# Introduction to GARCH

## Mean Equation:

Before estimating GARCH, we must have a mean equation that can be either be AR, MA or ARMA.

When should we use ARCH or GARCH models?

There are two pre-conditions that must be satisfied to go to ARCH / GARCH modelling.

1. The estimated residuals of the mean equation should exhibit clustering of volatility.
2. Before estimating GARCH, we must have ARCH effects in the residuals.

## What is Volatility Clustering?

When period of high volatility is followed by subsequent periods of high volatility and similarly, some period of low volatility is followed by some continuous periods of low volatility, this pattern of volatility is called volatility clustering.

It implies that the residuals are conditionally heteroskedastic and it can be represented by ARCH and GARCH model.

## ARCH Effect

$H_0$: There is no effect.

Note: We can estimate ARCH-GARCH models, if the estimated residuals of the mean equation will have volatility clustering and there is ARCH effect. The ARCH-GARCH model will be estimated from variance equation.

Suppose … … … … … (1)

Where , where and

Then … … … … … (2)

That means GERCH extends ARCH model by including i.e. lagged variance of error.

Here (omega) > 0

$\geq 0$ and $\geq 0$.

Here note that a GARCH (1, 1) model like an ARCH (1, 1) is based on a mean equation.

It first specifies an equation for conditional mean i.e. eq$^n$ 1 and q$^{nd}$ specifies an eq$^n$ for condition variance i.e. eq$^n$ (2).

(1) From eq$^n$ (1), we know that i.e. we are scaling the random innovation with to get .

Let us look at its conditional expectation i.e.

Note that the information set is conditional on and i.e. .

Hence, . That is conditional normality of given with conditional variance .

(2) Secondly define

Where is a surprise term correlated with the information set .

Now let us decompose

$\boxed{1}$ $\boxed{2}$

We have from the first part:

Since ,

We have … … … … … (4)

Incorporating eq$^n$ (2) here we can rewrite eq$^n$ (4) as:

… … … … … (5)

Remember here that in GARCH (1, 1) model, we have effectively specified ARMA (1, 1) process for the squared innovation.

(3) Here if weakly stationarity solution

the unconditional variance of

(4) Now rewrite

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Hence has an infinite memory in terms of . Here all the past matter, and the conditional variance at the time *t* is simply a function of all the past square residuals.

Giving the stationarity condition i.e. , the coefficients will be decaying. This is the reason why GARCH (1, 1) is a much better model than ARCH.

(5) A GARCH (2, 0, 2) can be written as –

Here variance of i.e. E

Or

White noise error independent of

Previous period's square residue