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Homework 2 12/7/2020

Q1.] Ordinary least Squares (OLS) can be expressed as a minimization problem

$$\min_{\beta} \|X\beta - y\|_2^2$$

$\hat{y} = X\beta$ is the general linear model

X is the design matrix having predictors

β is vector of regression coefficients or beta parameters.

To find the value of β above eqn can be written in the form

$y = X\beta$, where y is the dependent or response variable.

Now, $y = X\beta$ — (1)

Taking left inverse on both sides

$$(X^TX)^{-1}X^T y = ((X^TX)^{-1}X^T) \times \beta \quad (2)$$

Now, $(X^TX)^{-1} = (X^TX)^{-1}$

$\therefore ((X^TX)^{-1}X^T)X = (X^TX)^{-1}.(X^T.X) = I$

On Simplifying, eqⁿ (2) becomes

$$\boxed{\beta = (X^T X)^{-1} X^T \cdot y} \quad (3)$$

for $(X^T X)^{-1}$ to exist, mandatory condition is rank $(X^T X)$ should be same as number of columns of X. This means X should not have any column which is linearly dependent on other column of X.

Thus, if there are any dependent column, the design matrix becomes close to singular and as a result, the least square estimate becomes highly sensitive to Random errors in the observed target, producing large variance.

Q2.] Corrective measures:

i) Design matrix should be a tall matrix, ie, no. of rows should be more than no. of columns.

ii) Using VIF to identify inderpendent columns.

iii) Using L_2 (Ridge) and L_1 (Lasso) Regularization techniques.
iv) Removing highly correlated variables
v) Linearly combine the correlated variables, such as addition
vi) Use Principal Component Analysis (PCA) to find variables (Principal Components) which are linearly independent.
vii) Using other estimators like Partial least square regression.