Likelihoods

(It’s all relative)

A likelihood gives the function of a parameter given the data.

Binomial Likelihood:

$$L(\theta) = \frac{n!}{x!\,(n-x)!} \times \theta^{x} \times (1-\ \theta)^{n-x}$$

You flip 8 out of 10 heads. The likelihood of $\theta = 0.8$ is 0.30.

The likelihood of

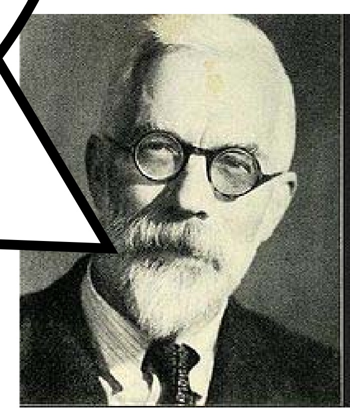
$\theta = 0.7$ is 0.23

$\theta = 0.6$ is 0.12

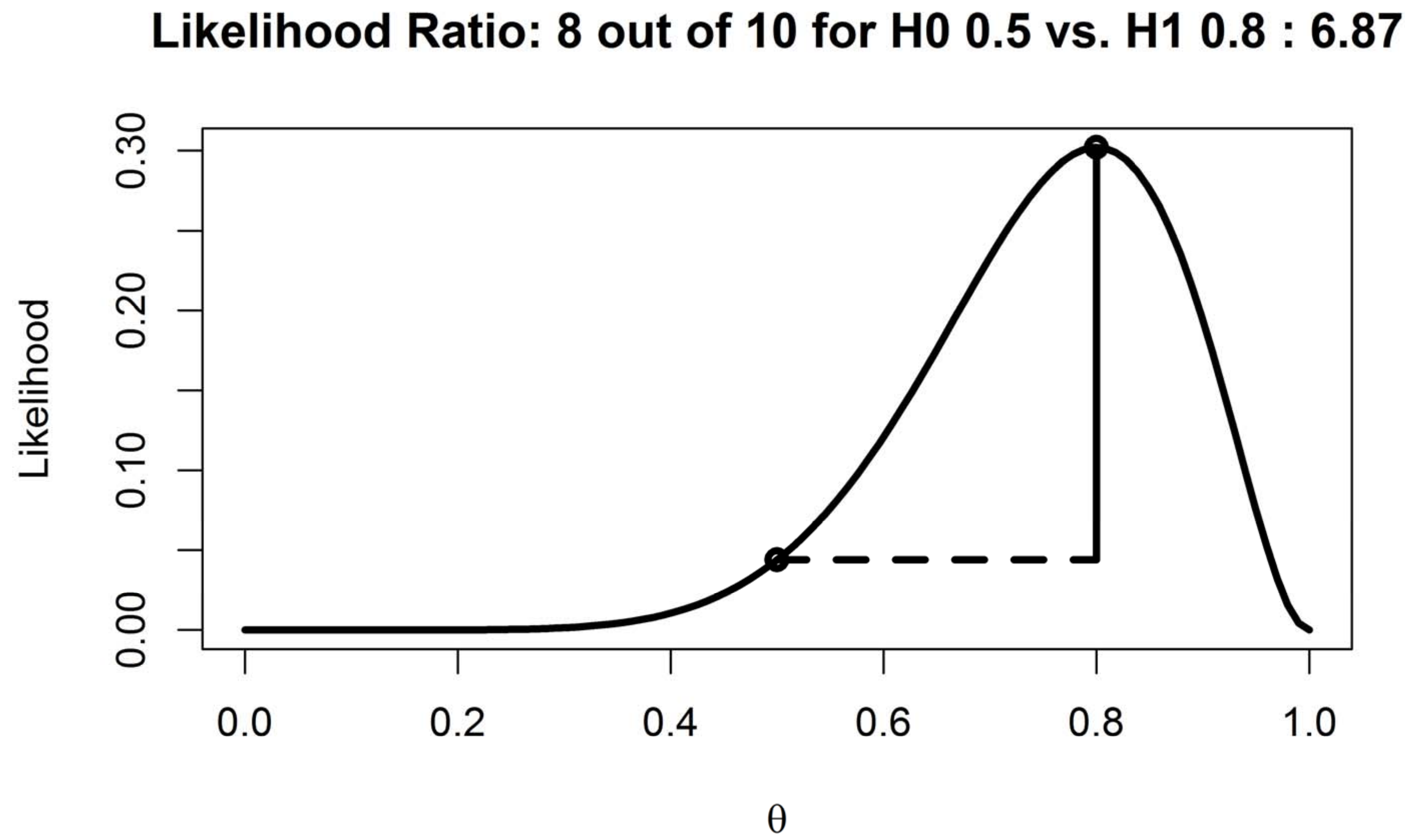
$\theta = 0.4 = 0.01$

$\theta = 0.2 = 0.001$

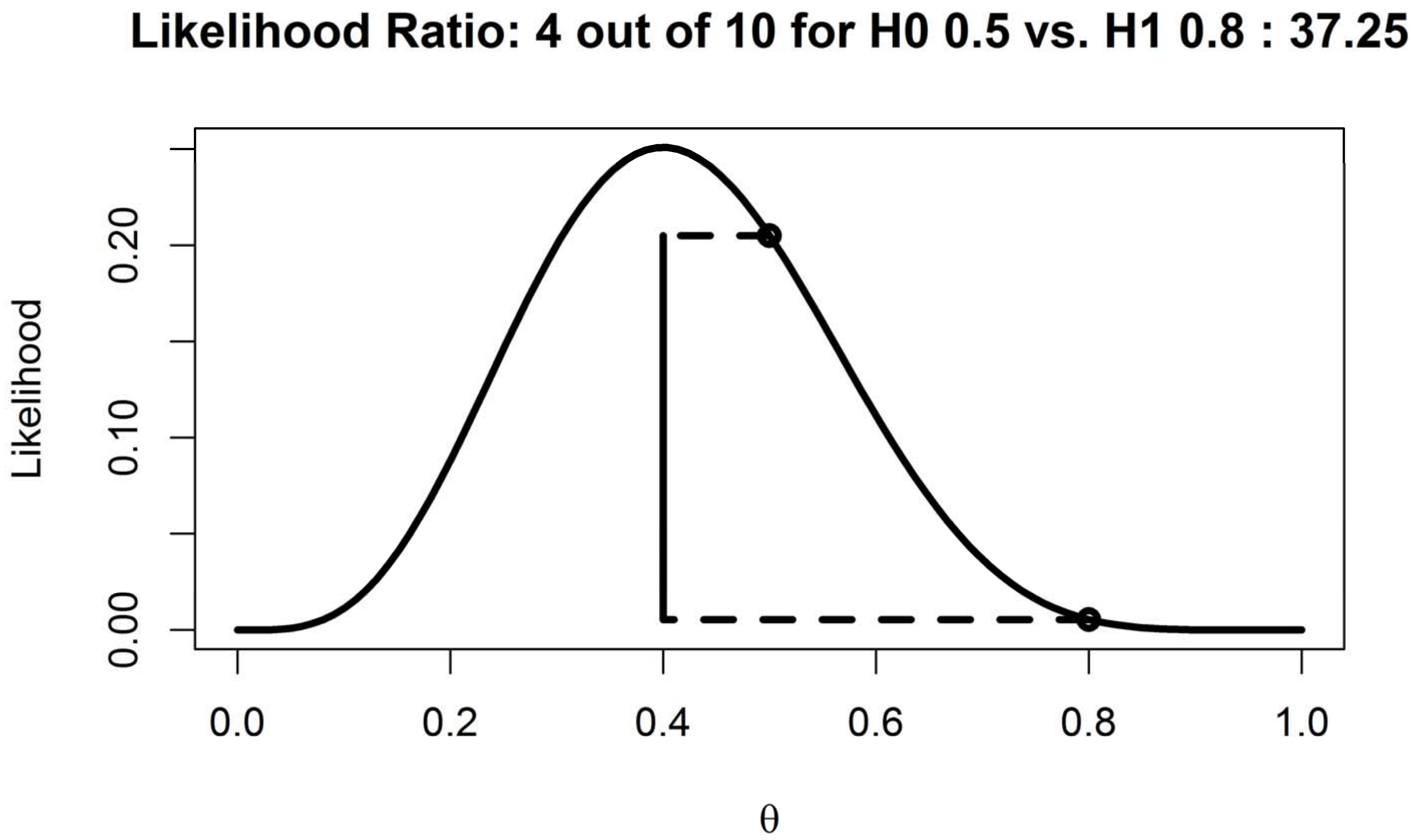
I invented likelihoods when I was 22, and a 3rd year undergraduate.



We can use the likelihood under H0 and H1 to calculate the **likelihood ratio**



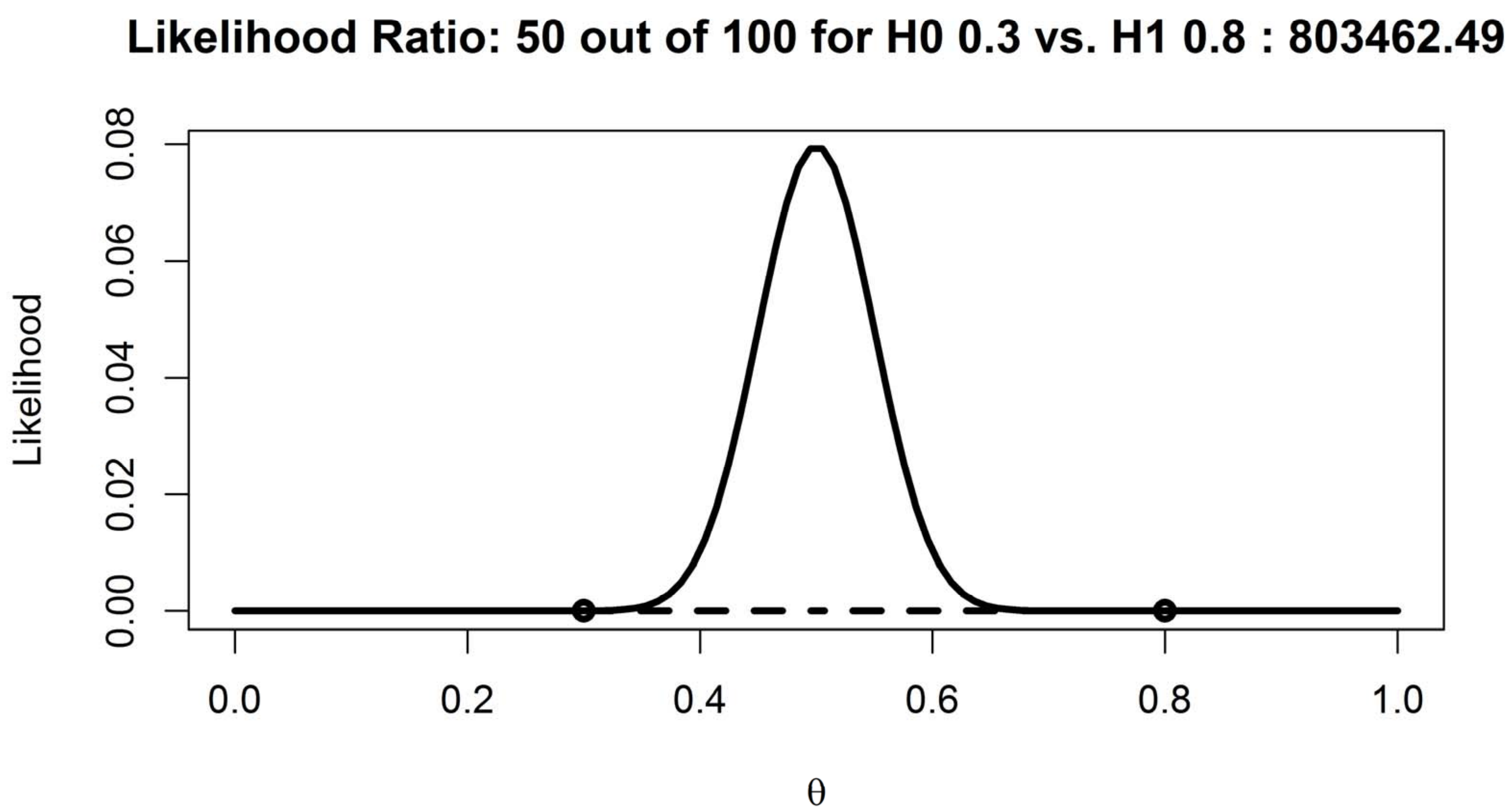
Likelihood Ratio: 8 out of 10 for H0 0.5 vs. H1 0.8 : 6.87
Likelihood
0.00
0.10
0.20
0.30
0.0
0.2
0.4
0.6
0.8
1.0
θ


Likelihood Ratio: 4 out of 10 for H0 0.5 vs. H1 0.8 : 37.25
Likelihood
0.00
0.10
0.20
0.0
0.2
0.4
0.6
0.8
1.0
θ

Likelihood ratios of **8** and **32** are moderately strong and strong evidence.

(Royall, 1997)

Likelihoods are **relative** evidence for H1 vs. H0. H0 and H1 might be unlikely.


Likelihood Ratio: 50 out of 100 for H0 0.3 vs. H1 0.8 : 803462.49
Likelihood
0.00
0.02
0.04
0.06
0.08
0.0
0.2
0.4
0.6
0.8
1.0
θ

We can compare the likelihood under H0 $\theta = 0.05$ and H1 $\theta =$ power (i.e., 0.8).

You perform 3 studies:

- 0 could be significant
- 1 could be significant
- 2 could be significant
- 3 could be significant

2 out of 3 significant:

When H0 is true:

$$0.05 \times 0.05 \times 0.95 = 0.0024$$

2 out of 3 significant:

When H1 is true

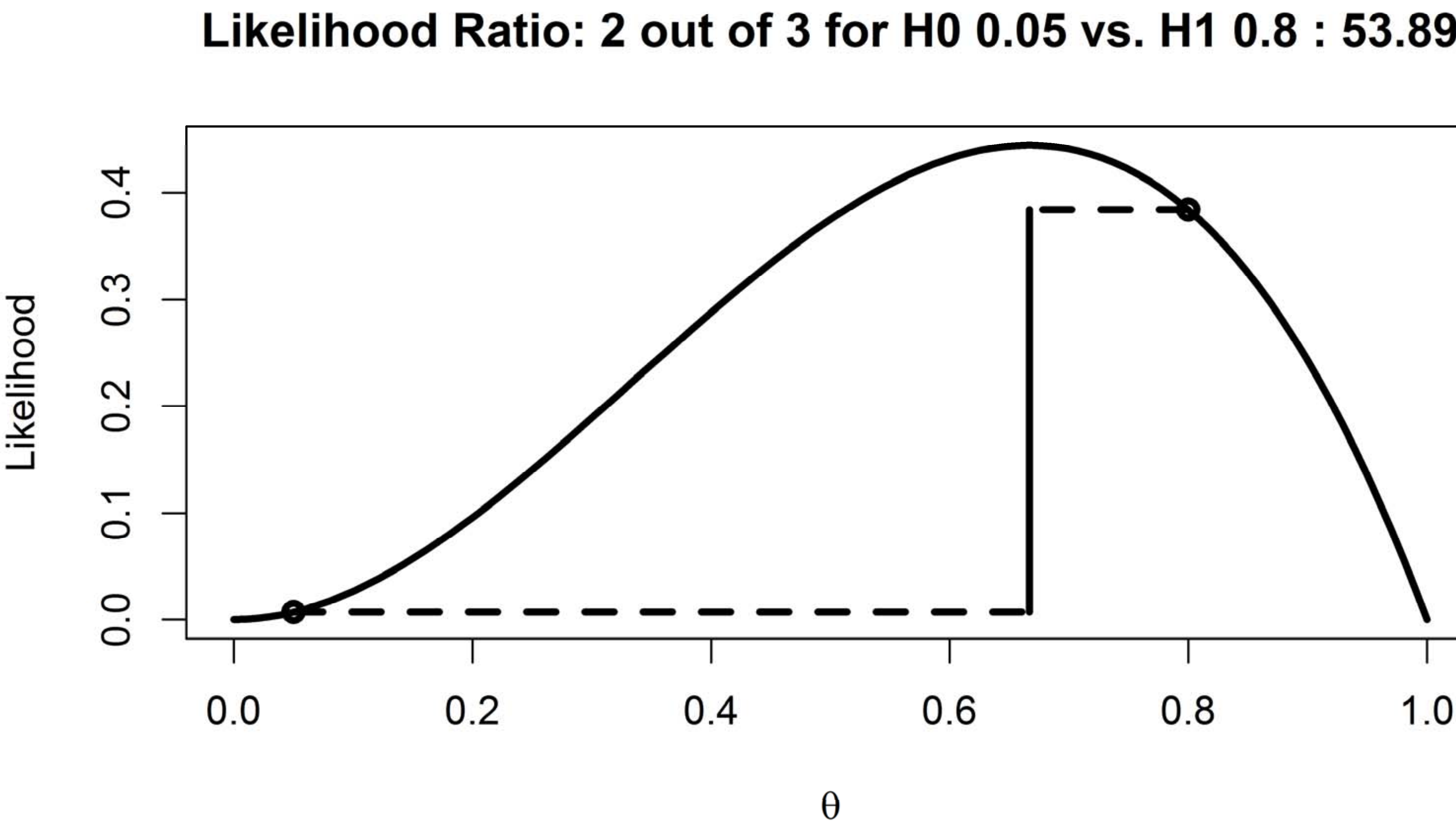
And power = 0.8

$0.8 \times 0.8 \times 0.2 = 0.128$

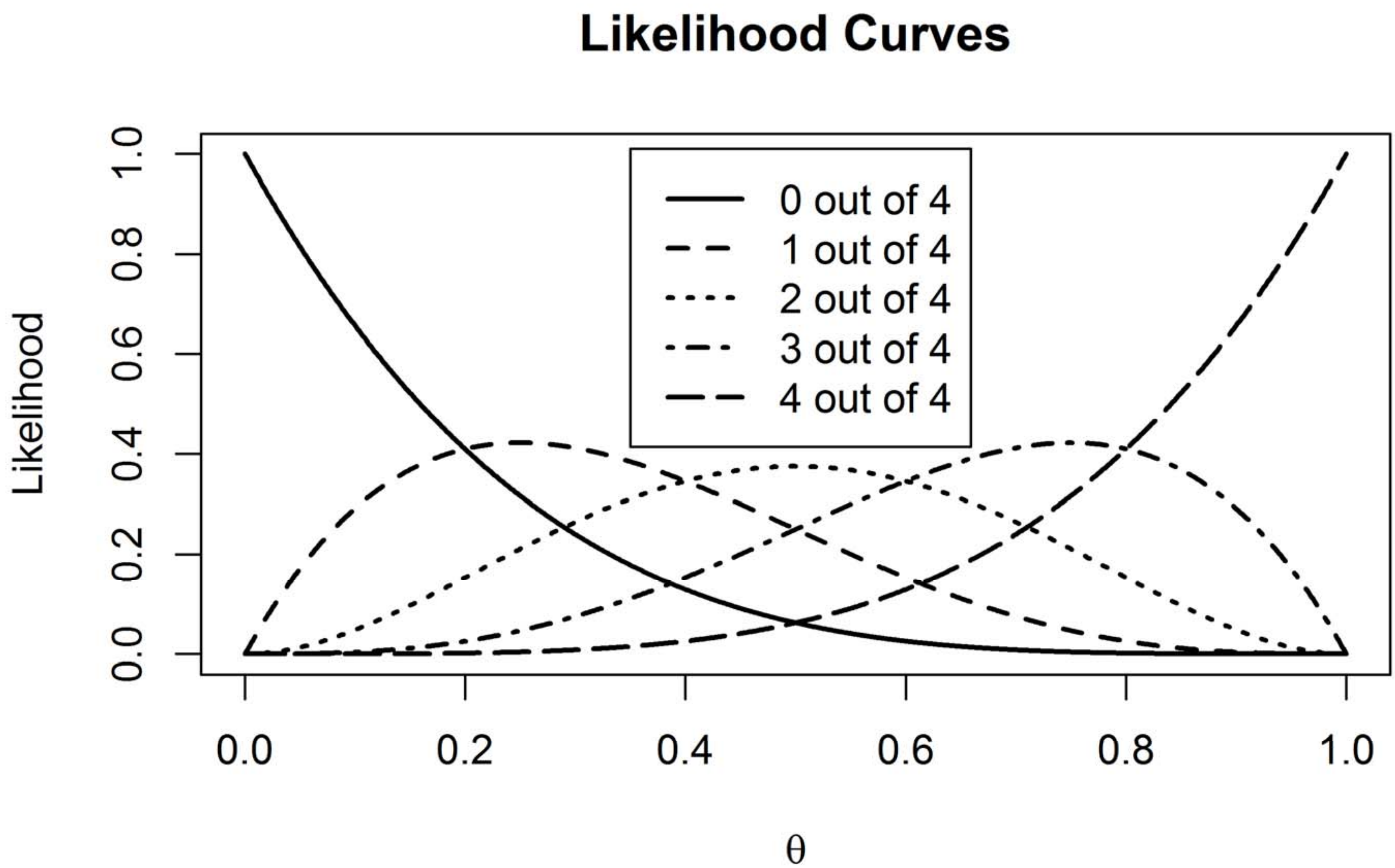
2 out of 3 significant:

0.128/0.0024=54

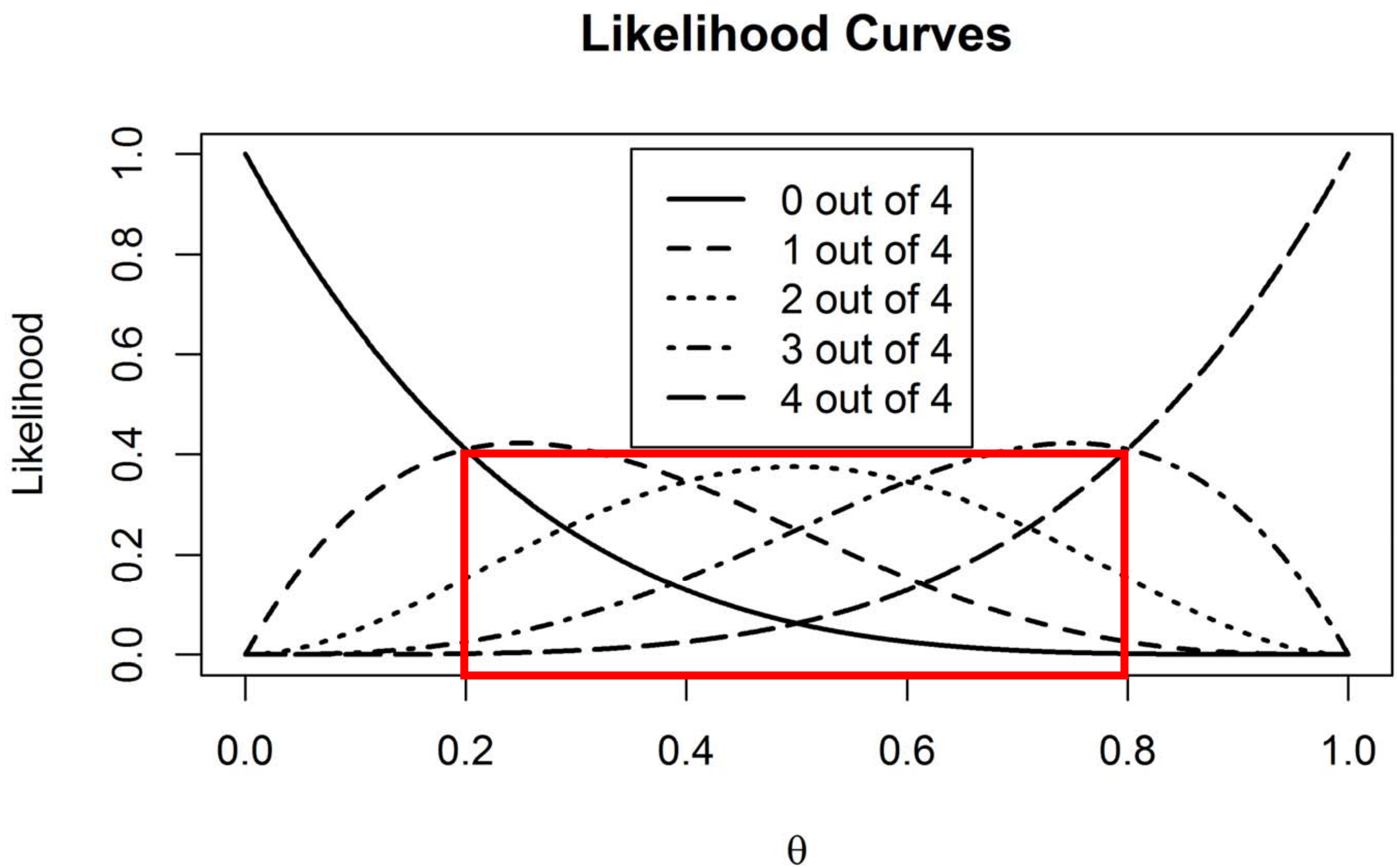
H1 is 54 times more likely than H0


Likelihood Ratio: 2 out of 3 for H0 0.05 vs. H1 0.8 : 53.89
Likelihood
0.0
0.1
0.2
0.3
0.4
0.0
0.2
0.4
0.6
0.8
1.0
θ

Multiple studies should give mixed results when H1 is true & 80% power: $0.8 \times 0.8 \times 0.8 = 0.51$.



Likelihood Curves
0 out of 4
1 out of 4
2 out of 4
3 out of 4
4 out of 4
Likelihood
0.0
0.2
0.4
0.6
0.8
1.0
θ


Likelihood Curves
0 out of 4
1 out of 4
2 out of 4
3 out of 4
4 out of 4
Likelihood
θ
0.0
0.2
0.4
0.6
0.8
1.0

Likelihood ratios are a way to quantify the relative evidence for H1 vs. H0.