

# Explaining Human Choice Probabilities with Simple Vector Representations

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## Abstract

When people pursue rewards in stochastic environments, they often match their choice frequencies to the observed target frequencies, even when this policy is demonstrably sub-optimal. We used a “hide and seek” task to evaluate this behavior under conditions where pursuit (seeking) could be toggled to avoidance (hiding), while leaving the probability distribution fixed, or varying complexity by changing the number of possible choices. We developed a model for participant choice built from choice frequency histograms treated as vectors. We posited the existence of a probability antimatching strategy for avoidance (hiding) rounds, and formalized this as a vector reflection of probability matching. We found that only two basis policies: matching/antimatching and maximizing/minimizing were sufficient to account for participant choices across a range of room numbers and opponent probability distributions. This schema requires only that people have the ability to remember the relative frequency of the different outcomes. With this knowledge simple operations can construct the maximizing and minimizing policies as well as matching and antimatching strategies. A mixture of these two policies captures human choice patterns in a stochastic environment.

## Author summary

We have mathematically formalized participant behavior in probability matching using histogram vectors. This representation covers not only the well-studied case of seeking a reward, but also the rarely studied scenario where people try to *avoid* consequences, and permits us to define *probability antimatching* in avoidance contexts. The similarities and differences between the seeking and avoiding scenarios, as well as their matching and antimatching counterparts, motivates our theoretical proposal for how decision making under uncertainty varies with the decision

maker’s role. Seekers, for whom the cost of failure is usually smaller than their “prey”, choose not merely to succeed, but also to learn what to expect as revealed by probability matching. Whereas in an avoidance scenario, minimizing behavior predominates. A simple vector representation of probability that combines these two modes of responding accounts for the array of human participant choice frequencies seen in our experiments despite varying outcome distributions, whether a participant was prey or predator, and across variation in environment complexity.

## Introduction

We implicitly learn to exploit the statistical structure of our environments. We do this to build context-dependent conceptual repertoires (e.g. in language the transition probabilities between syllables are dependent on the language (Saffran et al., 1996)). We do this to optimize the match between circumstance and action (this is the basis for reinforcement learning as a model for behavior (Sutton & Barto, 2018)). It might be tempting to assume that the way we do this is by representing the underlying probability distributions directly. However, this assumption seems at odds with all the probability fallacies that we are heir to (nicely reviewed by (Huang et al., 2024) in their introduction). And on reflection it seems obvious that we have neither the computational power nor storage to represent continuous distributions with high fidelity.

We do have evidence though that people can represent probabilities with reasonable fidelity when the option set is small. This evidence comes from the phenomenon of probability matching (Vulkan, 2000). Probability matching is a phenomenon found in repeated choice experiments where people select options in proportion to their occurrence. That is, they match their choice frequencies with their observed target frequencies. In many of the empirical studies demonstrating the phenomenon, participants are making their selections to obtain a reward. As one concrete example, Koehler and James (2009) found a predominance of probability matching for a marble guessing game where participants were told that there were 30 green and 10 red marbles in a computerized urn task. Participants were given the motivation of 50 cents for each correct guess and asked to predict the outcome of subsequent trials. Whether participants were allowed to observe the sequence of draws to search for patterns did not change their propensity to probability match. Note the discrepancy between the “optimal” maximizing strategy and probability matching: Exclusively picking the most likely option wins you 50 cents 75% of the time ( $1 * 0.75 + 0 * 0.25 = 75\%$ ) while probability matching only wins you 50 cents 62.5% of the time ( $0.75^2 + 0.25^2 = 62.5\%$ ). For studying probability representation, this “bug is a feature.” If people always maximized in such scenarios we would know that they had learned the most probable option, but we would be blind to their representation of the other options.

Probability matching is typically measured in the setting of pursuit. Where is the candy bar? Which lever do I press for a food pellet? But if we are seeking a general understanding of probability learning and its representation then the inverted case should be important too. What about when the task is avoidance? Don’t press the lever

that shocks. Don't choose the goblet with the iocane powder. In this scenario would we see the phenomena of probability *antimatching*? The inversion of matching with two options is clear. If you see two options with a ratio of 3:1 where the more common one is the more noxious you pick it with a ratio of 1:3. Your choices are now the "flipped" version of the observed frequencies. Note, analogous to matching, antimatching is not the optimal strategy of picking exclusively the least likely option.

As generally framed in Bayesian decision processes (Savage, 1972) the representation of probability is independent of the cost function. For a particular distribution of events we can change the gains and losses without changing the frequencies. Thus, simply due to symmetry considerations we might expect in scenarios with a 3:1 ratio of outcomes to see choices switch from a matching pattern to antimatching pattern when we invert the gains to be losses. We are unaware of any empirical work testing such a manipulation.

Of course, assuming that people invert gains and losses symmetrically may well be too strong. Consider a predator-prey framing. The difference between failing to acquire a reward of modest value and succumbing to a catastrophic cost are manifestly asymmetric. Seeking to learn the probability contours of an environment as a predator, where mistakes cost nothing (or cost little when viewed as a missed opportunity for a gain of a food resource), seems intuitively quite different when framed from the perspective of the prey where failure might mean death. Participants may assume a predator/prey mindset depending on whether the task is framed as gain pursuit or loss avoidance, especially if gains/loss values are not explicitly provided.

In loss avoidance, might we expect a *probability minimizing* pattern to emerge? This is where the prey always choose the option with the lowest probability, analogous to probability maximizing in pursuit contexts. The question of whether matching/antimatching (ma/am) is "dumb" and maximizing/minimizing (mx/mn) is "smart" (to use the phrasing of Koehler and James (2014)) is not straight forward empirically or conceptually. These two strategies form the basis for two distinct modes: Exploration - lots of ma/am, little mx/mn, and Exploitation - little ma/am, lots of mx/mn. Experimentally, we find that the emergence of Exploration or Exploitation modes might be the consequence of a pursuit/avoidance task framing and participant biases about a payoff asymmetry.

This leads to an analogy with Theory of Mind, call it a *Theory of the Probabilistic Mind* (ToPM). The predator chooses not merely to succeed, but also to learn what to expect. The predator does not freeze a choice policy that only estimates the current maximum, but also values information and deception. Perhaps this is beneficial in competitions with multiple rounds or as a strategy to track a drift in the prey's behavior (Koulouriotis & Xanthopoulos, 2008). This distributional learning is revealed by probability matching. On the other side, the prey might show a bias for more often choosing less frequent predator choices, and less often choosing more frequent predator choices. This could be called an "opposite" distribution. It is worth noting that mx/mn behavior does not always eliminate the opportunity for information gain. If an avoiding agent learns that the predator chooses one option more than all others, and has an estimate for the choice frequency of this one option, the prey may still be

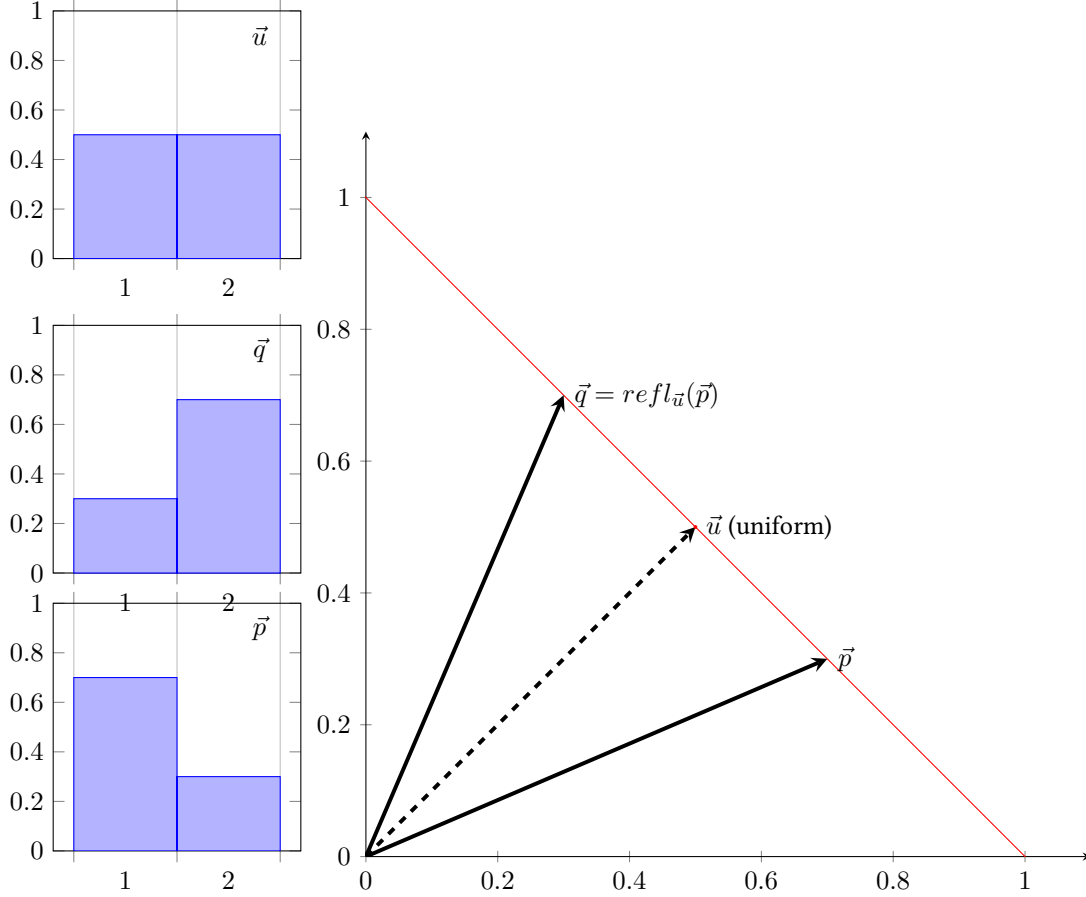
able to detect distributional shifts by the pursuer. Thus, it might detect when exploration is of sufficient benefit to switch to an antimatching strategy so as to renew its estimate of the the predator’s pursuit distribution.

For our theoretical developments along this line we take the choice history at face value and assume that *frequency histograms are the primitives for the internal representation of probabilistic information* (DiBerardino et al., 2022; Filipowicz et al., 2016; Fiser et al., 2010; Johnson-Laird, 2013; Tenenbaum et al., 2011). Intuitive? Perhaps. But the reason that the phrase “opposite” has been placed in scare quotes is because it is not easily defined for histograms. There are a number of approaches one could consider to find a maximally distant alternative (Bhattacharyya, 1943; Kullback & Leibler, 1951; Rubner et al., 2000). However, an information theoretic approach imposes unintuitive and complicated computations. Perhaps more importantly, the notion of “oppositeness” implies some relationship with recoverability. An agent using many existing methods for finding a distant distribution cannot easily recover its original distribution. As an analogy, imagine standing somewhere on an island and finding the point along the coast that is maximally distant. Now move to that point on the shore and apply your algorithm again. You will not return to your starting point. Oppositeness is not equivalent to maximumly distant, because oppositeness requires the additional feature of recoverability. Therefore, we need an operation for finding an opposite distribution that is it’s own inverse; reapplying the operation to the opposite will recover the original. Oppositeness with recoverability offers the cognitive benefit of reducing memory burden; you do not need to remember your original position if it can easily be recomputed from your current position.

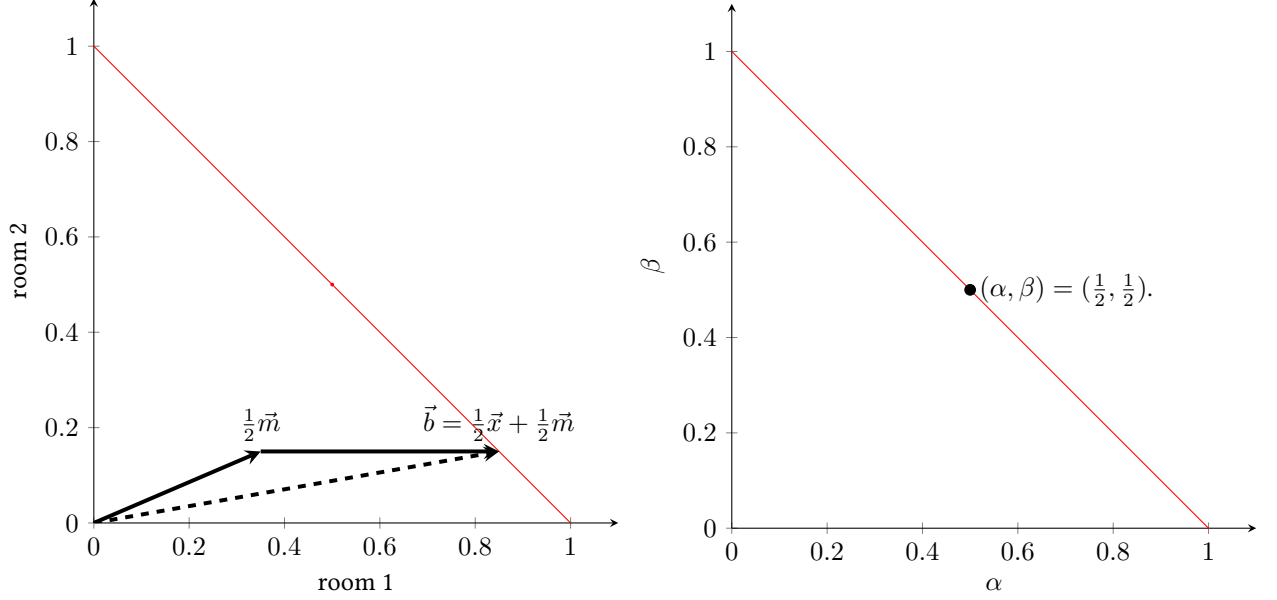
We offer a particular formalization that allows computing specific predictions for participant behavior in both pursuit and avoidance conditions. We treat presented probability distributions and participant choices as histogram vectors in Euclidean space. Since hiding is the opposite of seeking, we define the antimatching strategy to be the “opposite” of the matching strategy. That is, the vector representing a matching strategy is reflected across the uniform vector/distribution to produce the vector representing an antimatching strategy. The matching strategy vector can always be defined from experimental conditions (the presented distribution), and can always be recovered from the antimatching strategy vector by repeating the reflection. This leads to the practical benefits of an intuitive geometric interpretation and a simple visualization. This formalization also yields an analogy between psychological constructs and neural models, where geometric representations of various behavioral, cognitive and neural activity models are well established (Eliasmith, 2013; Gardenfors, 2004; Georgopoulos et al., 1989; Kriegeskorte & Kievit, 2013). The next several paragraphs and figures illustrate this idea graphically, and show how it leads to simple low-dimensional visualizations for even high dimensional choice spaces.

Consider the case of a two-dimensional histogram, like the one you would build when playing matching pennies ([https://en.wikipedia.org/wiki/Matching\\_pennies](https://en.wikipedia.org/wiki/Matching_pennies); Figure 1). The choices you make for any two-choice task creates a two-dimensional histogram. This can be represented as a vector on a two-dimensional plane. All permissible distributions live in the positive quadrant of 2D-space (since you can never make a negative number of choices).

The “maximally uncertain” distribution is the uniform, i.e. a histogram with all bars of equal height. It too is a vector in the positive quadrant of this space. The opposite choice vector/histogram would be the vector/histogram resulting from reflecting your choice vector across the uniform. The choice of the uniform vector as the reflection axis is in analogy to the uniform probability distribution in a Bayesian analysis as the distribution of maximal uncertainty.



**Fig 1.** The simplest case: a two-dimensional histogram. On the left there are three histograms showing the proportion of events for each of two bins. The upper histogram is the *uniform*, denoted as  $\vec{u}$ : all bins have the same probability. Histograms (such as  $\vec{p}$ ) can be reflected over the uniform to obtain a  $\vec{q}$ , and vice versa. The angle between  $\vec{q}$  and  $\vec{u}$  is identical to the angle between  $\vec{p}$  and  $\vec{u}$ . All of the vectors exist in the same plane. Two distributions are opposites if they are an equal angular distance away from the uniform vector and form a 2D-plane with the uniform.

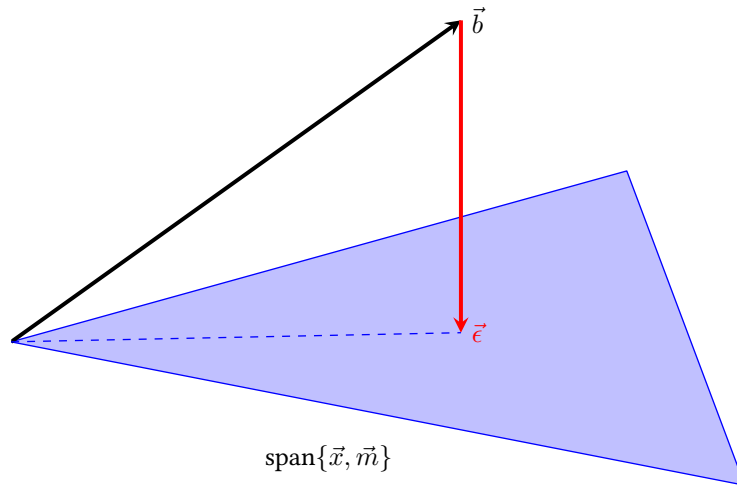


**Fig 2.** From the space of histograms to the space of strategies: The vector representing choice proportions or numbers is decomposed into a vector representing the matching and maximizing strategy combination. On the left you see  $\vec{b} = (0.85, 0.15)$  and its decomposition of  $\vec{b} = \alpha\vec{x} + \beta\vec{m}$  where  $\vec{x} = (1, 0)$  and  $\vec{m} = (0.7, 0.3)$ . The coefficients for each of these two strategies gives us a point on the  $(\alpha, \beta)$ , in this case  $(\alpha, \beta) = (\frac{1}{2}, \frac{1}{2})$ . Although the graph on the right looks similar to the one on the left the axes represent completely different things. On the left we are in the space of options: one axis for each option. On the right we are in the space of strategy. One axis for each strategy: maximizing ( $\alpha$ ) and matching ( $\beta$ ). The same method works for decomposing hiding choices into a combination of antimatching and minimizing.

While it is easy to depict visually the vectors of two or three dimensional distributions this becomes impossible as the number of options increases. In order to compare higher dimension histograms to each other and to compare conditions with varying numbers of options we would like to have a way to collapse performance to a consistent low dimensional representation. Our method for this is depicted in Figure 2 and embodies our intuition that most patterns of human choice in such scenarios are a near-exclusive combination of the ma/am strategy and the mx/mn strategy. Although the graph on the right of Figure 2 looks similar to the one on the left their axes represent completely different things. On the left is the space of options: one axis for each. In the case of the hide-and-seek game we run experimentally, each option is a room. On the right is the space of how much maximizing ( $\alpha$ ) and how much matching ( $\beta$ ) are combined to make the choice vector. While the dimensionality of the histogram representation changes as the number of choice options changes in a scenario, the size of the strategy space representation is always fixed at two, because we always consider a combination of two strategies. This is an advantage for cross scenario comparison, but it is also reflects a specific and concrete prediction about the nature

of the types of strategies that are used to form the individually variable choice proportions.

In the two-dimensional choice case it is always possible to re-frame our two-dimensional observed choice vector as the sum of two basis strategy vectors. Thus, we will always be able to construct a participant's seek choices in a two room hide-and-seek task as a combination of maximizing and matching basis vectors, and hide choices as a combination of minimizing and antimatching basis vectors. This is *not* guaranteed for higher dimensional problems. Figure 3 shows this diagrammatically. Conceptualizing human choice proportions as histogram vectors gives us a method of graphical representation, and a geometrical method for assessing the plausibility of hypotheses about the number of independent strategies that combine to describe peoples' choices. The closer participant choice vectors are to the plane formed by our proposed strategy combination, the better our strategy combination explains observed behavior. The observation of matching behavior and the optimality of maximizing behavior provide the principal starting points for exploring these ideas.



**Fig 3.** Testing: In higher dimensional spaces the vector representing a person's selection frequencies  $\vec{b}$  (black) may not exist on the plane created by a weighted sum of the maximizing  $\vec{x}$  and matching strategies  $\vec{m}$  (blue region). This error vector  $\epsilon$  (red) quantifies the discrepancy, and can be used as a convenient expression of the goodness of the model.

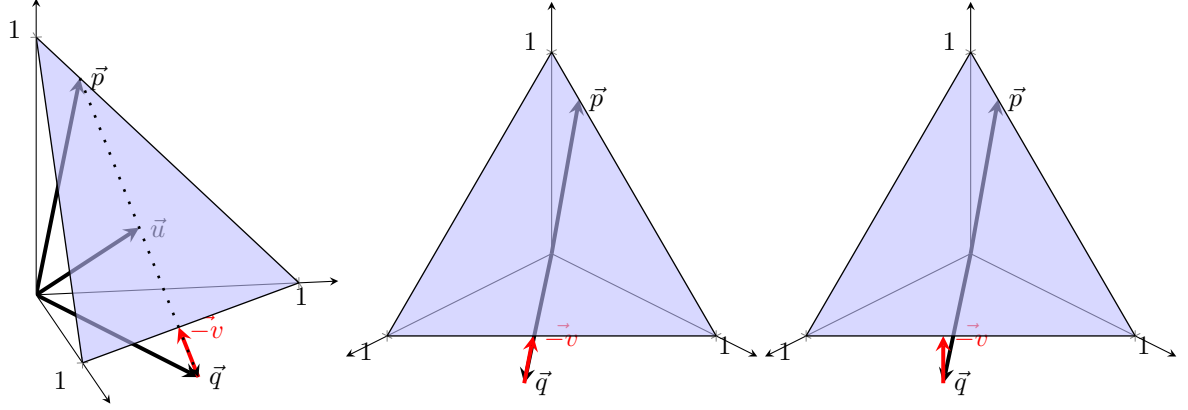
There are different ways in which the number of choices may be relevant. First, there is the question of whether the number of choices influences how many policies a participant combines to yield their particular choice histogram. We suggest that two policies are sufficient in all settings. The second way the number of choices is relevant is in changing complexity. Even if one only looks at the bookkeeping and ignores the idea of a high dimension vector reflection operation, tracking the number of choices made for each of seven rooms requires more memory than tracking the choices for three rooms. To successfully implement the ma/am policy a participant needs to track the frequencies of all choices. Intuitively, the mx/mn, which only requires determining the most frequently

chosen option, could be more tolerant of imprecision, and comparatively easier to implement. For these reasons our experiments have conditions with varying room numbers. If participants weight mx/mn strategies more as room number increases, we would conclude that computational complexity is an important driver of policy mixing.

To recapitulate, we know that matching behavior is common in human choice experiments. We also know that maximizing choice strategies are typically optimal under the usual experimental conditions. So, we conjecture that individual choice proportions in such tasks reflect for most people some combination of the two strategies. We describe this with Euclidean vector operations, and show how this formalization allows for easy visualizations and computations for assessing model goodness.

This consideration of context shift highlights another challenge. What is the right way to compute the opposite strategy vector needed to flip between hiding and seeking? We have specified it to be the matching strategy vector reflected across the uniform vector. This brings with it another interesting consequence that can be used to test this hypothesis: a valid reflection does not always exist. While it is always possible to generate the equal magnitude reflection mathematically, the resulting vector will not always exist in a permissible region of the space (Figure 4). That is, some vectors will produce negative values when reflected, and clearly people cannot choose an option less than 0 times. But this is actually a strength of our vector histogram formalization. With it we can propose specific testable alternatives for how one might get close to the optimal, but unobtainable, reflection. We highlight two such options in Figure 4. In practice the projection back to the closest point on the simplex and the retracing of the path of the reflection back to the border of the simplex usually lead to very similar choice histograms. However, there are certainly many other points in the simplex and along its border that are not at all near this location, and so if empirical data conform to these projections, it is supportive, though not conclusive, evidence that people may be doing something like reflecting a histogram representation to toggle between similar and opposite representations depending on task context.





**Fig 4.** Three different viewpoints of the probability simplex in 3-dimensional space. Here, the reflection of  $\vec{p}$ , denoted  $\vec{q}$ , lies outside of the probability simplex. Performing the Euclidean projection onto the simplex (right most panel) produces the shortest possible vector  $\vec{v}$  to the simplex, but does not always shift  $\vec{q}$  back along the original reflection trajectory from  $\vec{p}$  (as seen in the middle panel).

In summary, we wish to explore how probabilistic choices change when the context changes. We operationalize this context change in a game of hide-and-seek played on a computer against simulated human opponents. Further, we articulate a computationally simple model where players track the histogram of events observed and choices made to create a personal strategy that principally combines the matching and maximizing policies when seeking (or their “anti-” counterparts when hiding). The mathematical setting for this is of vectors and Euclidean geometry. We operationalize “opposite” distributions as the reflection across the uniform distribution arguing that it is a relatively straightforward computation that allows recovery of either the original or its opposite from repeated application of this single reflection operation. We show how this formalization allows us to produce a reduction of high dimensional choice vectors into a common two-dimensional strategy space and allows for quantification of error and a sense of model goodness of fit. We can also explore the direction of change in this space and compare the predicted “opposite” to participant performance in different contexts.

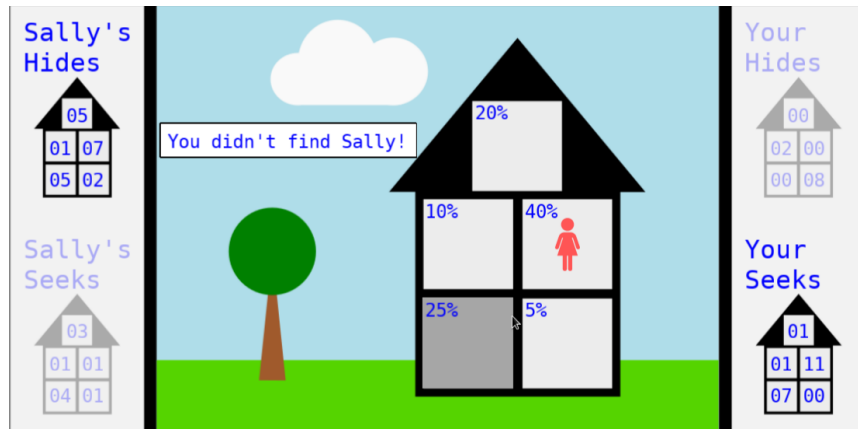
In the sequel we present the empirical data. We present three studies where participants played a children’s game of hide-and-seek on a computer. The game provided an intuitive way to explore both pursuit and avoidance and to assess for ma/am choice patterns (Chapman et al., 2014; Crawford & Iriberry, 2007). In Experiment 1, we deploy our methodology of representing participant strategies as a linear combination of Euclidean vectors. Using this methodology, we demonstrate participant seeking behavior is a combination of the optimal maximizing strategy and probability matching. We demonstrate that hiding behavior is a combination of the optimal minimizing strategy and probability antimatching. We show that when the problem complexity grows (from a 2 room to 5 room scenario in these experiments) the participant strategy mixes differ with context. In Experiment 2, we replicate Experiment 1 for online data collection. In Experiment 3, we present new probability distributions, some of which

have reflections that fall outside the probability simplex. We again demonstrate our expected hide/seek strategy mixes. We also show how the two projection methods for recovering from illegal reflections to the simplex give good approximations to participant behavior.

## Behavioral Methods

We developed a computerized version of the children’s game “hide-and-seek”. Each participant played a set of hide-and-seek games against simulated *predictable* opponents. They were predictable in that they hid and sought with a fixed probability distribution. This was known to all participants. We provided trial by trial counters to provide additional evidence to the participants that the information was accurate.

Our user interface (Figure 5) consisted of a cartoon scene of a house with either two, three, five, or seven rooms. The house was of a ‘doll house style’ where each room was visible to the participant. We presented a percentage within each room. Each percentage represented the probability that the opponent would hide/seek in that room. Room proportions always summed to 100%. Participants played against a fixed set of distributions, but the mapping between rooms and proportions for a given distribution were randomized for each participant. Presenting the actual choice counts to participants allowed us to attribute performance to their strategy construction and not ambiguity or imprecision in their estimates of opponent choices or memory for their own past choices.



**Fig 5.** Participant view of screen: This is the five-room condition. After selecting the bottom left room in a seek trial the selected room turned grey. The child is then revealed, a notification is presented, and the counters are updated.

## Task Procedure

Each experiment began with a short practice round consisting of three seeking trials followed by three hiding trials. As in standard hide-and-seek games, each participant was instructed to find each opponent while seeking, and avoid each opponent while hiding.


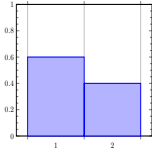
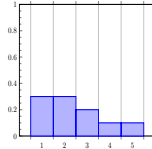

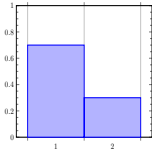
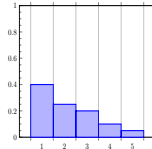

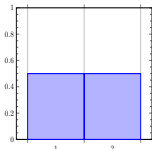
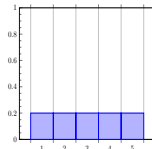

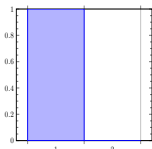
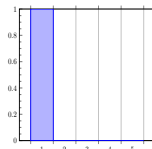
A dialogue box notified the participant that the child had hidden. The screen displayed “Look for [child]!” (where [child] is the name of the current opponent). Using the mouse the participant selected a room to search. Afterwards, either a notification of success (“You found [child]!”) or a notification of failure (“You didn’t find [child]!”) was displayed. As the hider, the dialogue box read “Hide from [child]!” The participant selected a room to hide using the mouse. Similarly, either “[child] didn’t find you!” was displayed on screen to indicate success, or “[child] found you!” to indicate failure. The child was always revealed after any successful or failed attempt for both trial types.

A single hide or seek choice of the participant was treated as a ‘trial.’ The child’s hiding and seeking locations were drawn from the distribution displayed over the rooms and were independent between trials. A series of 10 trials constituted a ‘round.’ The experiments alternated between seeking and hiding rounds, always beginning with seeking. A set of 10 seeking rounds and 10 hiding rounds against a particular child constituted a ‘game.’ Participants played a total of three games in Experiments 1 and 2 and four games in Experiment 3. To prevent frustration and boredom from interfering with performance in any later games, the last game was always played against the opponent who always hid in the same one room.

The experiment in Experiment 1 was programmed in Python using the PsychoPy module (Peirce et al., 2019) and completed in-lab. Due to the COVID-19 pandemic, Experiments 2 and 3 were conducted online. We reproduced the task using JsPsych (De Leeuw, 2015). This JavaScript implementation allowed for online data collection. Other than a slight visual change in the house due to the different method of rendering the image, the only other procedural change was to have participants advance from one trial to the next by clicking a button on the screen rather than using the space-bar of the keyboard as was used in Experiment 1.

At the end of Experiments 1 and 2, participants answered demographic questions of gender, age, term of study, and academic program, along with their perceived competence in logical reasoning relative to other students in that program.

Despite being generally similar in all the procedural aspects of the task, Experiment 3 had a more complicated structure of rooms and distributions. The distribution each participant faced in the first game was randomly selected from one of three sets: 3-, 5-, or 7-room distributions. The second game distribution was randomly selected from the remaining two sets that did not contain the distribution selected for the first game. The third distribution was randomly selected from the remaining set. The fourth game was randomly selected from a set that only contained

| Opponent name (order)                                                                                | 2 Room                                                                              | 5 Room                                                                                |
|------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
|  Toby (Practice)    |    |    |
|  Sally (1st or 2nd) |    |    |
|  Kala (1st or 2nd) |  |  |
|  Bo (3rd)         |  |  |

**Table 1.** Opponents faced by participants in each condition. Each distribution is presented here in decreasing room probabilities, though probabilities were randomly assigned to rooms during the experiment.

distributions with probability mass all in one room.

The distributions and the selection procedure can be efficiently outlined with a vector notation. The symbol  $\in_R$  denotes ‘randomly selected from’. A backslash in set notation denotes exclusion. For example,  $\{A, B, C, D\} \setminus \{A, C\} = \{B, D\}$ .

Let

$$D^3 = \left\{ \begin{bmatrix} 50 \\ 25 \\ 25 \end{bmatrix}, \begin{bmatrix} 42 \\ 42 \\ 16 \end{bmatrix}, \begin{bmatrix} 80 \\ 10 \\ 10 \end{bmatrix} \right\},$$

$$D^5 = \left\{ \begin{bmatrix} 35 \\ 30 \\ 15 \\ 15 \\ 5 \end{bmatrix}, \begin{bmatrix} 35 \\ 25 \\ 25 \\ 10 \\ 5 \end{bmatrix}, \begin{bmatrix} 45 \\ 35 \\ 10 \\ 5 \\ 5 \end{bmatrix} \right\},$$

$$D^7 = \left\{ \begin{bmatrix} 25 \\ 25 \\ 20 \\ 14 \\ 10 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 26 \\ 24 \\ 18 \\ 15 \\ 9 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 50 \\ 18 \\ 12 \\ 8 \\ 5 \\ 5 \\ 2 \end{bmatrix} \right\}, \text{ and}$$

$$D' = \left\{ \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}.$$

- Each participant plays 4 games:  $G_1, G_2, G_3, G_4$

- Each game  $G_i$  has one opponent that follows a particular hide-and-seek distribution,  $\vec{g}_i$
- For any game  $G_i$  with distribution  $\vec{g}_i$ , each element of  $\vec{g}_i$  is randomly assigned to each room of the house presented on the screen
- $\vec{g}_1 \in_R D^n$ , where  $n \in_R \{3, 5, 7\}$
- $\vec{g}_2 \in_R D^m$ , where  $m \in_R \{3, 5, 7\} \setminus \{n\}$
- $\vec{g}_3 \in_R D^k$ , where  $k \in_R \{3, 5, 7\} \setminus \{n, m\}$
- $\vec{g}_4 \in_R D'$

## Participants

We recruited a total of 281 University of Waterloo students to participate in exchange for a course credit. All participants gave informed consent and the study was cleared by a University of Waterloo Research Ethics Board (REB 41316). Experiments 1, 2, and 3 took approximately 25, 21, and 32 minutes to complete, respectively. For study 1 there were 50 participants (38 female). For study two there were 54 participants (44 female) and for study 3 there were 177 participants (134 female). No participants declared any other genders than male or female.

## Data Analysis

This section describes the motivations for our analytical procedures designed to evaluate the ideas outlined in the introduction: that people represent frequency information as probability histograms, and that they combine a small number of canonical reference histograms to create their strategy selection policy. Consider all the possible frequency histograms that could potentially be experienced. Recognize that all histograms are of dimension  $N$  where  $N$  is, in this case, is the number of rooms in which hiding or seeking is possible. We can attempt to represent any of the potentially infinite number of possible histograms as a weighted sum of a much smaller number of basis histograms. We can test the legitimacy of our proposed basis histograms by seeing how well linear combinations of basis histograms approximate participants' performances. We use the demonstration of probability matching and maximizing behavior seen previously to motivate an analysis of how accurately a combination of those two strategies alone could do in approximating our participants' seeking behavior. And if participants change their choice behavior between hiding and seeking conditions, will they use our proposed counterparts of matching and maximizing - antimatching and minimizing - in a similar way? Generally, we will express participant performance as

$$\text{participant strategy} = \text{exploitation strategy} + \text{exploration strategy}$$

where exploration is matching when seeking, antimatching when hiding, and exploitation is maximizing when seeking and minimizing when hiding.

The span of these two input histograms gives us a 2-dimensional linear sub-space. For any participant choice behavior, we can define a 'best fit' representation within this subspace. Any imprecision between best fit within the subspace and a participant's actual choice behavior can be summarized by the error vector we get from subtracting our best fit from the participant's choice histogram. The magnitude of that vector is a proxy measure of goodness of fit (Figure 3).

Seeking and hiding (avoidance) are two sides of the same coin and can be analyzed similarly. *Exploit* and *explore* hiding strategies must first be defined. The former is self-evident; when hiding the optimal (and therefore exploitative) strategy is to always select a room with the lowest probability of being found (if there is more than one room of this type, the choice among them can be arbitrary as our simple framework and task exerts no cost to, for example, climbing up to the attic versus staying on the ground floor). The explore strategy is the trickier one. Motivated by the reasoning outlined in the introduction we defined this strategy as the reflection of the opponent's seeking histogram reflected across the uniform distribution to produce a unique candidate distribution "opposite" to the original.

For seeking games, we plotted all participant seek frequencies overlaid with the opponent's hide frequencies. Similarly, for hiding games, we plotted all participant hide frequencies overlaid with the opponent's seek frequencies. Using these data, we computed participant strategy mixes. We tested how closely they approximated our best fit using matching + maximizing for seeking and antimatching + minimizing for hiding. We assessed how strategy mixes changed across condition: hiding or seeking, and complexity (different number of rooms). The above models were constructed using the Stark-Parker algorithm (Stark & Parker, 1995) for bounded-variable least squares via the 'bvls' package in R (Mullen, 2013). That is, we used the Stark-Parker algorithm to determine the mix of ma/mx and am/mn that most closely approximates participant choices. We used bounded least squares to ensure our model coefficients were between 0 and 1 (inclusive) because no strategy can be used less than 0% of the time or more than 100% of the time. Variables and abbreviations are summarized in Table 2.

| Variable       | Definition                                                   |
|----------------|--------------------------------------------------------------|
| $\vec{b}$      | Recorded participant behavior                                |
| $x_{s h}$      | Maximizing or minimizing strategy                            |
| $m_{s h}$      | Matching or antimatching strategy                            |
| $\alpha_{s h}$ | Amount of $\vec{x}$ needed to best describe $\vec{b}$        |
| $\beta_{s h}$  | Amount of $\vec{m}$ needed to best describe $\vec{b}$        |
| $\vec{e}$      | Error in representing $\vec{b}$ with $\vec{x}$ and $\vec{m}$ |

*Note:* Vectors and parameters with the subscript  $r$  are randomly generated for sensitivity analyses.  $\vec{u}$  denotes the uniform distribution.

**Table 2.** Definition of variables. Subscripts indicate “(s)eeeking” or “(h)iding”.

### Accounting for infinite optimal strategies

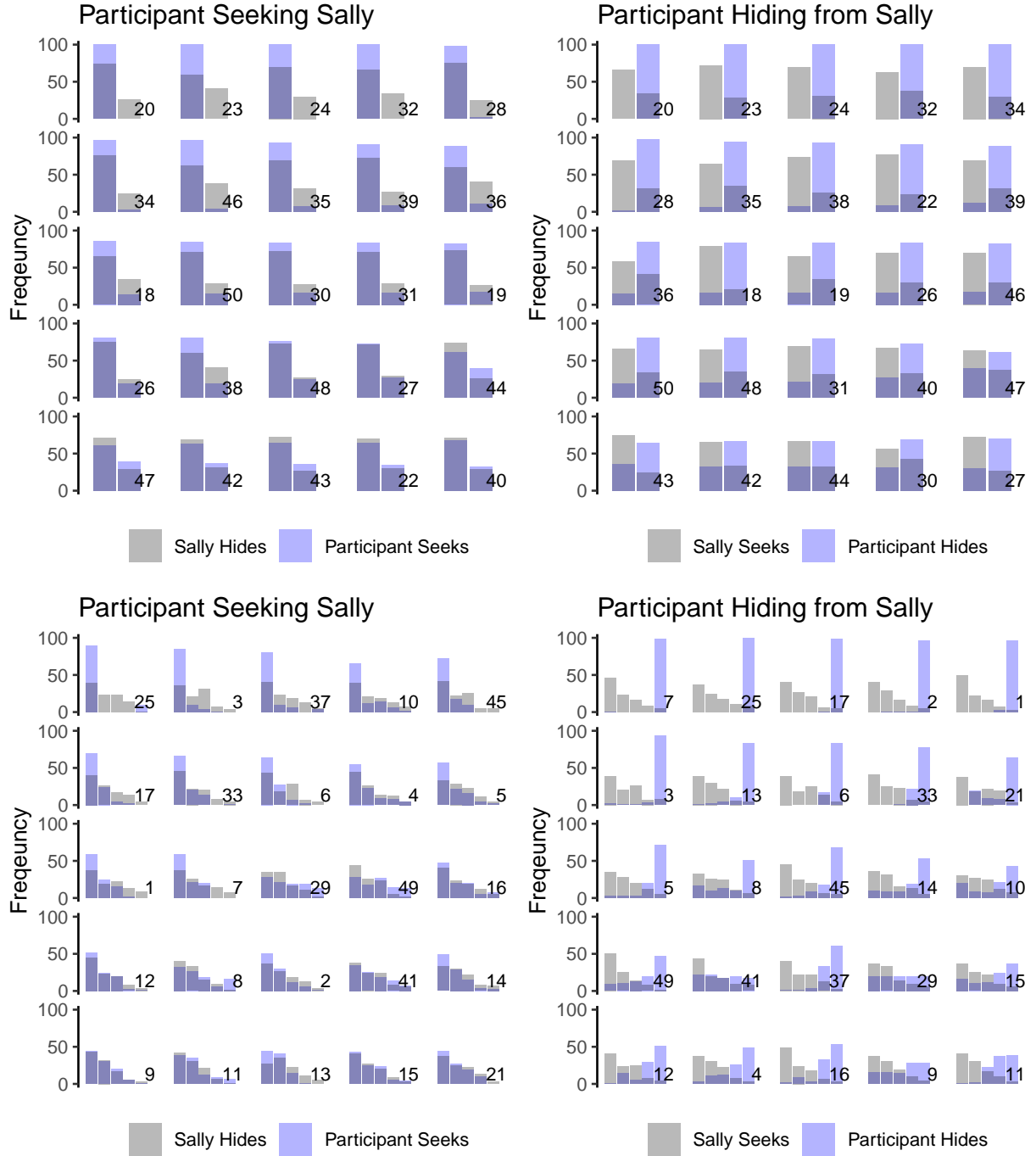
In Experiments 1 and 2, the optimal strategies were unique. For Experiment 3 it was necessary to account for the fact that some distributions had an infinite number of optimal strategies. This occurs for seeking when the maximum probability room was not unique (a tie for the maximum probability room), and for hiding where the minimum probability room was not unique (a tie for the minimum probability room). Interestingly, if there is no unique optimal strategy, then there are theoretically an infinite number of optimal strategies - any mix choosing exclusively from the maximum probability room when seeking (or minimum probability room when hiding) is equally optimal. Since participants only completed 100 trials for seeking/hiding for each distribution, there are actually finitely many optimal choice frequencies, as is true for any finite experiment. Our analysis also accommodates the infinite theoretical case, making this analysis generalizable for future work.

For any distribution that has a non-unique maximum or minimum room, the maximizing and minimizing strategies are generalized from the vectors  $x_s$  and  $x_h$ , respectively, to the sets of vectors  $X_s$  and  $X_h$ , respectively. The number of vectors in  $X_s$  is determined by the number of the rooms that share the maximum probability value. Analogously, the number of vectors in  $X_h$  is determined by the number of rooms that share the minimum probability value. Both  $X_s$  and  $X_h$  contain only standard unit vectors, denoted  $e$ , where the  $i$ 'th element is 1 and all other elements are 0. The set  $X_s$  ( $X_h$ ) contains vectors  $e$ , where  $i$  is selected from the room numbers that have the maximum (minimum) probability value. For example, if  $\vec{m}_s = (0.42, 0.42, 0.16)$ , then  $X_s = (1, 0, 0), (0, 1, 0)$ . If  $\vec{m}_h = (0.26, 0.24, 0.18, 0.15, 0.09, 0.04, 0.04)$ , then  $X_h = (0, 0, 0, 0, 0, 1, 0), (0, 0, 0, 0, 0, 0, 1)$ .



## **Results Experiment 1 (in-lab) Two Room and Five Room (Sally)**

The experiment that comes closest to traditional two-alternative forced choice demonstrations of probability matching is the two-room version of seeking Sally. A plot of the frequency with which each of our participants searched each of the two rooms confirms the predominance of probability matching in our task, along with the optimal maximizing strategy (Figure 6). We also see that participants are sensitive to context - when hiding, some participants use a mix of the optimal minimizing strategy and the antimatching strategy.

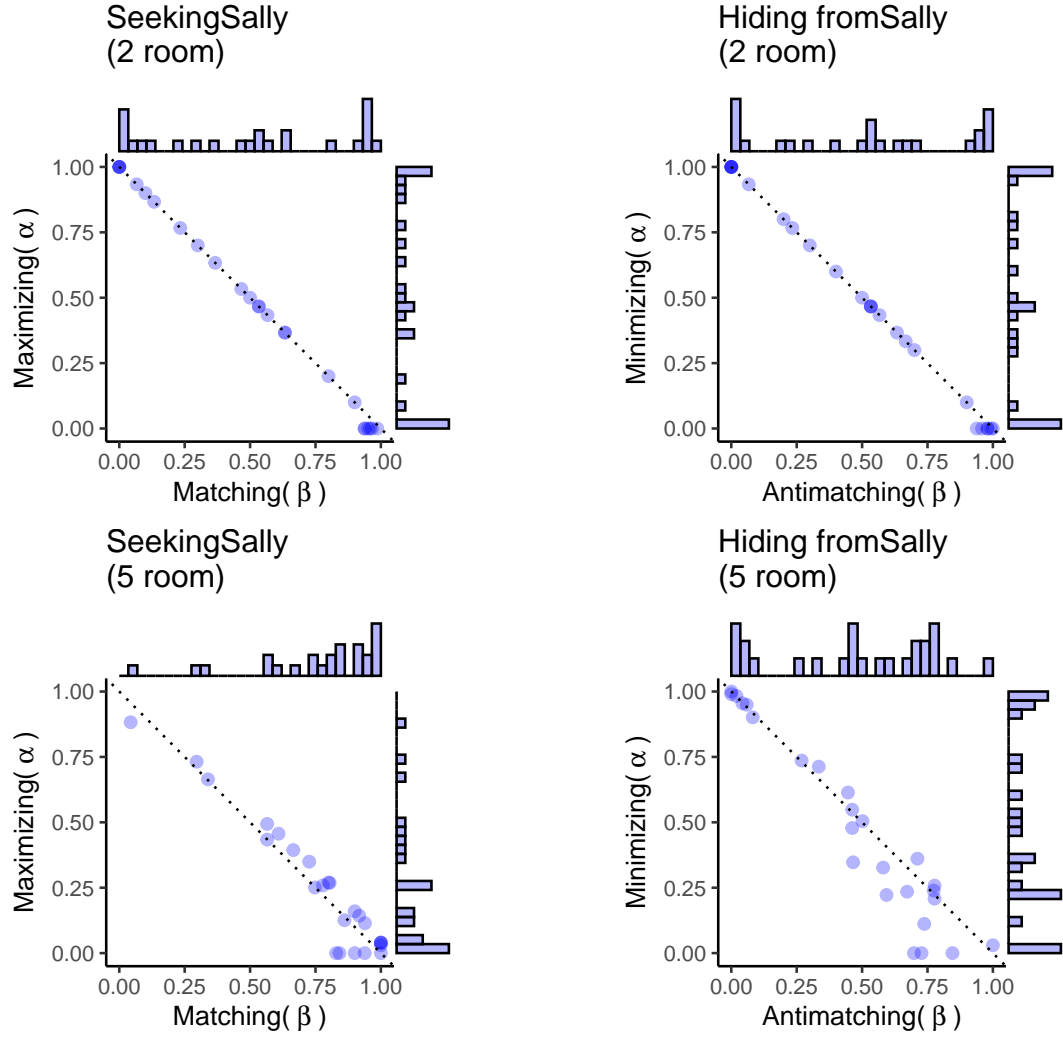


**Fig 6.** Frequency histograms for participants seeking/hiding with the Sally opponent in the two room (top row) and five room (bottom row) scenarios: The left column shows histograms for seek choices, and the right column for hide choices. The participant numbers are arbitrary, but consistent across panels. The gray bars show programmed opponent room frequencies. Darker areas are areas of overlap. Participants are plotted in order of strategy use, with mostly mx/mn in the top left of each panel, and mostly ma/am in the bottom right of each panel. When seeking, matching produces entirely dark bars.

The plots in Figure 6 confirm the prevalence of probability matching in our paradigm when it is conventionally structured. But the data also show that even though participants have identical probability information in the hide and seek conditions they use that information differently depending on the context. That is, in hiding contexts, matching is replaced with antimatching, and maximizing is replaced with minimizing.

## Comparing Hiding and Seeking

While the plots of Figure 6 provide a conventional summary it is hard to compare participants across conditions or to see if there are response patterns consistent for groups of participants. As an alternative, we can collapse much of these data to a single representation by using the ideas introduced in the introduction. We posit that a small number of *basis* strategies with various weights mix to generate the idiosyncratic choice frequencies of individual participants. We can re-plot participant data in this low dimensional strategy space. We begin by fixing this strategy space to be that defined by the mx/mn and ma/am strategies (Figure 7). This strategy space places the  $\alpha$  on the y axis and  $\beta$  on the X axis. Consequently, any participant strategy that uses exclusively a mx/mn strategy will appear as a point placed in the top left of the plot (0,1). Any participant strategy that uses exclusively a ma/am strategy will appear in the bottom right of the plot (1,0). Any strategy that is made of some exclusive mix of mx/mn and ma/am will appear on the diagonal line connecting (0,1) and (1,0). Whether that two-dimensional representation is sufficient is an empirical question. Sufficiency in the two room case is a trivial result since two independent basis vectors will always be enough for a two-dimensional space (although as a practical matter our fits will have few points that drift off the line since we constrain the weights to be between zero and one). Sufficiency is not guaranteed in higher dimensional cases. Despite this, the five-room data depict a similar pattern to the two-room data: Both hiding and seeking strategies are well represented by a mixture of mx/mn and ma/am (Figure 6 and Figure 7)

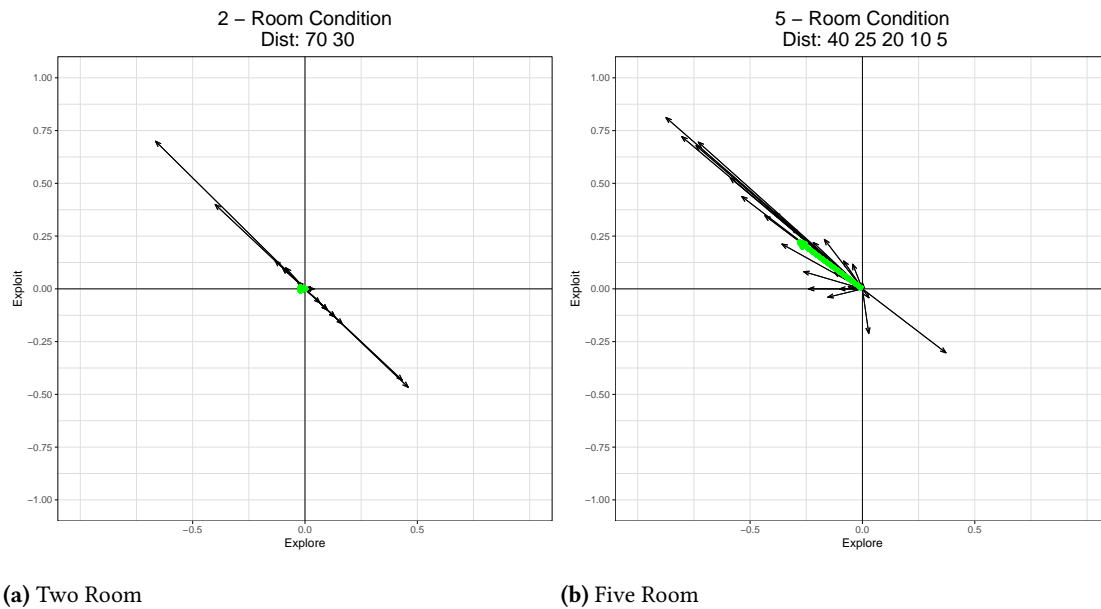


**Fig 7.** Strategy space plot: We fix two basis distributions: mx/mn and ma/am. We determine for each participant the optimal combination of these two vectors that gets as close as possible to their choice frequencies (in the two dimensional case we can almost do this perfectly). We label the mx/mn weight as  $\alpha$  and the ma/am as  $\beta$ . Each participant is a point in this space. For each panel, the closer they are to the upper left the more they mx/mn. The closer to the lower right the more they ma/am. The proximity to the line of slope -1 gives a measure of how well we can do approximating choice performance with just these two basic strategies (all participants lie on or essentially on the line  $\alpha + \beta = 1$  implying there is negligible error in this fit).

An alternative visualization that highlights the change in individual performance across contexts is to look at the strategy vector change. We have two points for each participant: the strategy mix for seeking and the strategy mix for hiding. Figure 8 shows the difference vector between these two points. For viewing the trend across the population we can plot the average of all these vectors (shown in green). If the average vector is very small than

change vectors are also generally small or the directions are evenly distributed angularly so that the summation cancels. If the summary vector is large then we can infer the consistent magnitude and direction of change between contexts.

Figure 8 shows the two room and five room Sally data. Nothing forces these change vectors to lie on the diagonal line from (0,1) to (1,0), but we observe this to be true since most seeking strategies are an exclusive mix of mx and ma, and most hiding strategies are an exclusive mix of mn and am. Moreover, we see that the average strategy mix is closer to 0,1 when hiding than when seeking, but only in the 5-room condition. This indicates that in the 5-room condition, participants generally use a greater mix of exploitation when hiding than when seeking.



**Fig 8.** Plotting changes in strategy vectors for the two and five room conditions (Sally distribution): The left plot shows the two-room condition and the right plot shows the five-room condition. For each plot the choice data for each participant was fit to best explain their choice data using a basis vector of the mx/mn and ma/am strategies. These two vectors are subtracted (hiding - seeking) to yield a change vector that originates at the observed seeking strategy, and points to the observed hiding strategy. They are then shifted to a common starting point to highlight the magnitude and direction of change. Change vectors of individual participants are shown as thin black lines. The average of all change vectors is shown as a thick green line.

The principle finding of Experiment 1 is that both seeking and hiding are well represented by a combination of only two basis vectors (mx/mn and ma/am) even for a condition (five rooms) that has considerably more degrees of freedom. We also find a greater use of mn than mx (the exploitation strategies), but only in the 5-room condition.

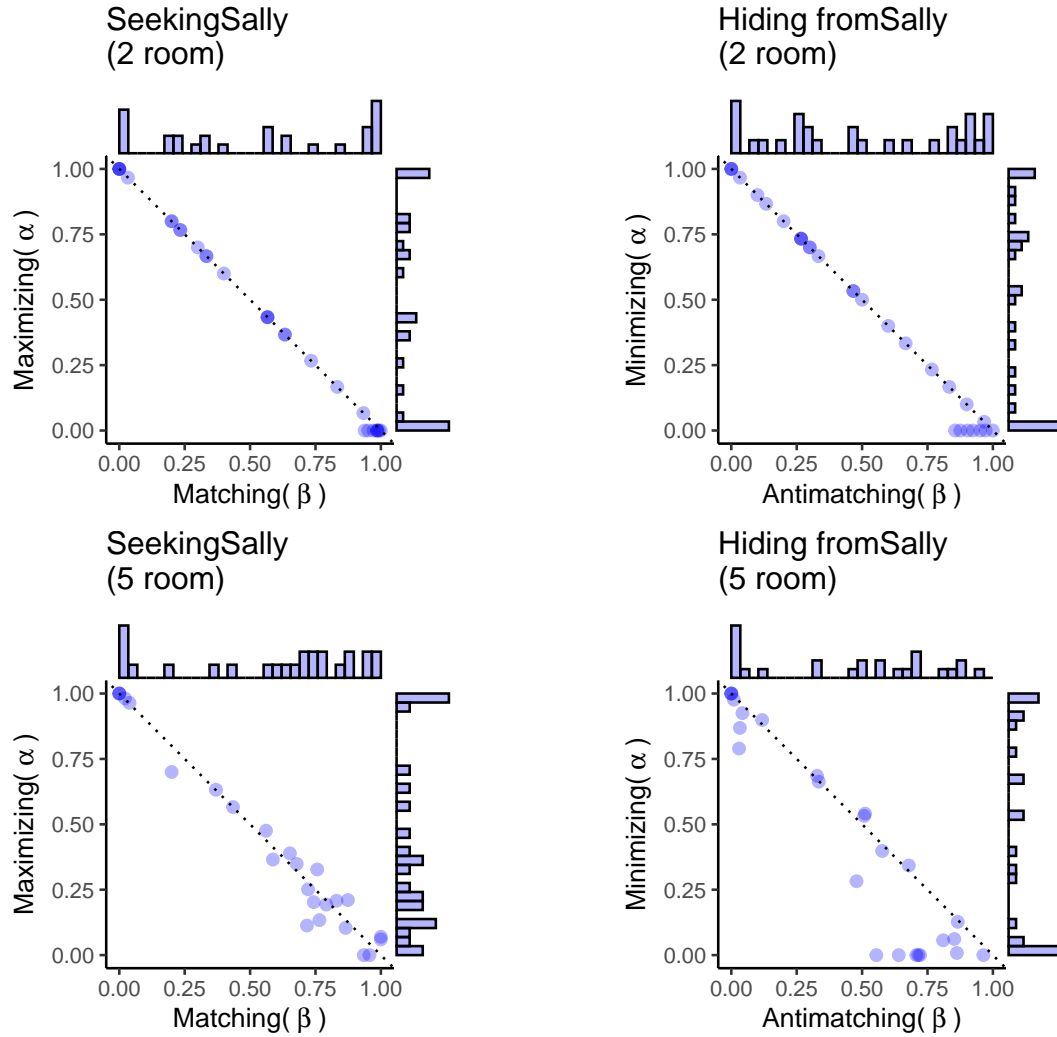
## **Additional Distributions**

In the first experiment participants not only played against Sally, but against two other players who either chose uniformly or chose one room exclusively (Table 1). These distributions are special. In the uniform case (Kala) it does not matter what strategy the participant employs: they will always theoretically win/lose the same number of rounds, and our plotting of strategy change vectors should be vacuous in this case. For the case (Bo) where one room is picked exclusively we should see a consistent change from only picking the 100% room (when they are seeking) to never picking that room (when they are hiding). Our strategy change plots confirm that participants are behaving as expected in these two degenerate cases (Figures 14 and 15). These extra distributions are not particularly theoretically informative, but they did serve as checks on the attentiveness of our participants, and as a check on our computer code. We will not be presenting further analyses of these distributions in the subsequent cases, although both distributions were included in all experiments.

## **Results Experiment 2 - Online Replication**

### **Sally: Two Room and Five Room Conditions**

Our first hide-and-seek experiment was done in person. Due to the pandemic we needed to transition to on-line administration. This experiment evaluated if our results replicated in an on-line setting.

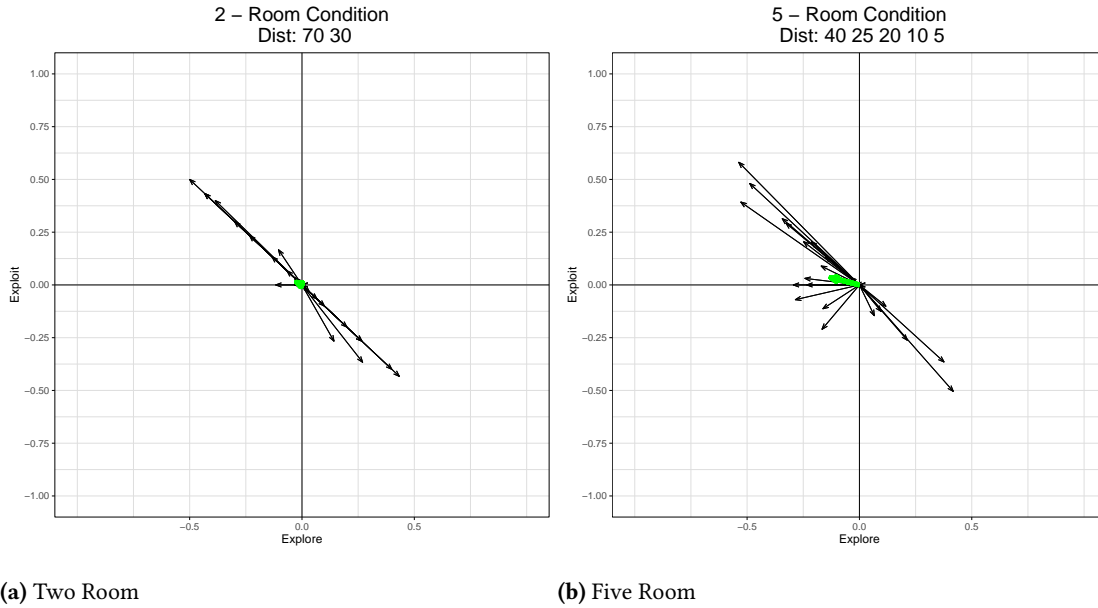


**Fig 9.** Strategy space of participant seeks against Sally. Bars along margin count participants using particular value of  $\alpha$  or  $\beta$ . Perfect maximizers use  $\alpha = 1$  and  $\beta = 0$  (top left). Perfect matchers use  $\alpha = 0$  and  $\beta = 1$  (bottom right). Participant strategies existing on or near line  $\alpha + \beta = 1$  implies near perfect model fit.

For comparison to Experiment 1 (Figure 7) the strategy space plots of  $\alpha$  and  $\beta$  are shown in Figure 9. This is largely a replication of Experiment 1 with many participants showing a preference for matching behavior when seeking. However, there is a mix of the two strategies: mn/mx (exploitation); and ma/am (exploration). The two strategies together provide a good account of the participants' choices (as they should; all participants lie on or essentially on the line  $\alpha + \beta = 1$ ).

For the five room condition (shown in the bottom of Figure 9) we see broadly similar results to Experiment 1 and to the predilection for participants' strategy mix to stay close to and walk along the theoretically proposed mx/mn - ma/am axis between (0,1) and (1,0) when being tested in hiding or seeking conditions. The change in the

location of the “blue dots” between conditions and away from the theoretically proposed axis suggests that there is error in expressing our theoretically proposed strategy mix, or alternatively, other strategies are being used, but only contribute a small and inconsistent influence to participant performance.<sup>1</sup> We do note though that we find more noise for Experiment 2 than Experiment 1. Since Experiment 2 was online it may be that these participants were less attentive or motivated, but ultimately the source of differences between in lab and online versions remain conjectural.



**Fig 10.** Plotting Changes in Strategy Vectors: Two and Five Room Conditions Sally Distribution. The left plot shows the two-room condition and the right plot shows the five-room condition. For each plot the choice data for each participant was fit to best explain their choice data using a basis vector of the mx/mn and ma/am strategies. These two vectors are subtracted to yield a change vector. This is plotted and the average of all the individual change vectors (thin black lines) is shown as a thick green line.

Unlike in Experiment 1, we did not see a notable difference in the strategy mix between the two-room and the five-room conditions. This appears to be largely driven by the fact that participants in Experiment 2 did not utilize the exploration seeking strategy (probability matching) as much as participants in Experiment 1. Probability matching was replaced with maximizing in Experiment 2, so we still observe that participant strategies lie close to the line  $\alpha + \beta = 1$  implying our model appropriately accounts for participant hiding and seeking behavior in both two-room and five-room conditions (Figure 9). Again, the line  $\alpha + \beta = 1$  represents a strategy mix of exclusively

<sup>1</sup>To test this we created random strategy spaces for plotting observed histograms and performed the same analyses (data and graphs not shown). True participant data looks very different when expressed as a linear combination of two random strategies. That is, of the thousands of random strategy vectors we generated, none of them expressed participant behavior as well as mx/mn and ma/am.



matching and maximizing for seeking, and exclusively antimatching and minimizing for hiding. If there is any consistent migration between the two conditions it is generally in favor of more minimization when hiding, and there is a numerically greater trend in the five room condition.

### **Special case opponent strategies**

Participant behavior against Kala (uniform) and Bo (100%) followed closely that of Experiment 1. While all strategies against a uniform distribution are equivalent, participants mostly produce a nearly uniform distribution, while hiding and seeking. Note that the reflection of the uniform is the uniform, thus the vector reflection hypothesis appears to still function in this ambiguous case.

Participant seeking against the 100% distribution utilized the unique reasonable strategy – only picking the 100% room. With respect to hiding, participants show a mix between exploiting a particular 0% room, and distributing hides across all of the 0% rooms, interesting since all of which are equally good strategies in our task environment.

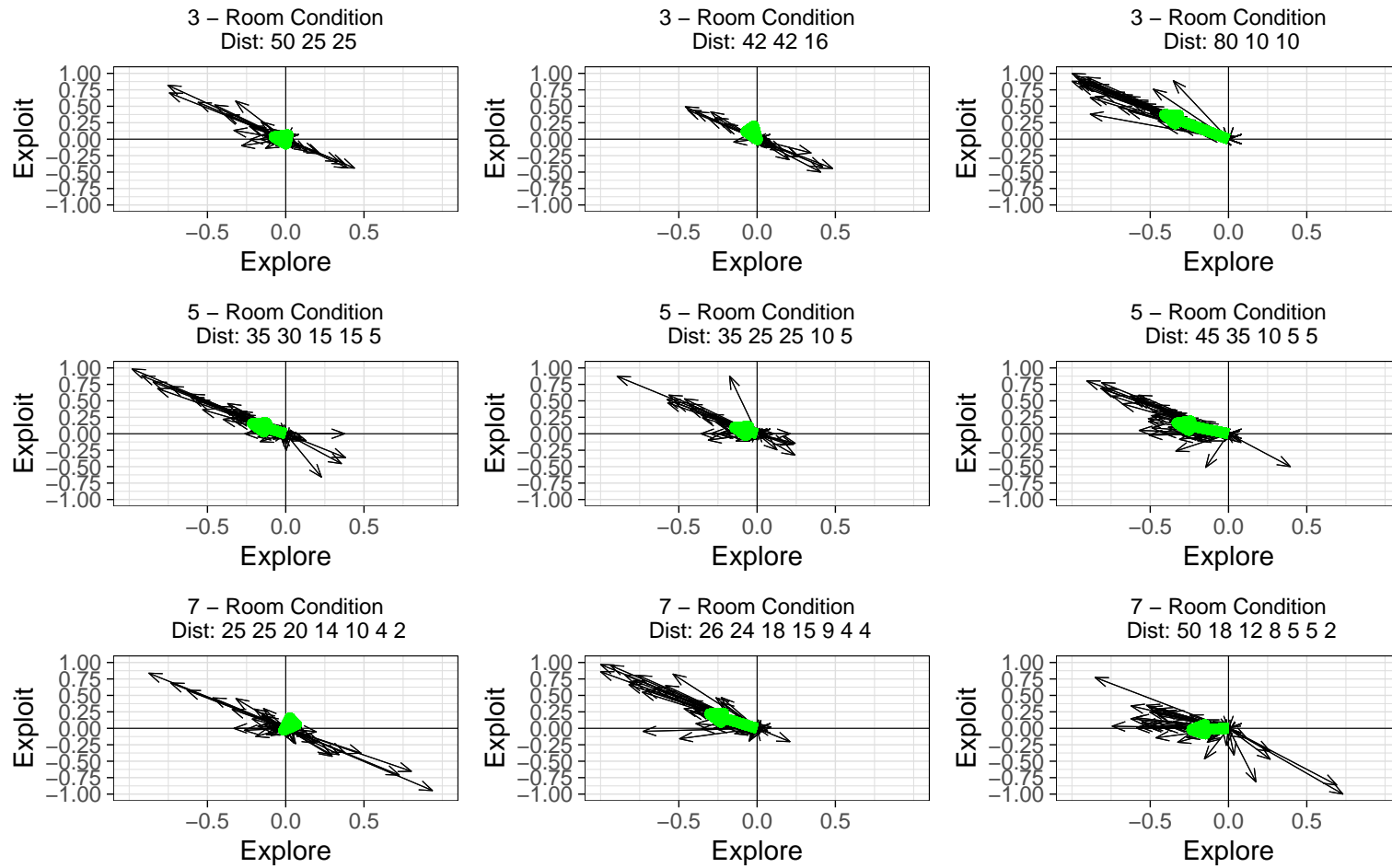
The first two experiments show broadly consistent results. Participants are sensitive to the specific probability distributions used by their simulated opponents for hiding and seeking. They modulate their use of the probability information depending on the context they find themselves in: hiding or seeking. A combination of ma/am and mx/mn strategies provides a good account of participant choices even in the five room situation when there is no mathematical reasons that it should. Additionally, the mixture of ma/am and mx/mn strategies participants deploy is modulated by the complexity of the distribution, in this case indexed by the number of rooms. Other distributions, like the uniform or one 100% room, show that our participants adapt sensibly. When there is no benefit to adjusting a mixture for either gaining information or improving outcomes (uniform) the participants mathematically remain stable in their choices across contexts. When there is little need to track opponent choices, and little ambiguity about consequences, participants make the optimal choice. Against the always-choose-the-same-room opponent, matching and maximizing are basically the same thing. These distributions generally don't do much to help us theoretically, but they do provide confirmation that participants are sensitive to the task manipulations and behave reasonably across various distributions, including these “degenerate” ones.

## **Results Experiment 3**

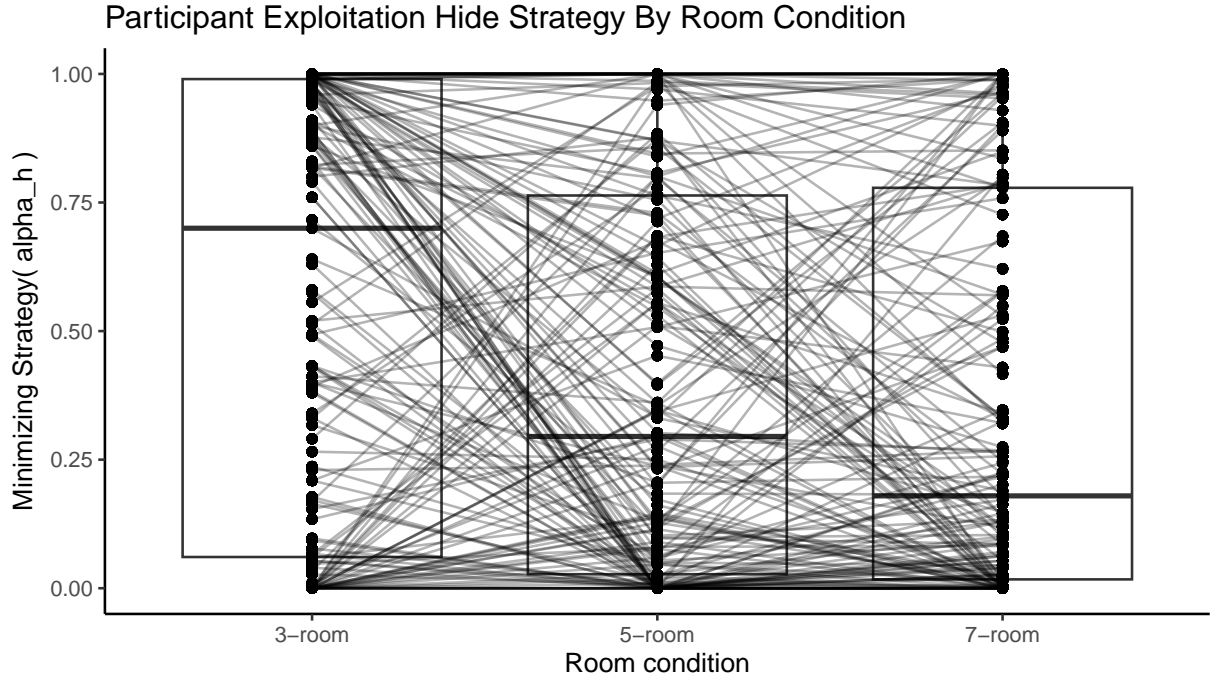
Experiment 3 increased the maximum number of rooms participants faced. This was to help distinguish if scenario complexity would influence how participants mixed ma/am and mx/mn strategies. Additionally, we included a wider range of distributions so as to have some distributions that had no permissible reflection. This was to probe our suggestion that the cognitive operation for calculating an opposite distribution when someone is switching between the roles of hider and seeker (or for pursuit versus avoidance) is akin to vector reflection.

## **Strategy mixes change between hiding and seeking and across room-count conditions**

Figure 11 provides an overview of how strategy mixes change between seeking and hiding. Comparing participant changes in strategy combination between hiding and seeking conditions reveals that, in general, participants use the computationally simpler, yet optimal, exploitation strategy more when hiding than they do while seeking. However, the strength of this effect does not obviously correlate to room-count as we might expect if computational complexity were a primary driver of policy mixing. Moreover, Figure 12 indicates that as situational complexity (room-count) increases, participants use the exploitative hiding strategy (minimizing) *less* in aggregate.



**Fig 11.** Strategy Changes for Different Numbers of Rooms. In general, there is more use of the optimal strategy when hiding than when seeking (green arrows point up and to the left). The increase in the number of rooms increases the complexity of the task. Participants shift their strategy mix based on decision context.



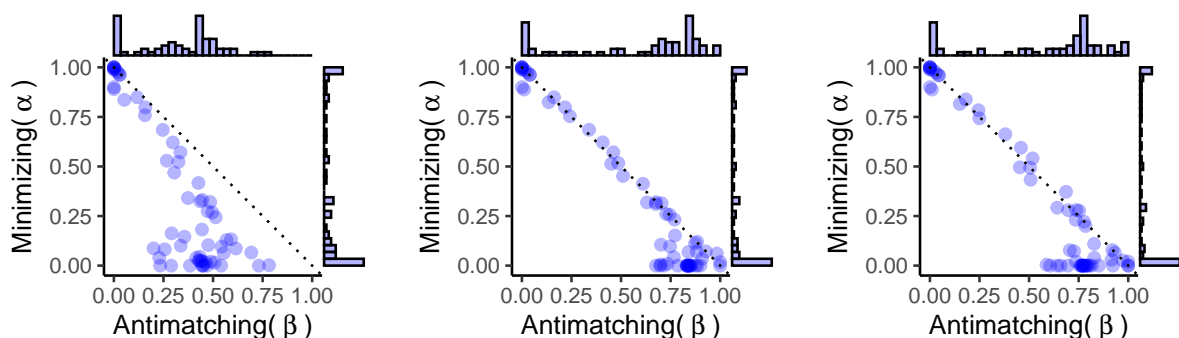
**Fig 12.** Participant use of the minimizing hiding strategy decreases as dimensionality increases. Boxplots the minimizing strategy component by room-number condition, and also plots the individual participant strategy mix for each condition. Lines connect individual participant strategies across conditions.

Taken together, we conclude that participants have a higher propensity to use an exploration strategy as situational complexity increases, yet at any given level of complexity, employ a exploitation more when hiding than when seeking. To explain this, we introduce an *assumed-payoff* hypothesis, where hiding strategies are generally more exploitative than when seeking because participants hold prior beliefs that a failed hide is worse than a failed seek. Perhaps with more room options, the perceived importance of each individual hiding trial outcome is lesser because the overall probability of being found is lesser, thus reducing motivation to select a strict minimizing strategy when hiding. This hypothesis can be tested in future studies by manipulating payoffs of successful/failed hide/seek via gamified points, money, and the like.

It may be that computational complexity and assumed-payoffs jointly influence player behaviour. It is still true that  $n$ -dimensional geometry is harder to work with than  $m$ -dimensional geometry where  $n > m$ . This increases the computational cost of determining the opposite probability distribution as room-count increases, thereby disincentivizing exploration when hiding. The reduction in risk of being found when hiding as room-count increases may simultaneously *incentivize* exploration, moderating the overall effect.

## Vector projection methods account for invalid reflections

Comparing the different approaches to handling invalid reflections: projection back to the simplex via the uniform shift method or back projection to the simplex via the closest point method reveals little difference. It is likely that any reasonable method to project an invalid reflection back to the simplex will result in very similar distributions. Future mathematical work is needed to determine which methods may differ the most, and which particular reflections may result in the largest discrepancy between projection methods. However, we do find that both projections produce dramatic improvements in our ability to model participant choice counts (for example see specifically the seven room condition in Figure 13). This implies that when a reflection does not exist within the simplex, and therefore the strategy cannot be expressed behaviorally (since it has negative values), participants instead use a distribution within the simplex that is near the invalid reflection.



**Fig 13.** Defining participant hide strategies using the original invalid reflection (left), unshift projection (center), and closest simplex point projection (right) in the 7 Room (50,18,12,8,5,5,2) condition. Both methods of projecting back to the simplex, either along the path of reflection or to the nearest point, result in considerable improvement in the fit to participants' choice probabilities.

While we may not be able to tell which projection is better here, each projection method comes paired with distinct theoretical implications. For the uniform shift method, any internal representations of probability distributions need not require that the invalid vector reflection (of negative entries) ever exist. That is, the adjusted reflection may be computed geometrically by following the projection path from the original distribution, over the uniform, until the boundary of the simplex is reached. This would imply humans only represent what exists within the probability simplex in order to compute opposite probability distributions.

Alternatively, the Euclidean simplex projection requires that the invalid reflection be explicitly represented. This would imply that humans can represent a broader interpretation of probabilities than those that are non-negative and sum to 1. Under this hypothesis, probability could be represented as any other geometric/visuospatial problems are, and finding the closest point on the simplex is only a means to behaviorally express this representation in a

task that is constrained by the formal definition of probability.

The benefit of our analysis here is that future researchers can propose new methods to account for invalid reflections. The effectiveness of alternative models can be directly compared to each other by examining the quantitative differences between strategy vector coefficients, and model error.

## Discussion

In this work, we found that modeling people’s choices as histogram vectors provided a concise summary of choice behavior. Whether seeking or hiding, regardless of room number, or opponent probability distribution, a linear combination of two canonical strategies (Exploration and Exploitation) did a good job of accommodating participant choice frequencies. Exploration when seeking is driven by traditional probability matching. Exploration when hiding is driven by our novel proposal of probability antimatching, formalized as a Euclidean vector reflection. Exploitation is selecting only the highest (lowest) probability room when seeking (hiding). Our results suggest that people deal with the complexity of random scenarios by counting outcomes, and if avoiding said outcomes, perform additional cognitive computations akin to a vector reflection to inform their behavior.

Experiment 1 established the efficacy of the hide-and-seek task in measuring pursuit and avoidance strategy. Experiment 2 was a replication of Experiment 1 using online materials. While there were some differences in the details, we found similar results to Experiment 1. The data from these two studies showed the effectiveness of histogram methods for describing human choice policies, but it did not test the consistency of this two strategy mixture account with varying complexity nor the “edge-cases” of the vector reflection account. Experiment 3 addressed this deficit. We adopted a within-subjects design so that every participant faced at least one 3-room, 5-room, and 7-room condition. In addition, some participants also faced distributions that had invalid reflections. Again, we found a near exclusive mix of maximizing and matching when seeking, and a near exclusive mix of minimizing and antimatching when hiding. We also found that when hiding against distributions with invalid reflections, participants’ antimatching strategy selected a distribution on the probability simplex that was “close” to the actual, but invalid, reflection. It is unclear exactly how the “close” distribution was derived. We considered two methods for this: backtracking along the line of reflection to the first “legal” histogram, and projecting from the point of reflection to the nearest point on the simplex. For our design, both selection methods led to very similar predictions. Finding a better way to separate these two accounts is a goal for future work, but the fact that two different geometrical accounts lead to, in practice, similar predictions means that the practical consequences for using either method is small.

In both Experiments 1 and 3 (but less so in Experiment 2) participants utilized a more optimal strategy mix when hiding than when seeking. We interpret this as an asymmetric weighting of gains and losses and consistent

with prospect theory (Kahneman & Tversky, 1979). It has also been suggested from choice behavior data that severe risk may influence estimates of probabilities (Weber, 1994), but our data and analyses suggest that frequencies can be estimated separately from outcome severity. We suggest that what happens when severity and risk intrude is that participants will mix their two basis policies,  $ma/am$  and  $mx/mn$ , differently.

One of our motivations for proposing our frequency histogram account is the fact that many real world processes are continuous and thus too complex to expect any precise estimates by human beings. We wanted to see how well frequency estimates worked and how they scaled when task complexity varied. We operationalized increasing complexity as increasing the number of rooms. Given how well a discrete model like ours works, we conjecture that when people are confronted by continuous real world problems there may be a feasibility constraint that leads to the adoption of a simpler, discrete model like the one we propose here.

Our work demonstrates that participant choice frequencies for seeking are adequately defined as a mix of two strategies: traditional probability matching and optimal maximizing. We describe a novel strategy “antimatching” revealed by hiding behavior. We articulate human stimulus avoidance strategies with just two dimensions: anti-matching and minimizing. Antimatching and minimizing are direct analogues of matching and maximizing. It is noteworthy that we can achieve such predictive success as there is no unambiguous mathematical definition of an opposite probability distribution. We highlight that the distinction between the two scenarios are more a reflection of task demands and the participant’s perspective than they are differences in a fundamental representation of probability.

Across our three studies, we observe a pattern of findings where participants use a more optimal strategy when hiding than they do when seeking. The strength of this finding, and the manner in which it is affected by the number of room choices (dimensionality) is unclear, and will require further study. Our current explanation for this shift towards optimality is an intuition from evolutionary considerations: a failed hide often carries greater consequence (eaten by a predator) than a failed seek (being hungry). This might induce a default payoff matrix that skews to value the outcomes of hide success over seek success. By modifying our hide-and-seek task to accommodate different payoffs, via arbitrary in-task points, monetary, or course credit it should be possible to test this intuition. And there may be practical benefits to confirming this idea. Many problems of daily life can be presented in a pursuit/avoidance dichotomy. It may be that simply framing a problem as something to hide from, rather than something to pursue, will make people act more optimally, at least in the short term. For example, would people make better financial decisions if the problem were framed as an avoidance of losses/costs (including opportunity cost) rather than a pursuit of gains (Kahneman & Tversky, 1979; Levin et al., 1998; Soman, 2004).

Our work presents a testable hypothesis for strategy utilization that accounts for both stimulus pursuit and stimulus evasion, therefore creating a meaningful extension to current probability matching literature. The formal geometric terms used to represent participant strategy mixes allow for richer analysis and quantitative theoretical

development. For example, if another researcher proposes an alternative definition of antimatching, we can again represent participant strategy mix as a linear combination of Euclidean vectors and compare model fit via the magnitudes of strategy mixture coefficients and model error.

With this work, we are not advocating that people are literally and explicitly computing vector reflections to determine their hiding strategies. Claiming this would imply that people are performing non-trivial algebra in their heads in order to complete our task. The benefit of a model with geometric interpretations is that it can be explained and conceptualized *without* algebra. This is similar to how catching a thrown ball is fundamentally different than solving a physics problem about a thrown ball. Our model with a built-in geometric interpretation, holds a “feasibility advantage” over other models such that the cognitive mechanisms required to emulate its output may already exist and be used for other visual-spatial capacities. Consider the analogy of Bayesian cognition. That is, the claim that people are “Bayesian” is infeasible as a literal claim since we cannot assume that people calculate the computationally difficult integrals in their head required by Bayes’ rule, despite often presenting behavioral patterns that loosely emulate Bayesian models (Jones & Love, 2011). It may be that some other mechanism is the driving force of this emulating antimatching behavior. Claiming people to be literally Bayesian is equivalent to claiming that people literally “use” vector reflections. While the geometric nature of vector representations provides some degree of feasibility over and above literal Bayesian cognition, we still stress that our model is effective in describing *behavior*. Cognitive and neural extensions of its implementation are left for future work.

## Acknowledgements

We appreciate the contributions of Vivian Diberardino for assistance with code editing and writing the code for strategy change plots. Britt Anderson was supported by an NSERC Discovery Award. The manuscript files, data analysis and plotting code, and the data are freely available at <https://codeberg.org/brittAnderson/prob-antimatch>.

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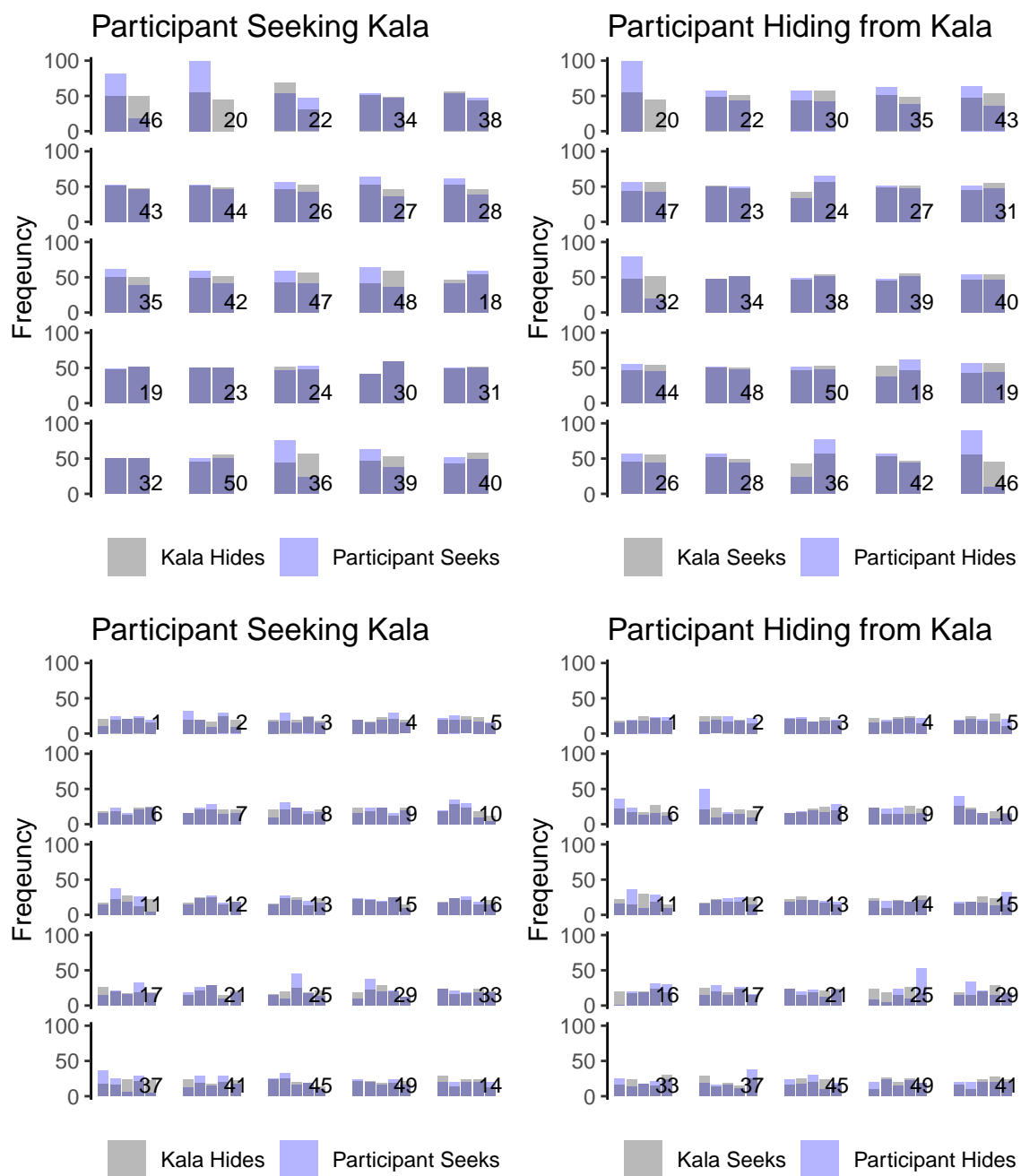
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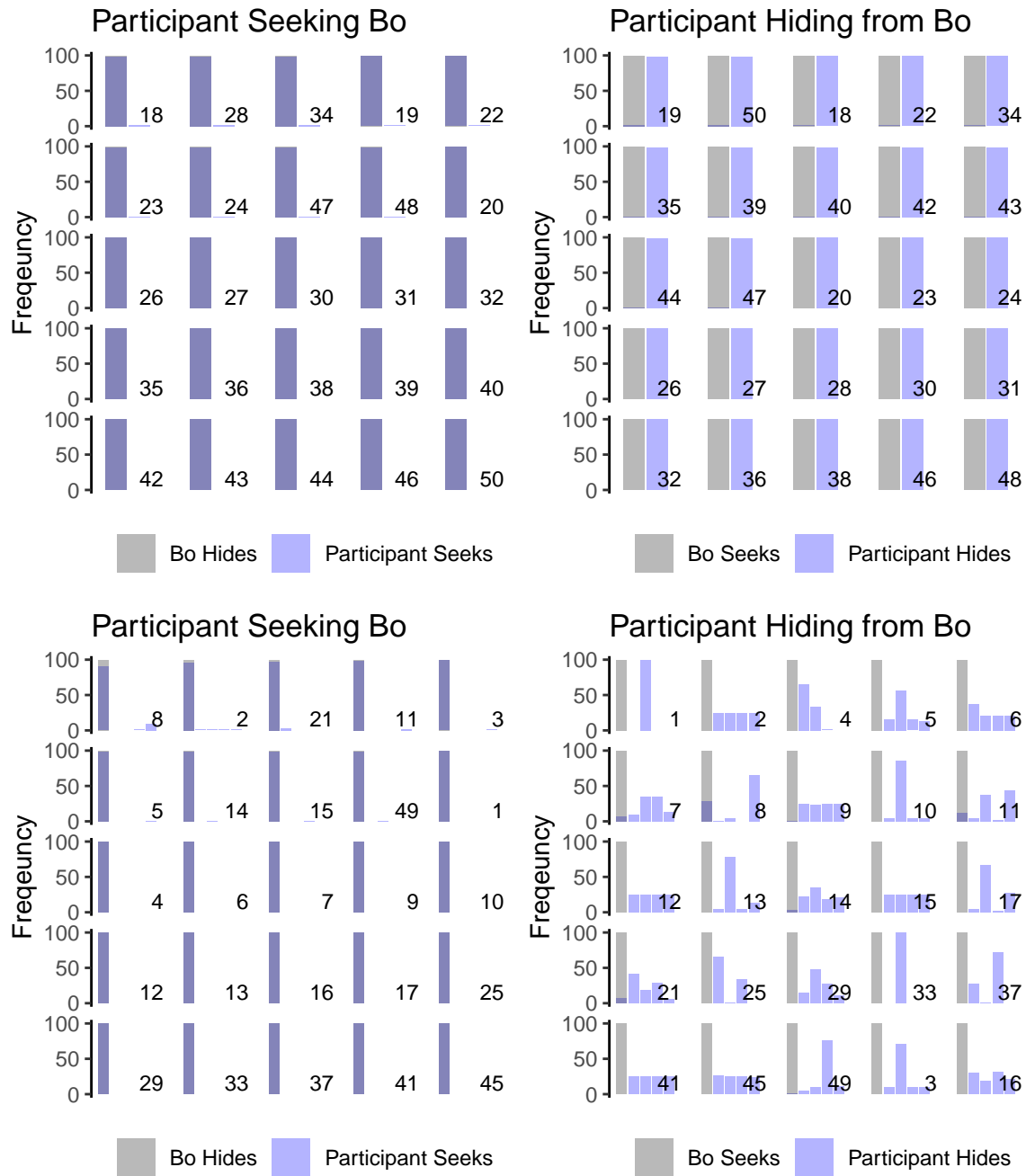
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## A Additional Distributions



**Fig 14.** Histograms for Alternate Distributions (Kala). Confirming that participants understood the task and behaved accordingly we find that for an opponent distribution that chooses all options equally often ('Kala'; Two Room Condition Row 1 and Five Room Condition Row 2) the choice of the participant has no effect on their probability of winning and their default choice histogram largely resembles matching with some individual heterogeneity.



**Fig 15.** Histograms for Alternate Distributions. Confirming that participants understood the task and behaved accordingly we find that for an opponent distribution that chooses one option 100% of the time ('Bo'; Two Room Condition Row 1 and Five Room Condition Row 2) the participants always look there and never hide there.