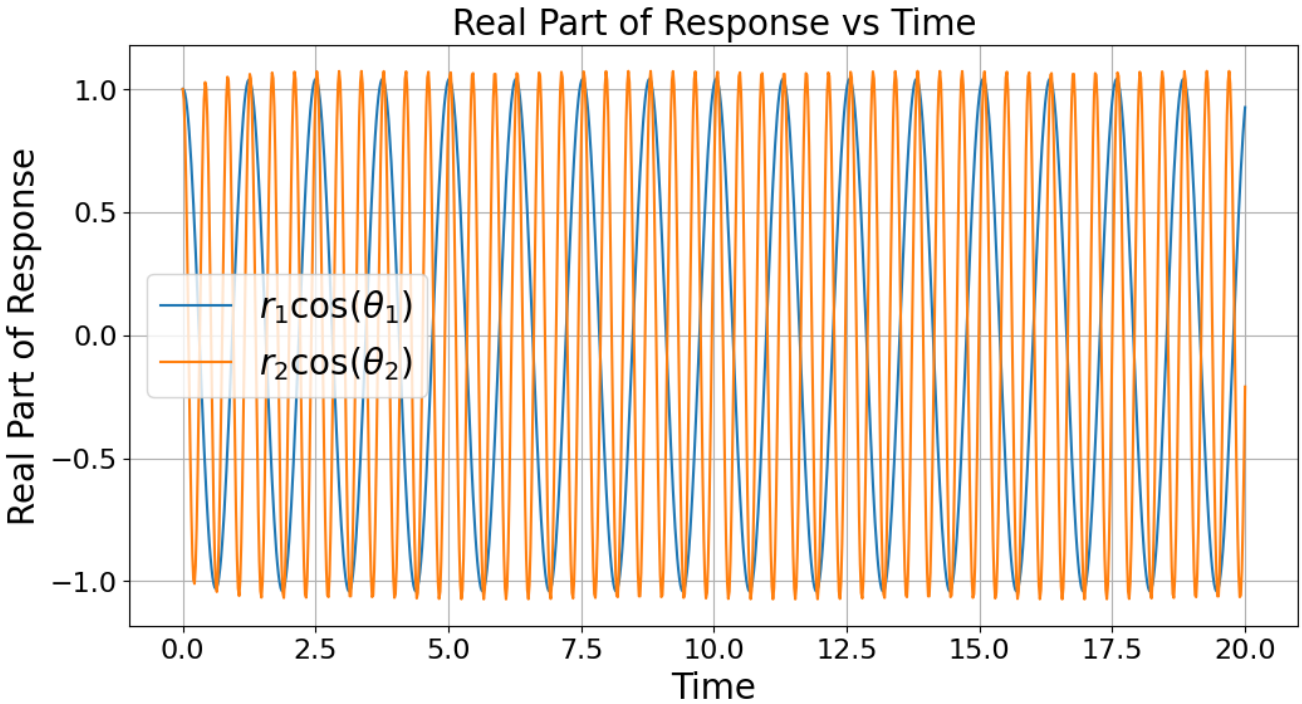

Figure 11: real part response of two hopf oscillators with power coupling of phase difference=98°

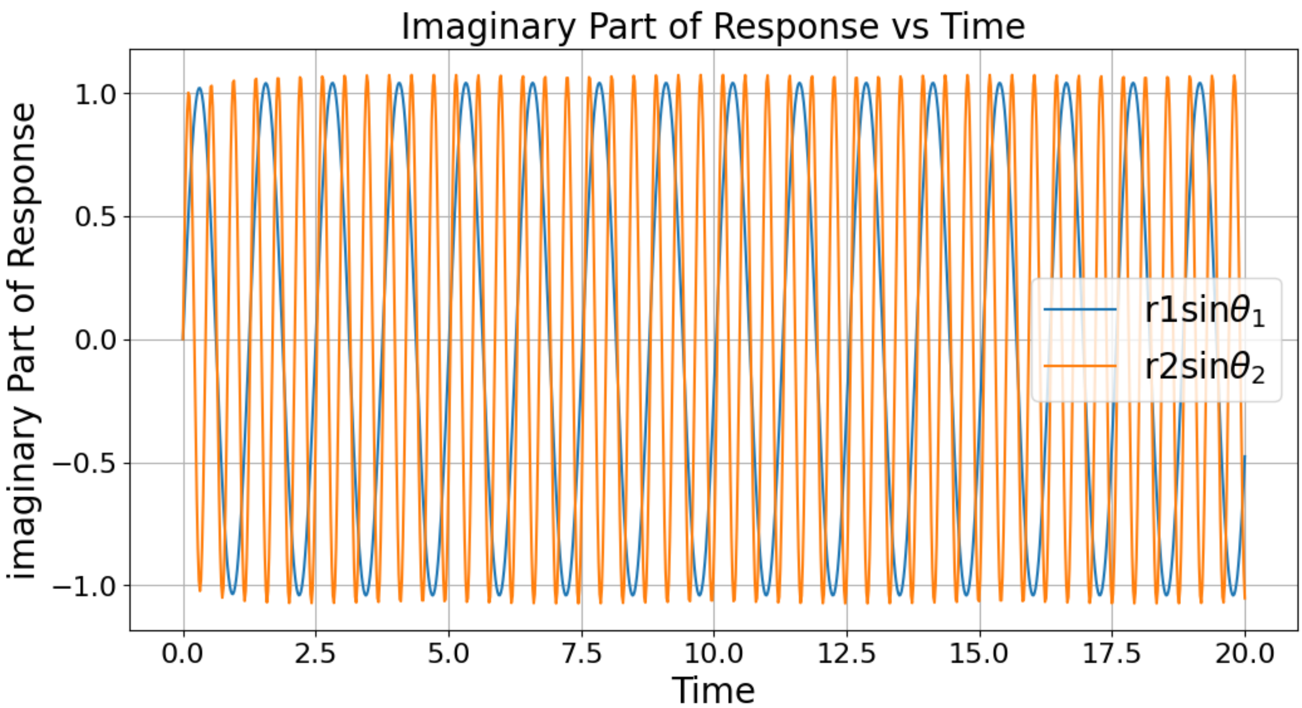

Figure 12: Imaginary part response of two hopf oscillators with power coupling of phase difference=98°