

Assignment

Vikeath — EE22BTECH11035

Question 9.3.13

Find the probability of getting 5 twice in 7 throws of a dice. **Solution:**

Gaussian:

TABLE 1: Variables

Variable	Value	Description
n	7	Number of throws
p	$\frac{1}{6}$	probability of getting five
$\mu = np$	$\frac{7}{6}$	mean of distribution
$\sigma = \sqrt{npq}$	$\sqrt{\frac{35}{216}}$	variance of distribution
X	$0 \leq X \leq 7$	Number of correct questions

$$Y \sim \mathcal{N}(\mu, \sigma^2) \quad (1)$$

CDF of Y is defined as:

$$F_Y(x) = \Pr(Y \leq x) \quad (2)$$

$$= \Pr\left(\frac{Y - \mu}{\sigma} \leq \frac{x - \mu}{\sigma}\right) \quad (3)$$

$$\frac{Y - \mu}{\sigma} \sim \mathcal{N}(0, 1) \quad (4)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{x - \mu}{\sigma}\right) \quad (5)$$

$$(6)$$

therefore the cdf will be:

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y - \mu}{\sigma}\right), & y \geq \mu \\ Q\left(\frac{\mu - y}{\sigma}\right), & y \leq \mu \end{cases} \quad (7)$$

1) Without correction:

$$\Pr(Y = 2) = 1 - \Pr(Y = 2) \quad (8)$$

$$= 1 - F_Y(4) \quad (9)$$

$$\Rightarrow \Pr(Y = 2) = Q\left(\frac{X - \mu}{\sigma}\right) \quad (10)$$

$$= Q(2.07036) \quad (11)$$

$$\Pr(Y = 2) = 0.019209 \quad (12)$$

2) With a 0.5 correction:

$$\Pr(Y = 2) = Q\left(\frac{X - \mu + 0.5}{\sigma}\right) \quad (13)$$

$$= Q(0.82824) \quad (14)$$

$$\Rightarrow \Pr(Y = 2) = 0.20377 \quad (15)$$

Binomial:

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \quad (16)$$

(17)

Probability that 4 or more are correct

$$\Rightarrow P(X = 2) = \sum_{k=2}^5 {}^7C_k \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{7-k} \quad (18)$$

$$= 0.234427244 \quad (19)$$

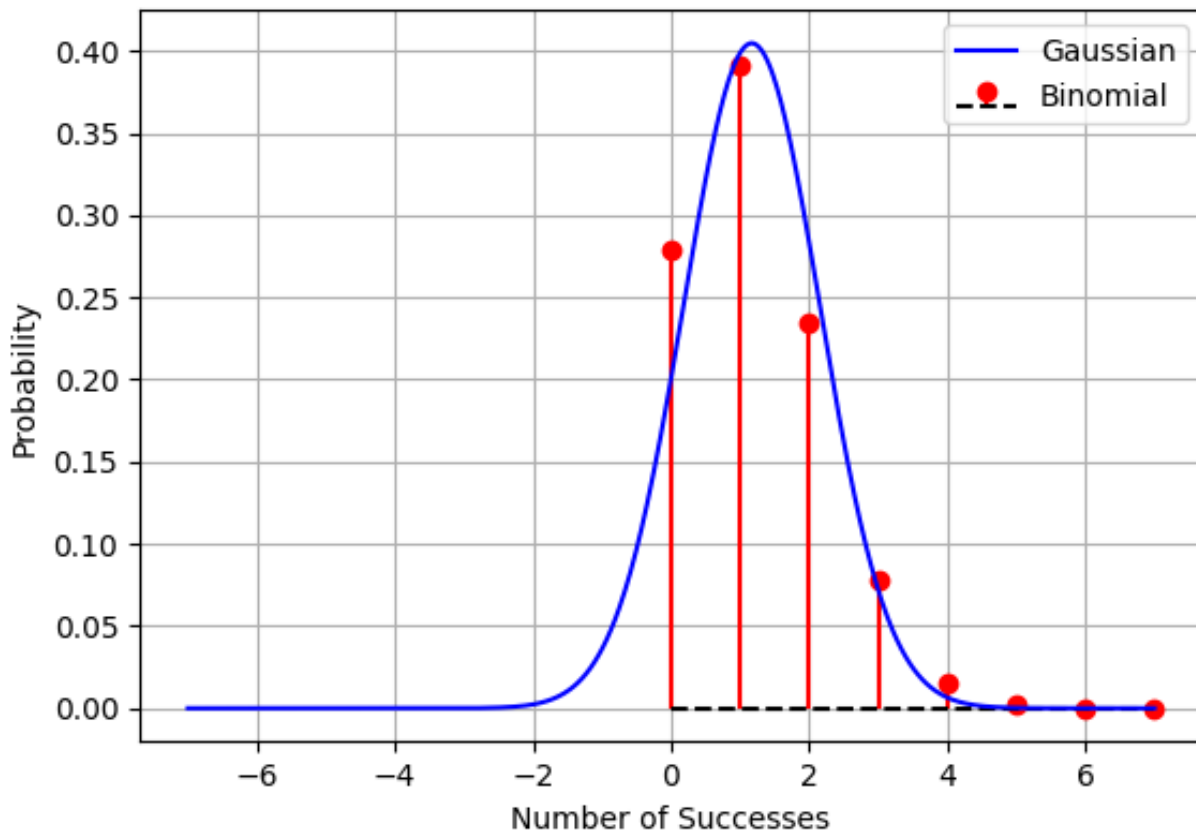


Fig. 1: Binomial vs gaussian