BT6270 Computational Neuroscience Assignment 2

Kaja Vikas | ED20B026

Department of Engineering Design,

Indian Institute of Technology, Madras

Introduction:

The Fitzhugh-Nagumo model is a simplified mathematical framework used to describe the dynamics of excitable systems, particularly in the context of neuronal activity. Introduced independently by Richard Fitzhugh in 1961 and John Nagumo in 1962, this model serves as a valuable tool for understanding the spiking behavior exhibited by neurons. It consists of a pair of coupled differential equations capturing the interplay between the membrane potential and a recovery variable. By elucidating the mechanisms underlying excitability, the Fitzhugh-Nagumo model has found wide application in neuroscience and related fields.

The Fitzhugh-Nagumo model is described by the following set of differential equations:

$$\frac{dv}{dt} = f(v) - w + I_{ext}$$

$$\frac{dw}{dt} = bv - rw$$

where:

v represents the membrane potential of the neuron.

w represents a recovery variable associated with the neuron's refractory period.

f(v) is a nonlinear function that characterizes the voltage-dependent gating mechanisms. It is defined as f(v)=v(a−v)(v−1).

I_{ext} denotes an external current applied to the neuron.

a, b, and r are parameters that influence the behavior of the model.

This system of equations captures the essential features of neuronal excitability. The variable v undergoes rapid changes, known as action potentials, while w regulates the recovery period following each spike. The interplay between these variables, influenced by the external current I_{ext}, gives rise to a rich array of dynamical behaviors, making the Fitzhugh-Nagumo model a cornerstone in the study of neuronal dynamics.

1 Case 1: I_{ext}=0

A) Phase plot

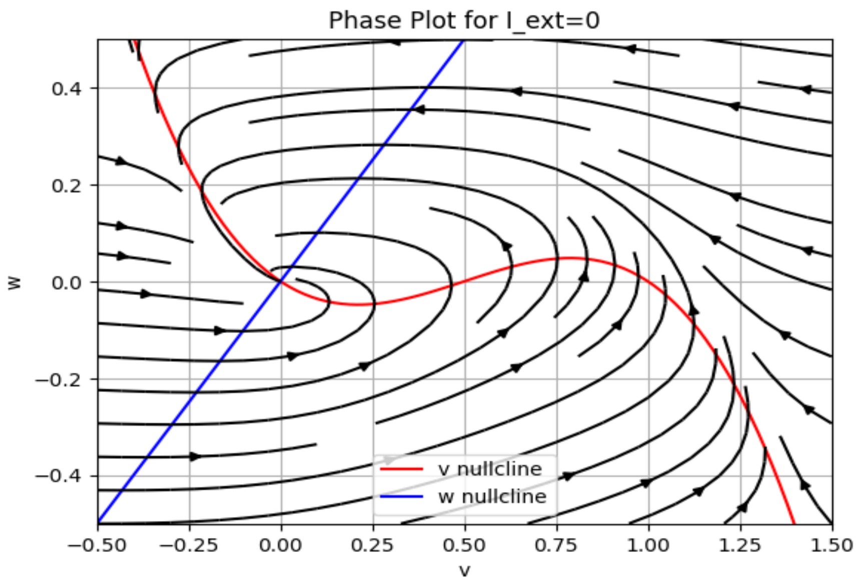

Figure 1: Phase Plot of the system when I_{ext} = 0. The stationary point obtained is a stable point.

observing the trajectories by using initial points v(0) = [0, 0.4, 0.6], w(0) = 0, we can see that even for smaller perturbations in the initial start point, we return to the fixed point at [0, 0]. Hence, the point [0, 0] is a stable fixed point.

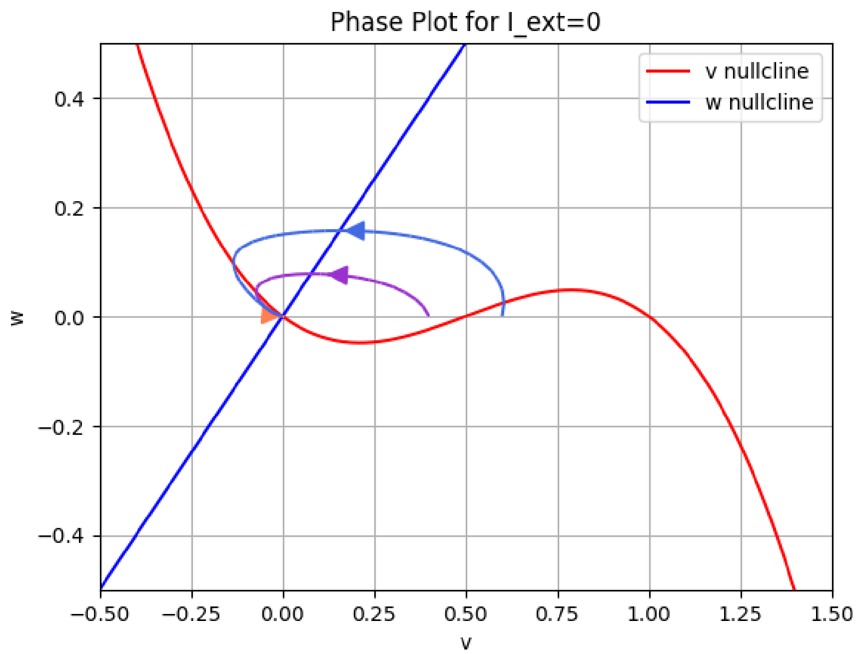

Figure 2: The model approaches the fixed point irrespective of the initial conditions. Hence, the fixed point is a stable fixed point

B)

For a I_{ext} = 0, no action potentials are observed.

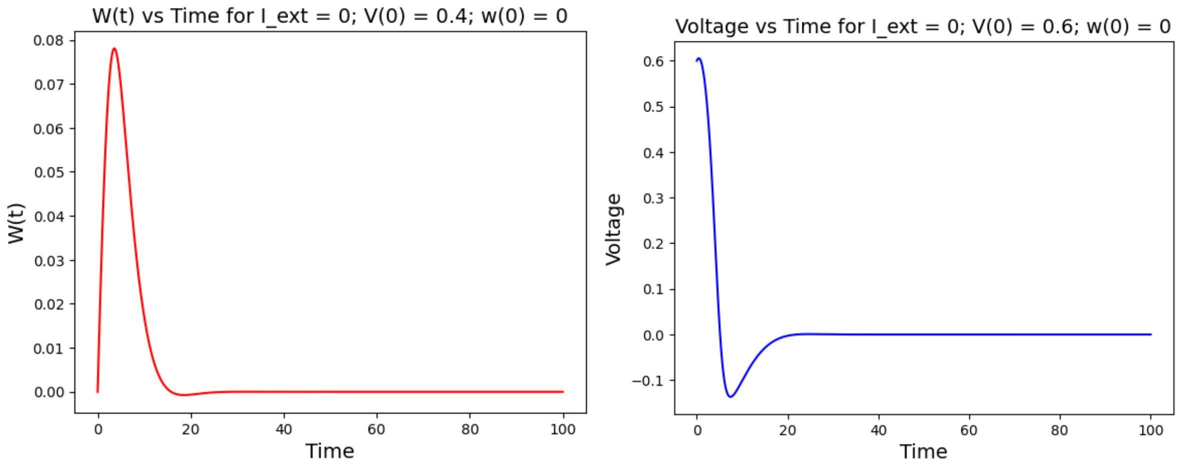

Figure 3: V (t), W(t) across t, when V (0) < a. With sub-threshold pulse injections, no action potentials are observed.

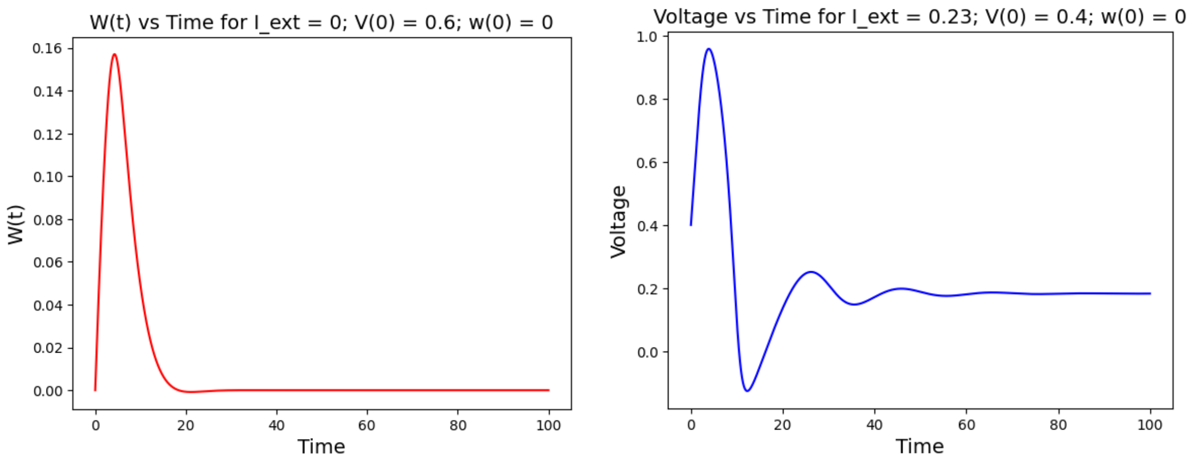

Figure 4: V (t), W(t) across t, when V (0) < a. With sub-threshold pulse injections, no action potentials are observed.

2 Case 2: $I_1 < I_{ext} < I_2$

To find I_1 & I_2, the I_{ext} was varied such that after I1, there were oscillations and after I2 there are no oscillations. So, I_1 is 0.23 and I_2 is 0.80.

A) I_1 **&** I_2 **plots; V(0) < a i.e., V (0) = 4**

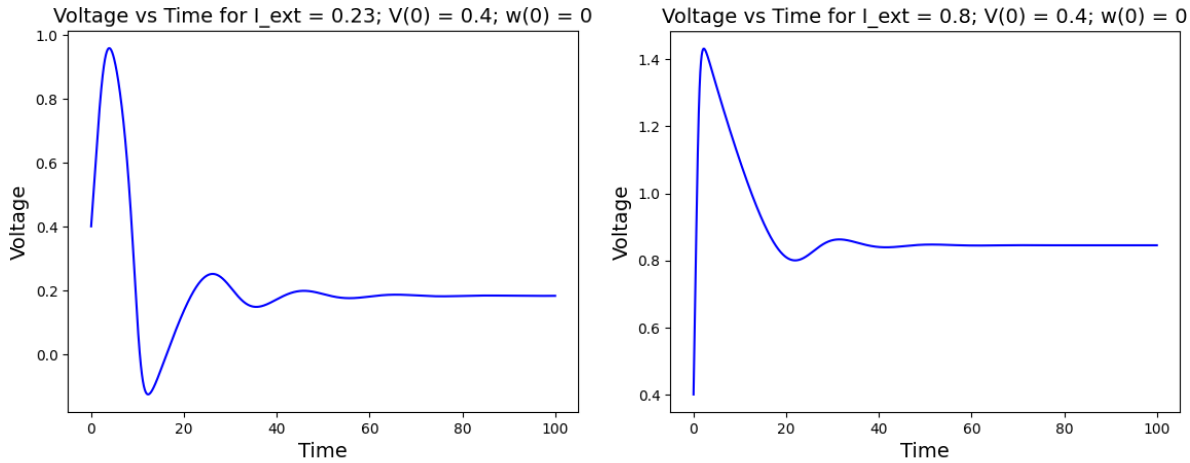

Figure 5: V(t) vs t at I_{ext} = 0.23 and I_{ext}=0.80

The figures 5 shows the plots at I_1 = 0.23mV & I_2= 0.80mV respectively at the V (0) = 0.4 mV. So, after 0.23 mV, there will oscillations and after 0.80 mV, there will be no oscillations at all.

B) Phase plot for I_{ext} = 0.6

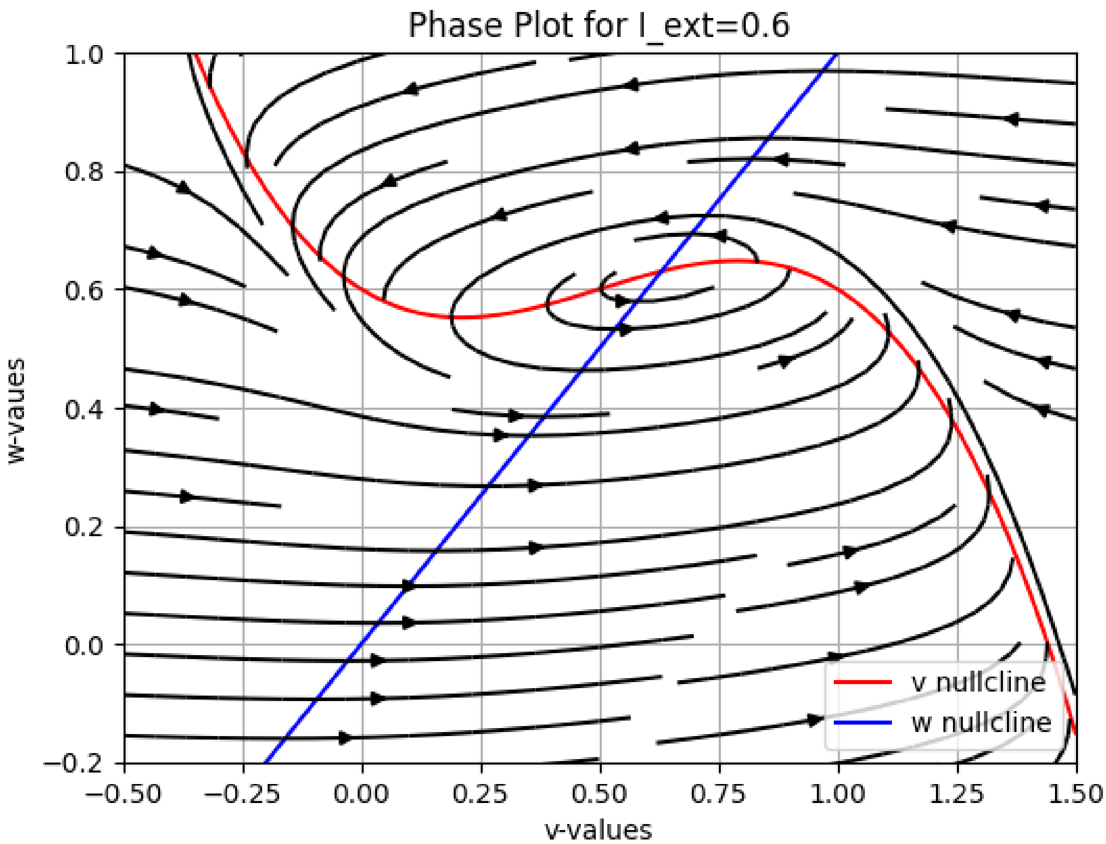

Figure 6: Phase Plot of the system when I_{ext} = 0.6. This fixed point is found to be an unstable point.

observing the trajectories by using initial points [v(0), w(0)] = `[0.75,0.8]`, `[0.4,0.4]`, `[0.6,0.4]`, `[0.6,0.8]`. We can see that at the point of intersection of the nullclines, there are circulating fields around the unstable fixed point. we also see limit cycle enclosing the fixed point.

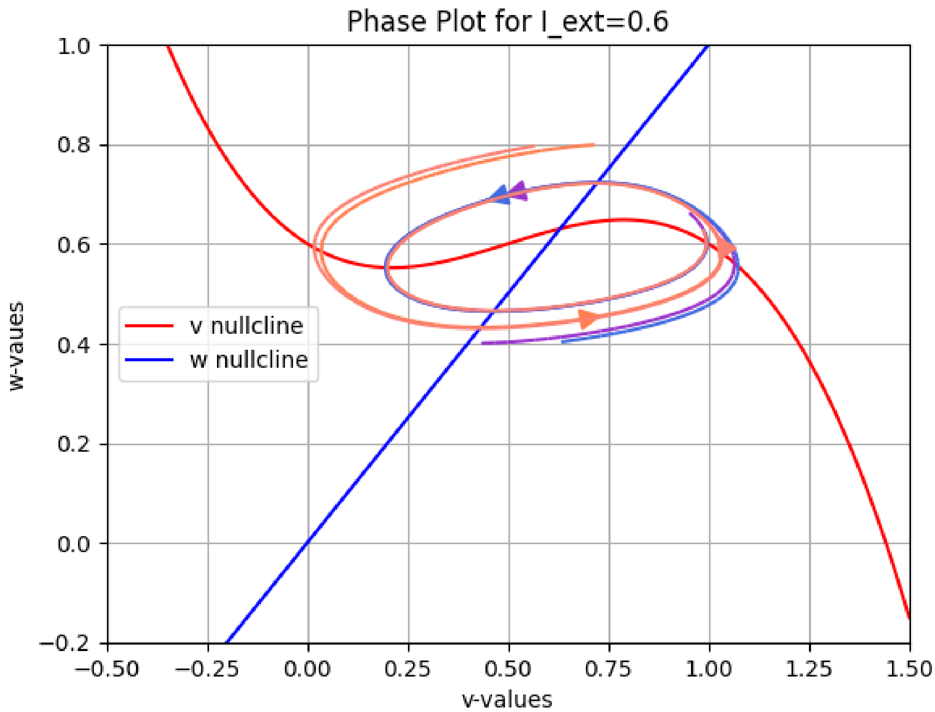

Figure 7: The stationary point is found to be unstable because for smaller perturbations it never returned to fixed point and limit cycle behavior is also observed.

C) V(t), W(t) across t

For I_{ext} = 0.6, oscillatory membrane potential is seen in the limit cycle region.

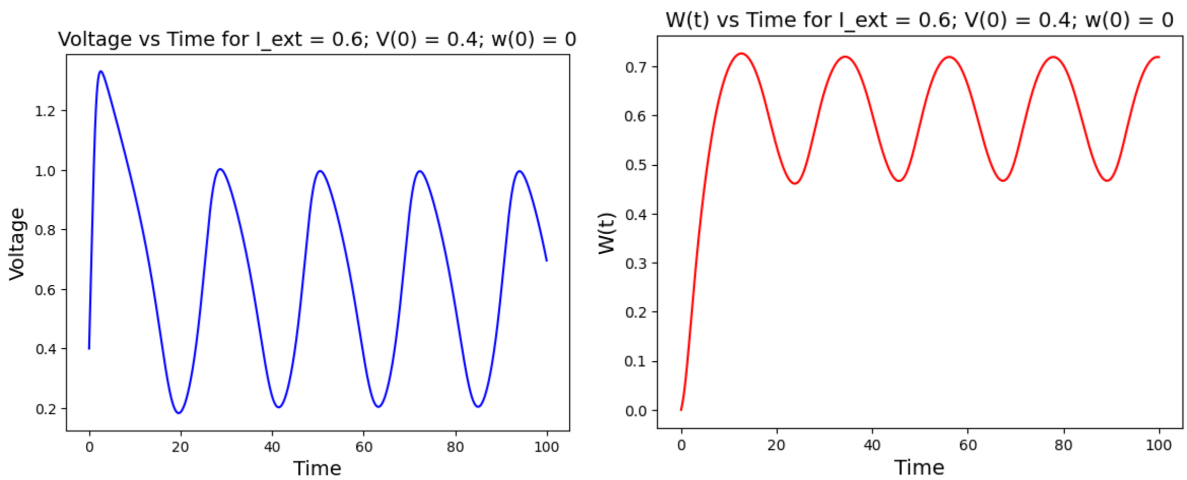

Figure 8: V (t), W(t) across t, when V (0) < a. Sustained oscillations are observed.

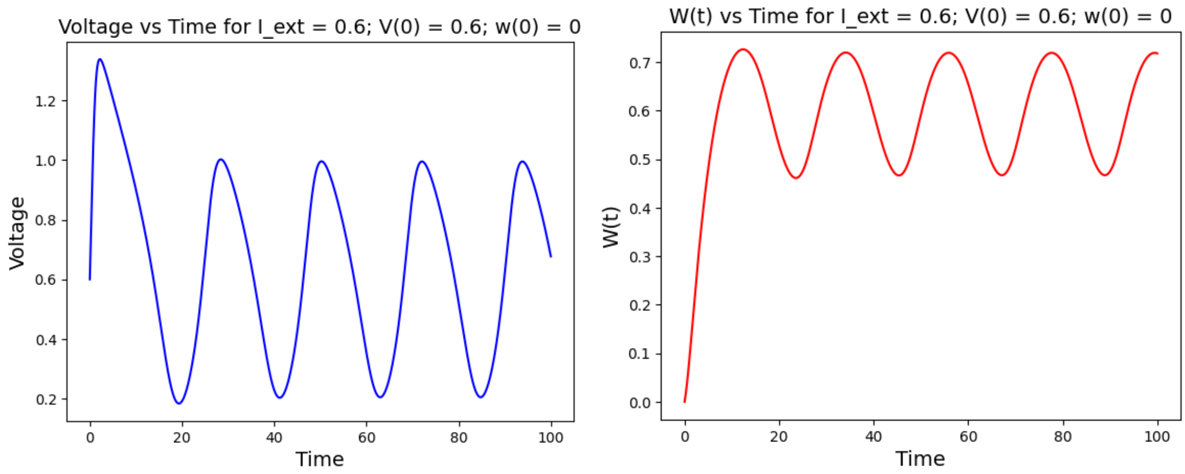

Figure 9: V (t), W(t) across t, when V (0) > a. Sustained oscillations are observed.

3 Case 3:$I_{ext} > I_2$,I_{ext}=1

A) Phase Plot

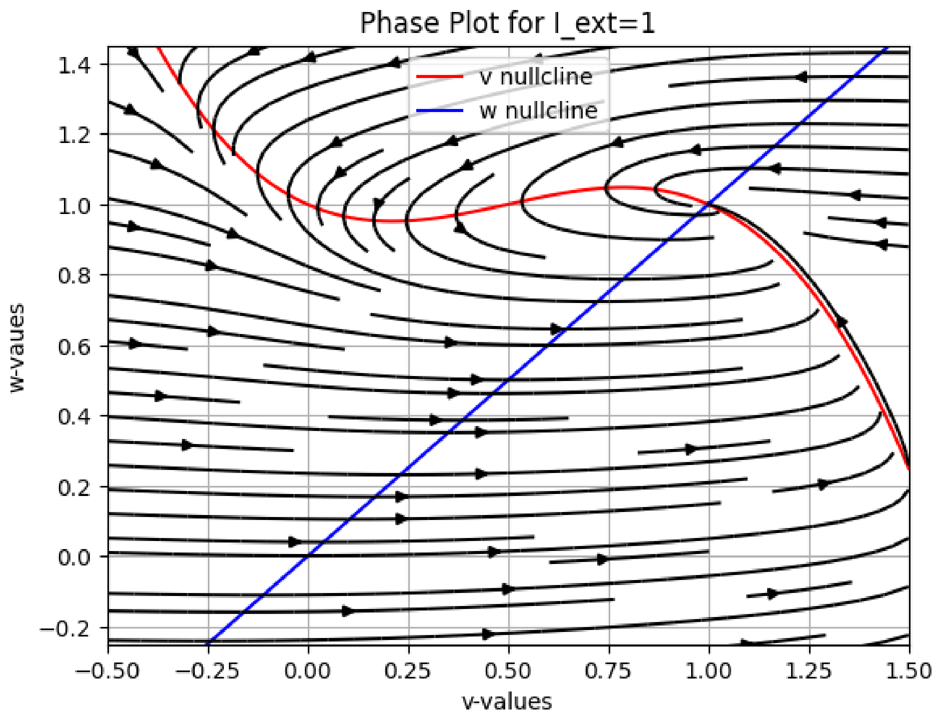

Figure 10: Phase Plot of the system when I_{ext} = 1. This fixed point is found to be a stable point

The trajectories were observed by using initial points [V (0), W (0)] = `[1.2,1]`, `[0.8,1]`, `[1,0.8]`, `[1, 1.2]`. We can see that even for small perturbations in the initial start point, we approach the fixed point at [1, 1]. Hence, the point [1, 1] is a stable fixed point.

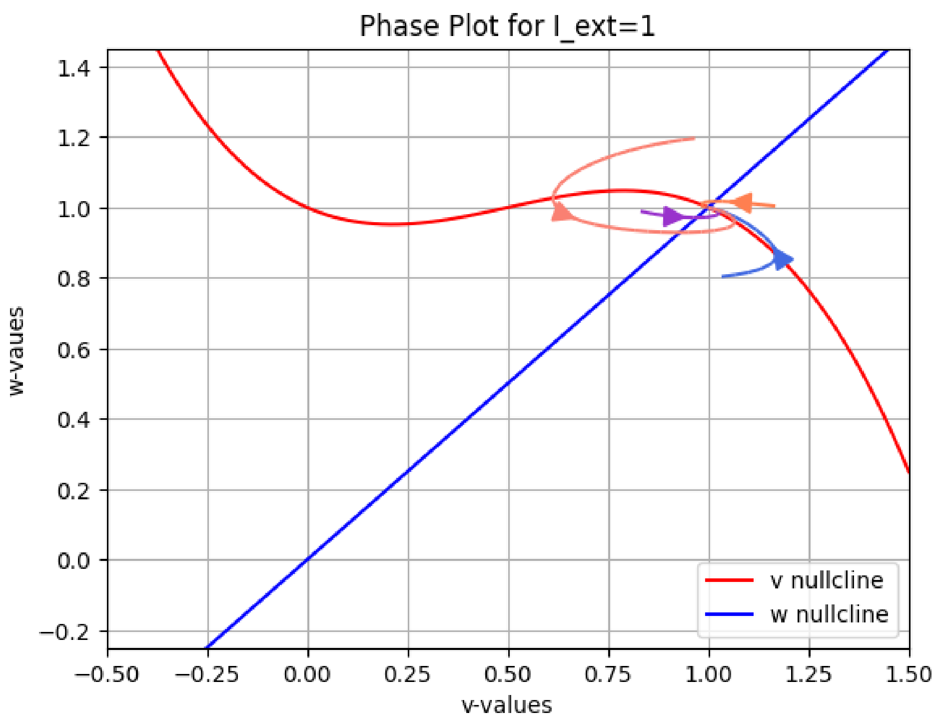

Figure 11: This fixed point is found to be a stable point.

B) V (t), W(t) across t

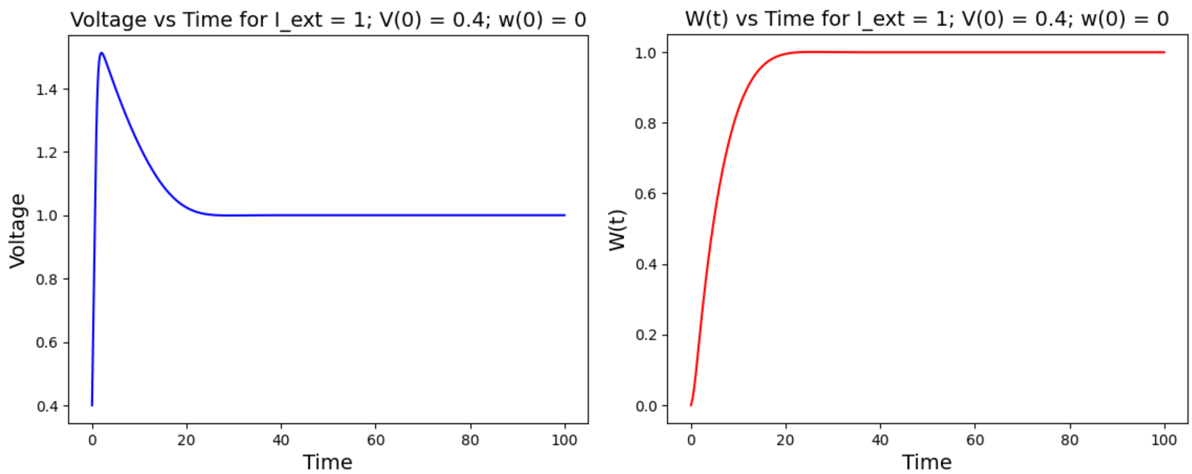

Figure 12: V (t), W(t) across t, when V (0) < a. With sub-threshold pulse injections, depolarization in the action potential can be observed.

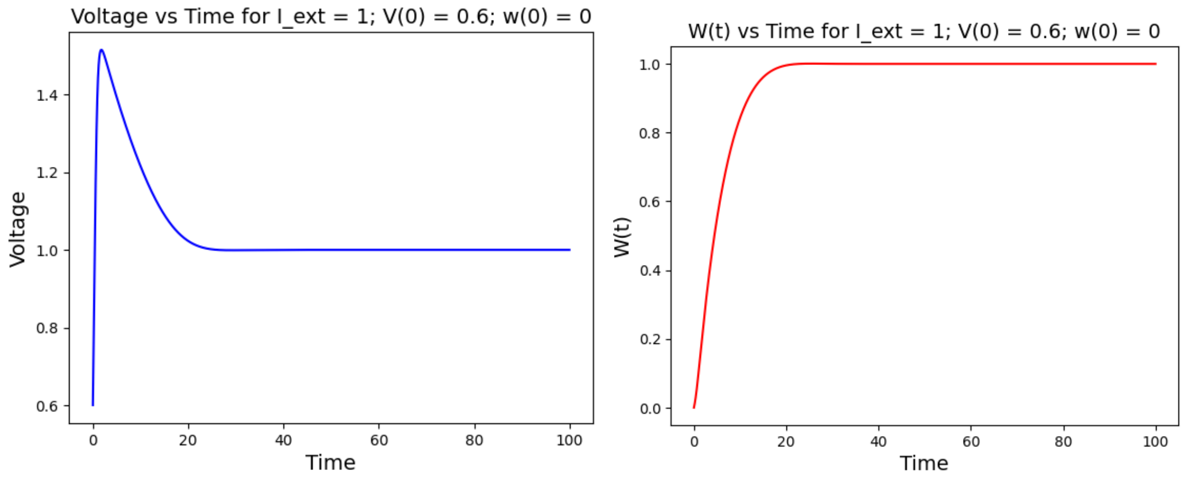

Figure 13: V (t), W(t) across t, when V (0) > a. With sub-threshold pulse injections, depolarization in the action potential can be observed.

4) Case 4: I_{ext} = 0.02

The parameter values used to simulate this case are b = 0.02, r = 0.7. Hence, b/r = 0.0285

A) Phase Plot

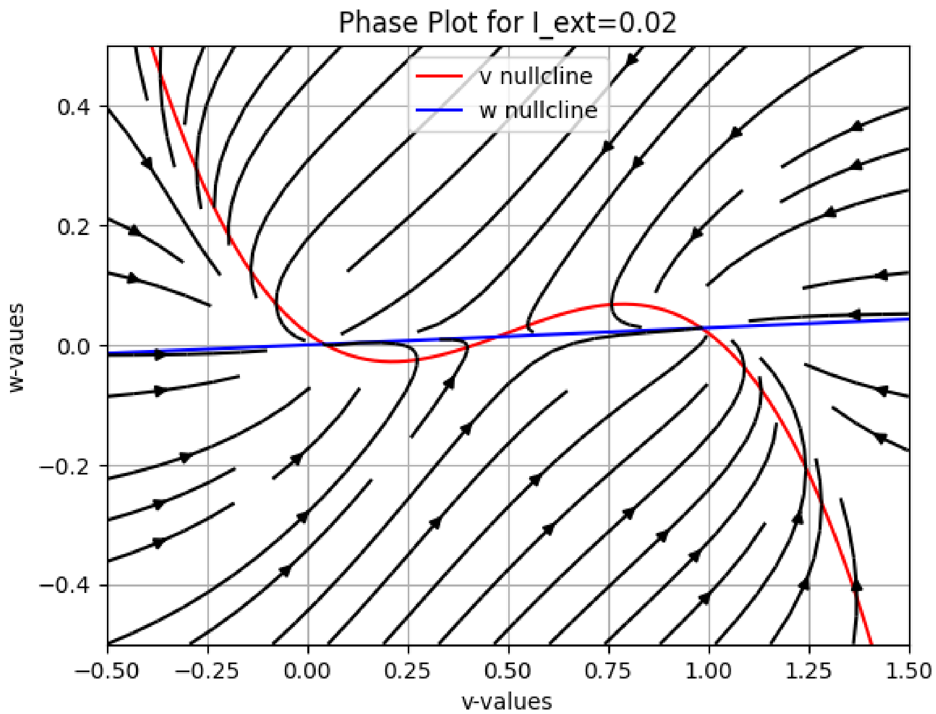

Figure 14: Phase Plot of the system when I_{ext} = 0.02.

The trajectories were observed by using initial points [v(0), w(0)] = `[0,0.4]`, `[0.1,0.4]`, `[-0.1,0.4]`, `[0,-0.4]`, `[0.4,0.4]`, `[0.6, 0.4]`, `[-0.03,-0.4]`, `[1, 0.4]`,`[0.1,-0.4]`, `[-0.1,-0.4]`, `[0.4,-0.4]`, `[0.6, -0.4]`, `[1, -0.4]`. The stationary points are P1, P2 and P3, in that order. In case of P1 and P3 - small and intermediate perturbations lead back to P1 and P3 respectively. Hence P1 and P3 are stable point. In case of P2, small perturbations along one axis leads to large change in final point. Hence, P2 is a saddle node

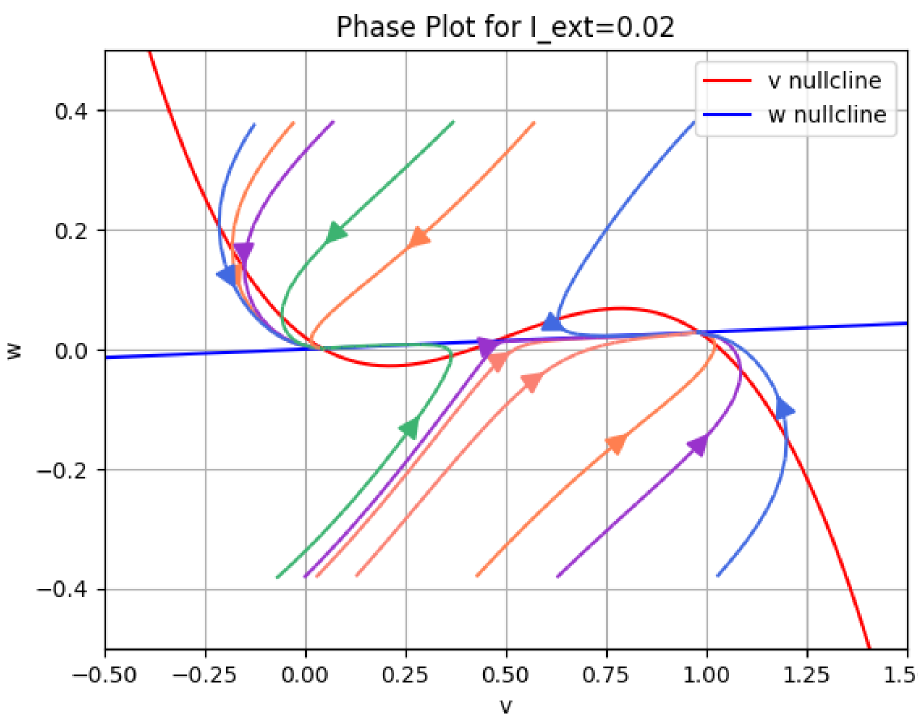

Figure 15: Stability analysis of the stationary point. The stationary points P1, P2 and P3 in that order are stable, saddle and stable points respectively.

B) V (t), W(t) across t

For I_{ext}= 0.02, r = 0.7, b = 0.02, bi-stability is observed.

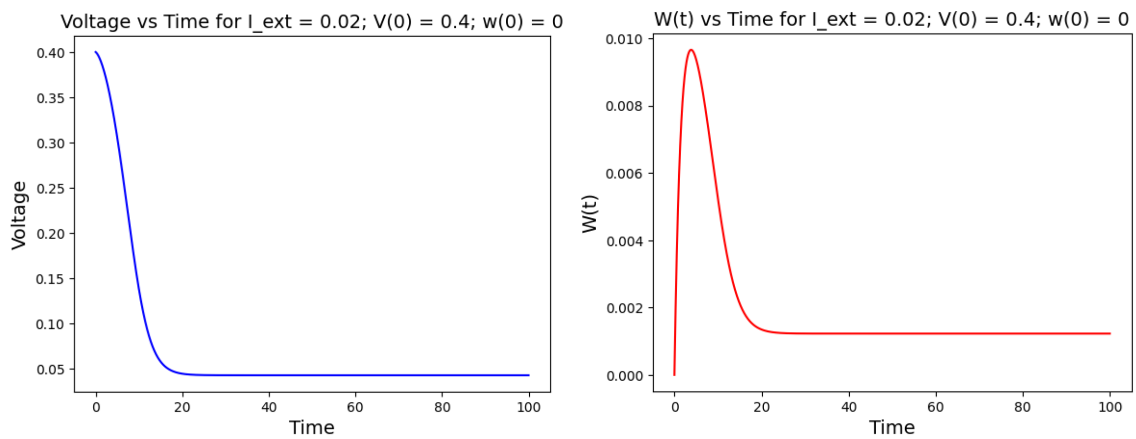

Figure 15: V (t), W(t) across t, when V (0) < a. The neuron exists in a tonically down state.

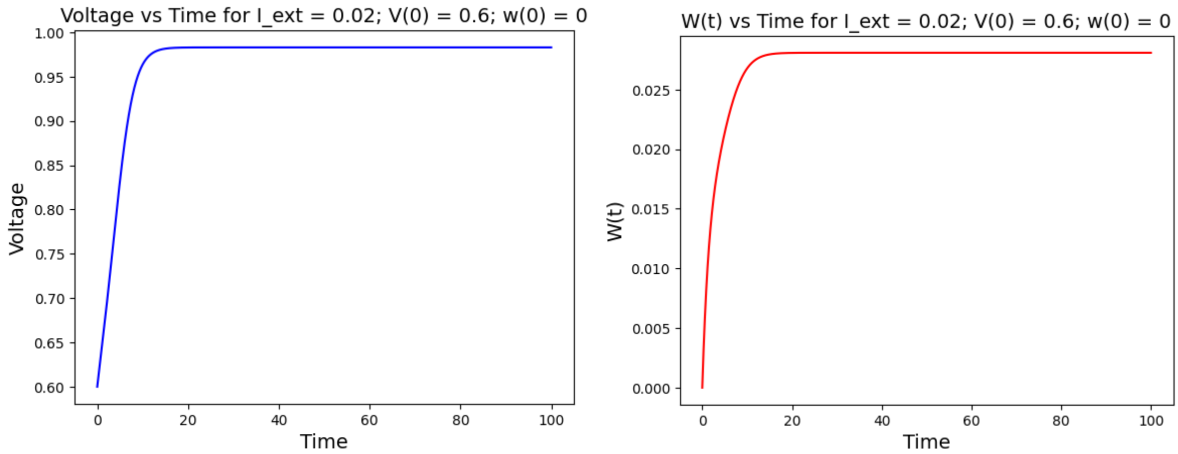

Figure 17: V (t), W(t) across t, when V (0) > a. The neuron exists in a tonically up state

Assumptions:

Parameter Values: The parameters a, b, and r are set to specific values except case 4:

- a=0.5
- b=0.1
- r=0.1

Integration Method: The simulation uses a forward Euler integration method with a time step size of dt=0.01.

Simulation Time: The simulation is run for T=100 units of time.