VALUE OF MONEY

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Exercise: A

- Suppose Mr. Ramesh wants to construct a pension fund for himself. At present he is 30 years old and he wants to receive pension of Rs. 50,000 per month starting from age of 50 to 90 years. If market interest rate is 9% p.a. for the entire investment period, how much he should invest per month?

Exercise: B

Suppose a 40 yr. old employee who is expecting to retire at age of 60 is planning for his retirement. At present he estimates that he can live comfortably with ₹24,000 per year in terms of present rupee value. How much should he save/invest each year until his retirement so that he can start getting pension at the end of 21st from now till 10 years, that can allow him to live as comfortably as he desired now.

Future Value (FV):

FV = Principal + interest,   if i = 10%, P = 100

$F_1 = 100 + (0.10 \times 100) = 100 \times (1.10) = 110$

$F_2 = [100 + (0.10 \times 100)] + 0.10\ [100 + (0.10 \times 100)]$

$= 100 \times (1.10) \times (1.10) = 121$

Or $F_2 = F_1 \times 1.10 = 121$

In case of simple interest: F2 = 100 + [2 × (0.10 × 100)] = ₹120

In case of compounding interest: F2 = ₹121,

i.e. excess ₹1 is interest on interest.

Symbolically:

$FV_1 = PV + (PV \times i)$ or $PV (1 + i)$

$FV_2 = FV_1 \times (1 + i)$

Or $FV_2 = PV \times (1 + i) (1 + i) = PV \times (1 + i)^2$

.

$FV_5 = PV (1 + i)^5$

.

.

.$FV_n = PV (1 + i)^n$

$(1 + i)^n$ = Compound value factor (CVF)

$$FV_n = PV \times CVF_{n,i}$$

Example:

If you deposit ₹1000 in a bank at 5% interest rate P.A. Then in future

$FV_1 = 1000\ (1.05) = ₹1050.00$

$FV_2 = 1000\ (1.05)^2 = ₹1102.50$ or $1050\ (1.05) = 1102.5$

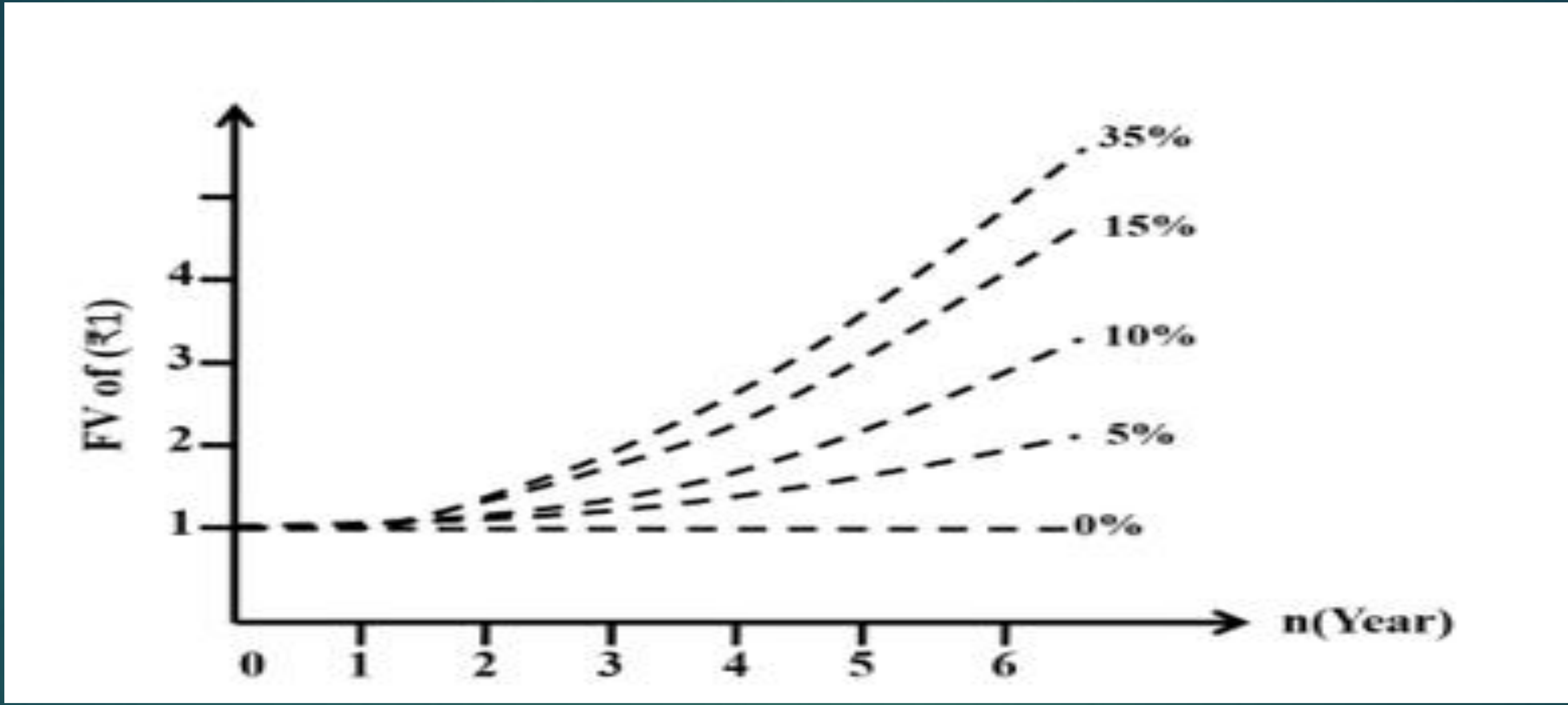
$FV_3 = F_2\ (1 + i)$ or $PV\ (1 + i)^3$

$= 1102.5\ (1.05) = 1157.6$ or $1000\ (1.05)^3 = 1157.6$

Similarly,

$FV_{15} = 1000\ (1.05)^{15}$

Growth of Rs. 1 at different n and i



Example 1:

You have ₹10,000 and you want ₹16,105 after 5 year. Then at what interest rate you should invest it?

Ans: The ratio of cash flow consists,

$16{,}105 / 10{,}000 = 1.6105$

That is, you must earn an interest that will allow ₹1 to grow up to ₹1.6105 at 5 years. That is

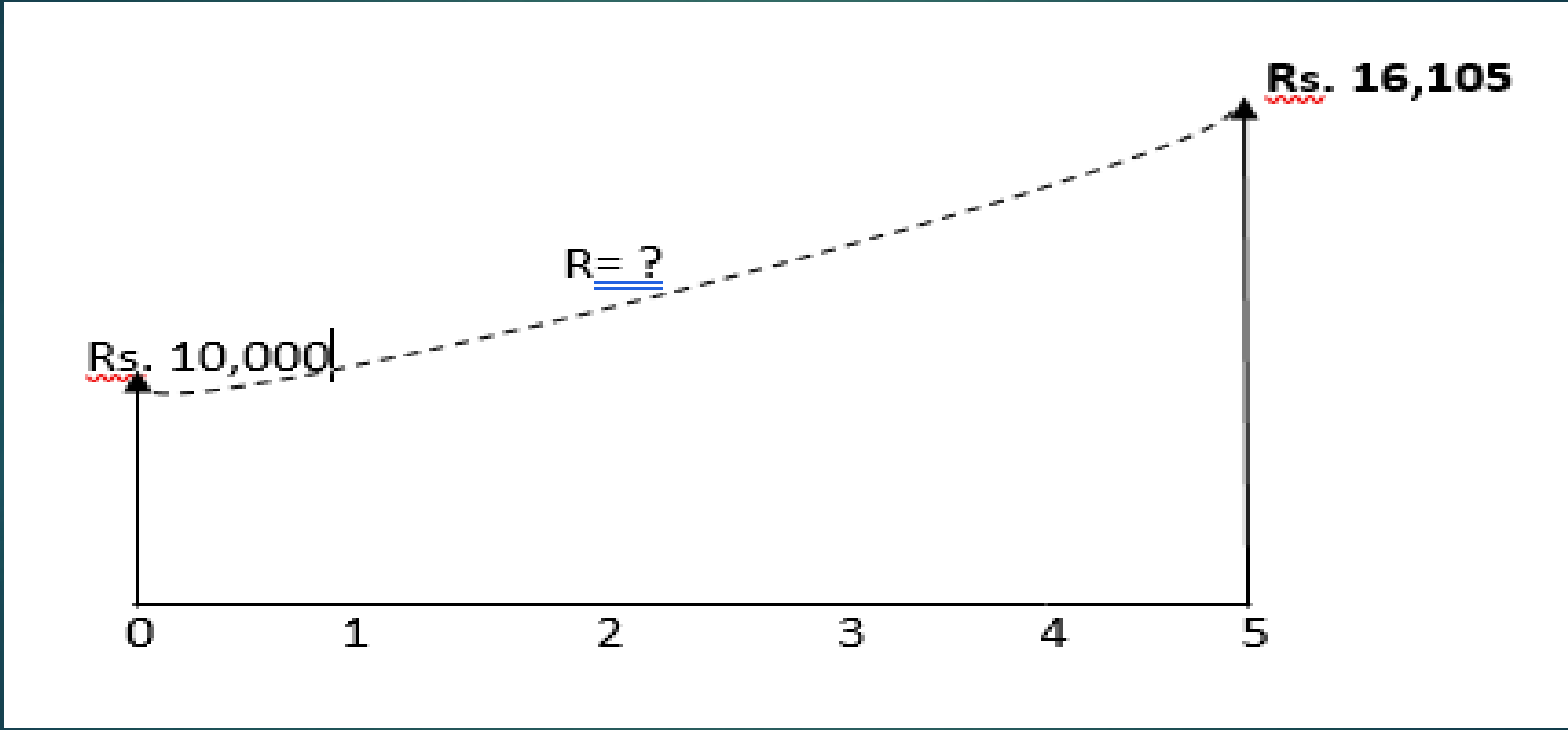
$10{,}000\,(1 + i)^5 = ₹16105$

$(1 + i)^5 = 1.6105$

$i = 10\%$

$10{,}000\,(1.1)^5 = 16105.1$

At What rate Principal should Grow?



Present Value (PV):

$FV_1 = PV (1 + i)$

$PV = FV_1 / (1 + i)$

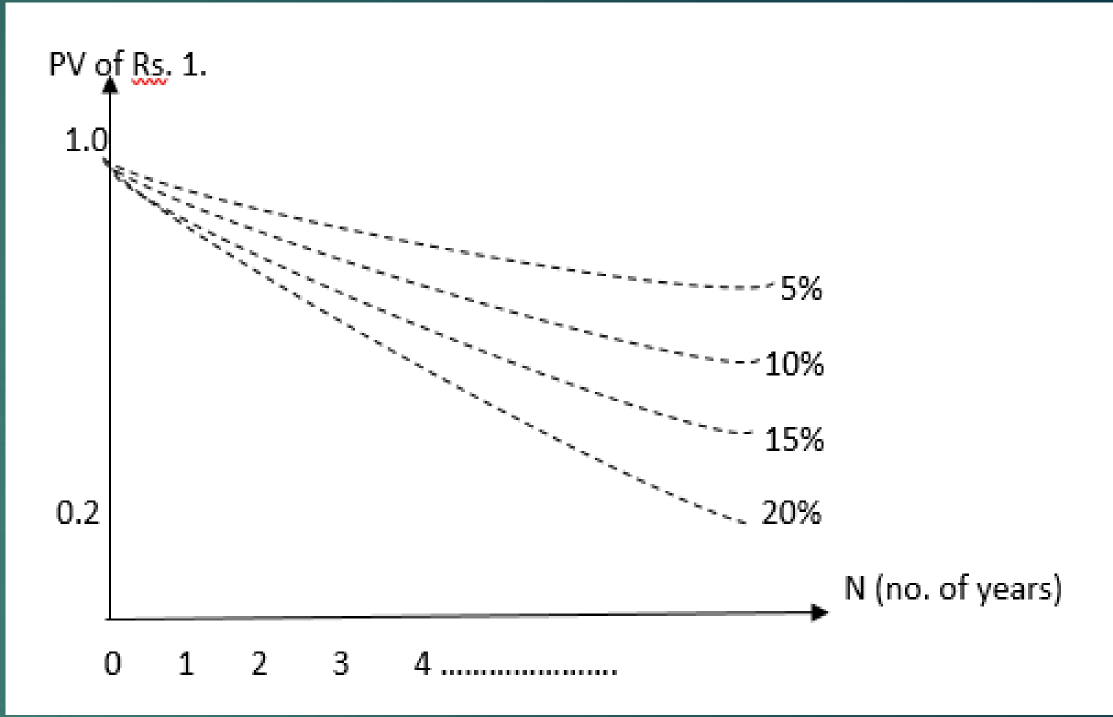
If $FV_1 = 100 (1.10) = 110$

Then $PV = 110 / (1.10) = 100$

$FV_2 = PV (1 + i)^2 = 100 (1.10)^2 = 121$

Then $PV = FV_2 / (1 + i)^2 = 121 / 1.21 = 100$

$PV = FV_n / (1 + i)^n$ or $FV_n \times [1/ (1 + i)^n]$



$[1/ (1 + i)^n]$ = Discount factor or present value factor (PVF)

$PV = FV_n \times PVF_{n,i}$

PV of ₹1 over n time period

Example 2:

Suppose you have an expected Cash Flow of ₹50,000 to be received after 5 years from now. What is interest PV, if i = 9% ?

Ans: PV = 50,000 / $(1 + 09)^5$ = 50,000 / $(1.09)^5$

= 50,000 / 1.5386 = ₹32,497.07

1 / 1.5386 = 0.6499 = $PVF_{5,\ 9\%}$ for n = 5 and i = 0.09

Similarly, for 10 Years;

PV = 50,000 / $(1.09)^{10}$ = 50,000 / 2.3074 = ₹21,1201.21

1 / 2.3674 = 0.4224 = $PVF_{10,\ 9\%}$ for n = 10 and i = 0.09

Example 3:

Mr. X want to sell his land. Buyer 1 offers ₹10,000 now, whereas buyer 2 offers ₹11424 after 1 year. If market interest rate is 12%, what should Mr. X do?

Ans: PV of 11424 = 11424 / 1.12 = ₹10,200

Secondly, if Mr. X prefers to sell it to buyer 1 at 10,000, then after 1year.

FV of 10,100 = 10,000 × (1.12) = ₹11,200

According to PV, buyer 2 is giving more (10,200) than buyer 1 (10,000).

According to FV of buyer 1, it is less than buyer 2. Hence, Mr. X should prefer to sell it to buyer 2.

Example 4:

Mr. X, a financial analyst thinks that price of land that cost ₹85,000 will certainly grow up to ₹91,000 by next year and recommends Mr. Y to buy it. Given market interest rate of 10%, should Mr. Y accept the advice of Mr. X to buy the land?

Ans: (a) As per FV = 85000 × (1.10) = ₹93,500

Which is much more than appreciated price of land after 1 year (91,000).

(b) As per PV, 91,000 / (1.10) = ₹82,727.27

which is much less than initial investment.

(c) As per the interest gain

$85{,}000\,(1 + i) = 91{,}000$

$\Rightarrow (1 + i) = 91000 / 85000 = 1.0706$ (i.e.) $i = 0.0706$ or 7.06%

Finding out No. of Periods (n):

Example 5:

You wanted to have ₹50,000, but at present you have ₹25,000. If you invest your ₹25,000 at 12% interest rate per annum, then how long it will take you to make your investment ₹25,000.

Ans: $25{,}000 \times (1.12)^n = 50{,}000$ or $50{,}000 / (1.12)^n = 25{,}000$

- $(1.12)^n = 2$
- $n = 6$ years

Example 6:

You have ₹2.3 million. If you invest it at 5% P.A interest rate, then how long it will take you to make your investment ₹10 million.

Ans: ₹23,00,000 × $(1.05)^n$ = 10,000,000

- 1.05^n = 10,000,000 / 2,300,000 = 4.35
- n = 30 years

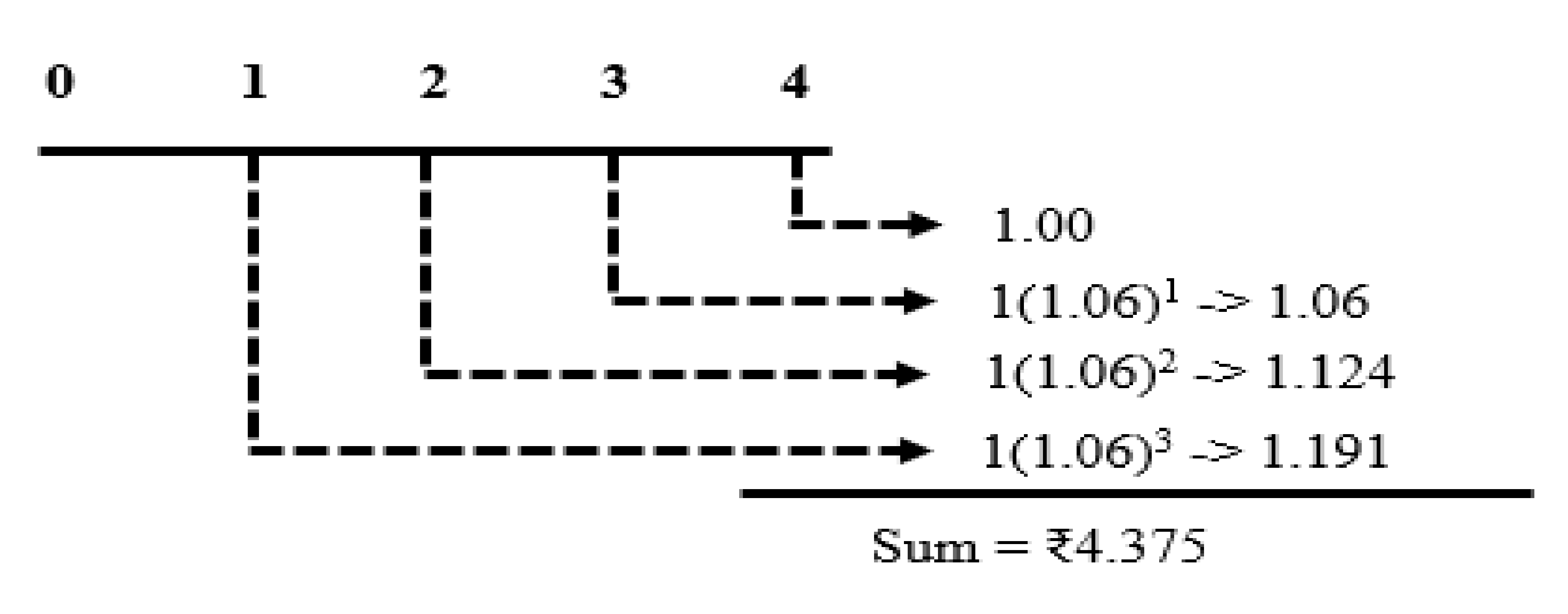
Time Value in case of continuous CF

1. Annuity 2. Growing Annuity 3. Perpetuity and 4. Growing Perpetuity

Annuity: Annuity is a fixed cash flow (payments/receipts) for a specified no. of times. House rent, car loan, house loan etc. are examples of annuity.

Suppose a fixed sum of rupees i.e. ₹1.00 is deposited in a savings bank a/c @ 6% interest P.A for 4 years. Then how the annuity will grow.

Cont…..



CF_1 has a scope to grow for 3 years at 6% P.A rate i.e.

$1(1.06)^3 = 1.191$

CF_2 has a scope to grow for 2 years at 6% P.A rate i.e.

$(1.06)^2 = 1.124$ and

Similarly, $CF_3 = (1.06)^1 = 1.06$

$CF_4 = 1.00$

The aggregate compounded value of ₹1.00 deposited at the end of each year for 4 years would be 1.191 + 1.124 + 1.06 + 1.00 = ₹4.375.

The above example can be expressed

$FV_4 = A\,(1 + i)^3 + A\,(1 + i)^2 + A\,(1 + i)^1 + A$

$= A\,[(1 + i)^3 + (1 + i)^2 + (1 + i)^1 + 1]$

.

$FV_n = A + A\,(1 + i)^1 + A\,(1 + i)^2 + \ldots\ldots + A\,(1 + i)^{n-1}$

$FV_n = A\,[1 + (1 + i)^1 + (1 + i)^2 + \ldots\ldots + (1 + i)^{n-1}]$.............(1)

- Now multiply (1 + i) in both sides of equation (1)

$FV_n\,(1 + i) = A\,[(1 + i) + (1 + i)^2 + (1 + i)^3 + \ldots\ldots + (1 + i)^n]$.............(2)

Subtract equation (2) from equation (1)

$FV_n (1 + i) – FV_n = A [(1 + i)^n -1]$

$FV_n + FV_n i - FV_n = A [(1 + i)^n -1]$ (3)

Rearranging the above equation 3;

$$\boldsymbol{FV_n = A\,[\frac{(1+i)^n - 1}{i}]} \quad \textbf{or} \quad \boldsymbol{A\,[\frac{(1+r)^T - 1}{r}]}$$

$$\boldsymbol{A\,[\frac{(1+i)^n}{i} - \frac{1}{i}]} \quad \textbf{or} \quad \boldsymbol{A\,[\frac{(1+r)^T}{r} - \frac{1}{r}]}$$

Hence FV_n = Annuity CF × Compounded value factor for annuity

$[\frac{(1+i)^n - 1}{i}]$ = Compounded value factor for annuity ($CVFA_{i,n}$)

Or Future value interest factor for annuity ($FVIA_{i,n}$)

Example 10:

Suppose, you are investing ₹5000 at the end of each year for 4 years at 6% P.A interest. Find out the future value of your investment at the end of 4 years i.e. FV_4?

Ans: $FV_4 = 5000\ [\frac{(1.06)^4-1}{0.06}]$ = ₹5000 × 4.3746 = ₹21,873.00

Example 11:

Suppose you invest ₹3000 per year in investment fund at 6% P.A interest rate for 30 years. How much will you have, when you retire in 30 years.

Ans: $FV_{30} = 3000\left[\frac{(1.06)^{30}-1}{0.06}\right] = 3000 \times 79.0582 = ₹2,37,174.56$

Sinking Fund Factor (SFF)

$FV_n = A \times CVFA_{n,i}$

$A = FV_n \times (1 / CVFA_{n,i})$

$1 / CVFA_{n,i} = SFF_{n,i}$

$A = FV_n \times SFF_{n,i}$

$SFF_{n,i} = [\frac{i}{(1+i)^n - 1}]$ or $[\frac{r}{(1+r)^T - 1}]$

$$\boldsymbol{A = FV_n[\frac{i}{(1+i)^n - 1}]}$$

- You want to accumulate ₹2,37,174.56 at the end of 30 years. How much you should invest every year at 6% P.A interest rate so that, you will be able to have the above stated amount at the end of 30 years?

$$A = FV_{30}\left[\frac{i}{(1+i)^{30}-1}\right]$$

i.e. $2,37,174.56\ \left[\frac{0.06}{(1.06)^{30}-1}\right]$

= 2,37,174.56 × 0.0126

= ₹2,999.999 or ₹3000.00 P.A

SFF is used to calculate annuity for a given sum.

Present value of an annuity

Suppose you have an opportunity to receive ₹1.00 for 4 years (at the frequency of end of each year for 4 years). If your required rate of interest is 10%, then calculate the PV of the annuity?

- **Ans:**

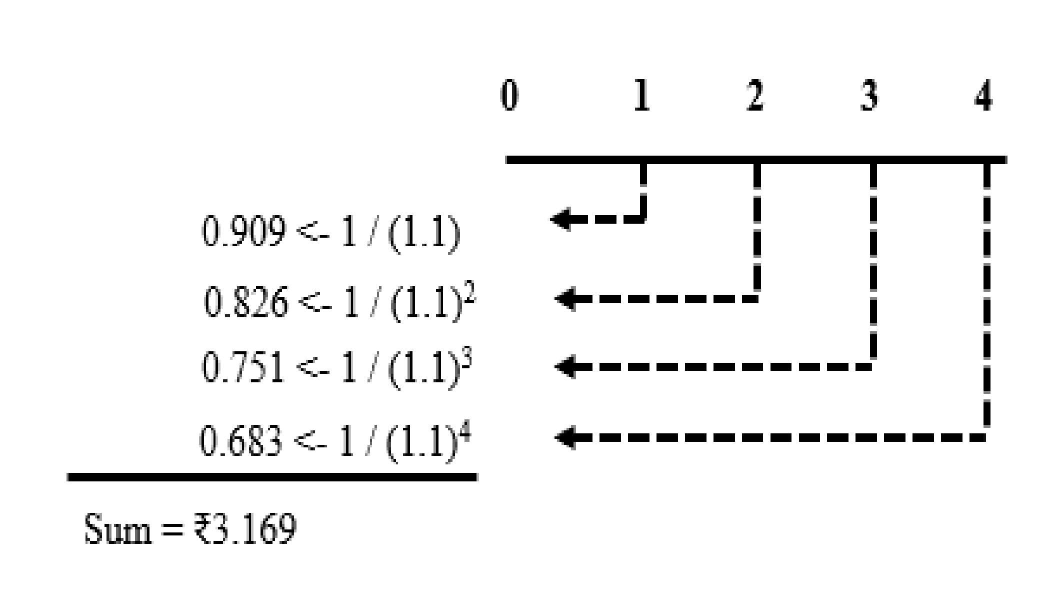
$P_1 = 1 / (1.1) = 0.909$

P_2 (PV of CF_2) $= 1 / (1.1)^2 = 0.826$

P_3 (PV of CF_3) $= 1 / (1.1)^3 = 0.751$

P_4 (PV of CF_4) $= 1 / (1.1)^4 = 0.683$

The aggregate PV = ₹3.169



The computation of the above PV of annuity can be written as.

$$P = \left[\frac{A}{(1+i)}\right] + \left[\frac{A}{(1+i)^2}\right] + \left[\frac{A}{(1+i)^3}\right] + \cdots + \left[\frac{A}{(1+i)^n}\right] \ldots\ldots. (1)$$

$$= A\left[\frac{1}{(1+i)}\right] + \left[\frac{1}{(1+i)^2}\right] + \left[\frac{1}{(1+i)^3}\right] + \cdots + \left[\frac{1}{(1+i)^n}\right] \ldots\ldots. (2)$$

Multiply 1 / (1 + i) in equation (1)

$$\frac{P}{(1+i)} = \left[\frac{A}{(1+i)^2}\right] + \left[\frac{A}{(1+i)^3}\right] + \cdots + \left[\frac{A}{(1+i)^{n+1}}\right] \ldots\ldots. (3)$$

Subtract equation (1) from (3)

$$P - \frac{P}{(1+i)} = \left[\frac{A}{(1+i)}\right] - \left[\frac{A}{(1+i)^{n+2}}\right] \;\ldots\ldots.\; (4)$$

Now multiply both side of equation (4) by (1 + i) we get;

$$P(1+i) - P = A - \frac{A}{(1+i)^n}$$

$$P + Pi - P = A\left[1 - \frac{1}{(1+i)^n}\right]$$

And $P = A\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

Hence Present value of an annuity can be calculates as:

$$P = A[\frac{(1+i)^n - 1}{i\ (1+i)^n}] \qquad \text{or} \qquad A\left[\frac{1}{i} - \frac{1}{i(1+i)^n}\right]$$

Where

$[\frac{(1+i)^n - 1}{i\ (1+i)^n}]$ = PV factor of annuity (PVFA)

or PV interest factor for annuity ($PVIA_{n,i}$)

Example 13:

Suppose a person receives an annuity of 5000/- at the end of each year for 4 years. If market interest rate is 10%, then calculate the present value of the annuity?

Ans: $P = 5000\left[\frac{1}{0.1} - \frac{1}{0.1(1.10)^4}\right]$

$= 5000\ [10 - 6.830] = 5000 \times 3.170 = ₹15{,}850.00$

Example 14:

Saurabh has got a scheme at the name of million rupee scheme as it pay ₹50,000 at the end of each year for 20 years. (50,000 × 20 = ₹1,000,000). If market interest rate is 8%, what is the PV of the scheme?

Ans:

$$PV = A \left[\frac{1-\frac{1}{(1+i)^n}}{i}\right] \quad \text{or} \quad A\left[\frac{(1+i)^n-1}{i\,(1+i)^n}\right]$$

$$= 50{,}000 \left[\frac{1-\frac{1}{(1.08)^{20}}}{0.08}\right]$$

$$= ₹4{,}90{,}905.00$$

Hence, can the scheme be named as “Million Rupee Scheme”.

Capital Recovery Factor

$P = A \times PVFA_{n,i}$

$A = P \times (1 / PVFA_{n,i})$ or $A = P \times (1 / PVIA_{n,i})$

$(1 / PVFA)_{n,i}$ = Capital recovery factor

$$= \left[\frac{1}{\frac{1}{i} - \frac{1}{i(1+i)^n}}\right] \text{ or } \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

Example 15:

Suppose you have a loan of ₹10,00,000 today for 4 years to your friend. If market interest rate is 10%, then how much money per year (EMI) you should receive to recover your investment?

Ans: $10,00,000 = A\left[\frac{1}{i} - \frac{1}{i(1+i)^n}\right]$

Or $10,00,000 = A\left[\frac{1}{0.10} - \frac{1}{0.10(1.10)^4}\right]$

$$A = 10,00,000\left[\frac{1}{\frac{1}{0.10} - \frac{1}{0.10(1.10)^4}}\right]$$

$= 10,00,000 \times 0.3155$

$=$ ₹3,15,500.00

Example 16:

If your employer is providing you an easy loan of ₹50,000 at 9% P.A interest rate to buy a laptop. If you are asked to repay the loan in 3 equal installments at the end of each year for 3 years, then how much will be the annual installment for you?

Ans: $50,000 = A[\frac{(1.09)^3-1}{0.09(1.09)^3}]$

Or $A = 50,000\ [\frac{0.09(1.09)^3}{(1.09)^3-1}]$

$= 50,000 \times 0.3951$

$=$ ₹19,755

Present Value of Growing Annuity

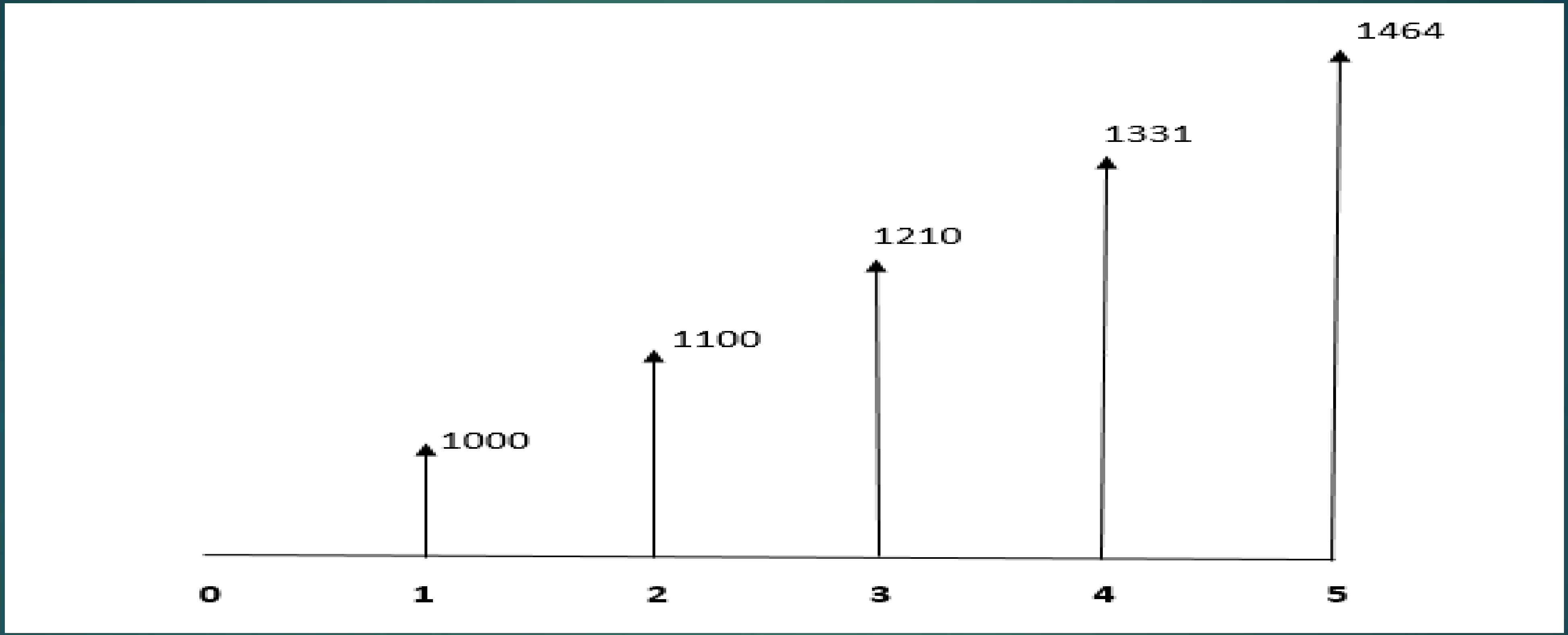
In financial decision making, there are number of situations where CFs grows at a constant rate. There are also inflation protected security where returns grow at the growth of inflation.

Example 17:

Suppose you have an annuity receivable of ₹1000 at the end of each year for 5 years and the annuity is expected to grow at 10% each year. If market interest rate is 12%, then calculate PV of the investment?

Ans:

Year	CF
1	$1000\ (1.10)^0 = 1000$
2	$1000\ (1.10)^1 = 1100$
3	$1000\ (1.10)^2 = 1210$
4	$1000\ (1.10)^3 = 1331$
5	$1000\ (1.10)^4 = 1469$


1464
1331
1210
1100
1000
0
1
2
3
4
5

$$PV = \left[\frac{1000}{(1.12)^1}\right] + \left[\frac{1100}{(1.12)^2}\right] + \left[\frac{1210}{(1.12)^3}\right] + \left[\frac{1331}{(1.12)^4}\right] + \left[\frac{1464}{(1.12)^5}\right]$$

$$= \left[\frac{1000(1.10)^0}{(1.12)^1}\right] + \left[\frac{1000(1.10)^1}{(1.12)^2}\right] + \left[\frac{1000(1.10)^2}{(1.12)^3}\right] + \left[\frac{1000(1.10)^3}{(1.12)^4}\right] + [\frac{1000(1.10)^4}{(1.12)^5}]$$

Representing this in term of formula

$$\mathbf{PV} = \frac{A}{(1+i)^1} + \frac{A(1+g)^1}{(1+i)^2} + \frac{A(1+g)^2}{(1+i)^3} + \frac{A(1+g)^3}{(1+i)^4} + \frac{A(1+g)^4}{(1+i)^5}$$

$$\mathrm{PV} = \mathbf{A}\left[\frac{1}{(1+i)^1} + \frac{(1+g)^1}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \frac{(1+g)^3}{(1+i)^4} + \frac{(1+g)^4}{(1+i)^5}\right]$$

- **Extending this up to n time period**

$$PV = \mathbf{A}\left[\frac{1}{(1+i)^1} + \frac{(1+g)^1}{(1+i)^2} + \cdots + \frac{(1+g)^{n-1}}{(1+i)^n}\right]$$

$$= \mathbf{A}\left[\frac{1}{(i-g)} - \frac{1\,(1+g)^n}{(i-g)(1+i)^2}\right]$$

$$= \mathbf{A}\left[\frac{1-(\frac{1+g}{1+i})^n}{i-g}\right]$$

$$PV = \frac{A}{i-g}\left[1-\left(\frac{1+g}{1+i}\right)^{n}\right]$$

$$PV = A\left[\frac{1}{r-g} - \frac{1}{r-g} \times \left(\frac{1+g}{1+r}\right)^{T}\right] \quad \text{OR} \quad A\left[\frac{1-\left(\frac{1+g}{1+r}\right)^{T}}{r-g}\right]$$

- Now solve the previous problem

$$PV = \frac{1000}{0.12-0.10}\left[1-\left(\frac{1.10}{1.12}\right)^{5}\right]$$

= Rs. 50,000 × (1 - 0.9138)

= Rs. 4310

Example 18

Mr. X, a 2nd year MBA student has got a job that pays 8,00,000 per annum and is expected to grow @ 9% every year till 40 years. Given an market rate of 20%, calculate the PV. of his lifetime income?

- **PV =** $\mathbf{8{,}00{,}000 \left[\dfrac{1-\left(\frac{1.09}{1.20}\right)^{40}}{0.20-0.09}\right] = \text{Rs. } 71{,}17{,}300.71}$

Present value of perpetuity:

Perpetuity is an annuity that occurs for indefinite time period. It is not very common in financial decision making. The British bond called "Consol" is an example of perpetuity. An investor holding a Consol is entitled to receive yearly interest from British Govt forever.

Look at the PV. of an Annuity formula

- $PV = A\left[\frac{1}{i} - \frac{1}{i(1+i)^n}\right]$

When n → ∞, then $(1+i)^n$ term reduced to zero and hence, PV. Of a perpetuity will be

- $PV = \frac{A}{i} \quad Or \quad \frac{CF}{i} \quad Or \quad \frac{CF}{r}$

Example 19

Consider a Consol is paying £100 per year. If market interest rate is 6% , then calculate the PV. of the Consol?

- $PV = \frac{£100}{0.06} = £1666.67$

Hence there exists an inverse relationship between value of perpetuity and its interest rate.

PV of growing perpetuity:

- $PV = \frac{A}{(i-g)}$ Or $\frac{CF}{(r-g)}$

In this case, the perpetuity is growing at a constant rate (g).

Loan amortization:

Loan amortization, is known for repayment schedule that specifies the time schedule for paying interest and principal. This can be done in two ways:

- Through Balloon payments.
- Through Capital recovery.

Through balloon payments, the loan requires the repayment of principal in same equal instalments and pay interest on the unpaid loan. Thus, interest payment will decline over the years and hence the loan payment (EMI) will not be equal for all the periods.

Example 21

Suppose Mr. X has taken a loan of 5000 @ 9% interest per annum. The loan has to be paid in 5 instalments where the principal and Interest will be repaid in 5 equal yearly instalments.

- In case of Balloon payments structure, the payment schedule will be as follow:

Year	Beginning balance	Total payment (P + I)	Interest paid (I)	Principal paid (P)	Year End balance
1	5000	1450 (1000 + 450)	450 (5000×0.09)	1000	4000 (5000 -1000)
2	4000	1360	360 (4000×0.09)	1000	3000 (4000 -1000)
3	3000	1270	270 (3000×0.09)	1000	2000
4	2000	1180	180 (2000×0.09)	1000	1000
5	1000	1090	90 (1000×0.09)	1000	0
	Total	**6,350**	**1350**	**5000**	

An alternative way to calculate loan amortization is through **Capital Recovery Factor**. In this problem we want to answer the annuity for 5 year @ 9% interest rate p.a. for the present value of Rs. 5000.

$$A = PV \times (1/PVFA_{\,n,i})$$

Or $$A = PV\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

$$= 5000\left[\frac{0.09(1.09)^5}{(1.09)^5-1}\right]$$

$$= 5000 \times \frac{0.1385}{0.5386} = 5000 \times 0.2571 = ₹\ 1285.5$$

Year	Beginning balance (₹)	Total payment (₹)	Interest paid (₹)	Principal paid (₹)	End balance (₹)
1	5000	1285.5	450 (5000 × 0.09)	835.5 (1285.5 - 450)	4264.5 (5000-835.5)
2	4164.5	1285.5	374.8 (4164.5 × 0.09)	910.7 (1285.5 - 374.8)	3253.8 (4164.5 - 910.7)
3	3253.8	1285.5	292.8 (3253.8 × 0.09)	992.7 (1285.5 - 292.8)	2261.1 (3253.8 – 992.7)
4	2261.1	1285.5	203.49 (2261.1 × 0.09)	1082.01 (1285.5 -203.49)	1179.09 (2261.1- 1082.01)
5	1179.09	1285.5	106.12 (1179.09 × 0.09)	1179.3 (1285.5 - 106.12)	**-0.21** (1179.09 - 1179.3)
	Total	**6427.5**	**1427.2**	**5000.3**	

Estimation Of a Pension Fund

Suppose Mr. Ramesh wants to construct a pension fund for himself. At present he is 30 years old and he wants to receive pension of Rs. 50,000 per month starting from age of 50 to 90 years. If market interest rate is 9% p.a. for the entire investment period, how much he should invest per month?

Ans: The problem has two parts.

Part1: From Age 30 to 50 i.e. 20 years or 240 monthly investments (- CF)

Part 2: From Age 50 to 90 i.e. 40 years or 480 monthly investments (+ CF)

And Annual interest rate is 9% or 0.75 % per month

Start with Part 2:

If A = ₹ 50,000 n = 480 i = 0.0075 PV = ?

$$PV_{40} = A\left[\frac{1}{i} - \frac{1}{i(1+i)^n}\right] \quad \text{or}$$

$$PV_{40} = 50{,}000\left[\frac{1}{0.0075} - \frac{1}{0.0075(1.0075)^{480}}\right]$$

$$= 50{,}000 \times 129.6409 = ₹\ 64{,}82{,}045/-$$

Part 1:

If FV_{20} = 64,82,045 n = 240 i = 0.0075

Then calculate the Annuity (A) = ?

$$A = FV_{20}\left[\frac{i}{(1+i)^n - 1}\right]$$

$$A = 64{,}82{,}045 \left[\frac{0.0075}{(1.0075)^{240} - 1}\right] = 64{,}82{,}045 \times 0.001497$$

$$= ₹\ 9703 \text{ per month}$$

That is Mr. Ramesh has to invest ₹ 9703 per month now till 20 years (i.e. up to 50 years of his age) to receive a per month pension of ₹ 50,000 for 40 years (i.e. up to 50 years of his age).

Inflation Adjustment of a Pension Fund:

Suppose a 40 yr. old employee who is expecting to retire at age of 60 is planning for his retirement. At present he estimates that he can live comfortably with ₹24,000 per year in terms of present rupee value. How much should he save/invest each year until his retirement so that he can start getting pension at the end of 21st from now till 10 years, that can allow him to live as comfortably as he desired now.

Note: *At present most of the long term investments in market are giving an average return of 15% annually, and assume an annual average inflation rate of 9% P.A for next 30 years.*

Answer :- The investor estimates that he can live comfortably with ₹24,000 per year in terms of present rupee value & annual average inflation rate of 9% P.A. Lets calculate the inflation adjusted expected cash flow that is his estimated annual pension amount.

Year	Age	Inflation adjusted future value of 24000	CF or Pension
21	61	$24000\ (1.09)^{21}$	146611.39
22	62	$24000\ (1.09)^{22}$	159806.41
23	63	$24000\ (1.09)^{23}$	174188.99
24	64	$24000\ (1.09)^{24}$	189866.00
25	65	$24000\ (1.09)^{25}$	206953.94
26	66	$24000\ (1.09)^{26}$	225579.79
27	67	$24000\ (1.09)^{27}$	245881.97
28	68	$24000\ (1.09)^{28}$	268011.35
29	69	$24000\ (1.09)^{29}$	292132.37
30	70	$24000\ (1.09)^{30}$	318424.28

Present Value of the Future CF

$$PV_{20} = \frac{146611.39}{1.15} + \frac{159806.41}{1.15^2} + \frac{174188.99}{1.15^3} + \frac{189866.00}{1.15^4} + \frac{206953.94}{1.15^5} +$$

$$\frac{225580}{1.15^6} + \frac{245881.97}{1.15^7} + \frac{268011.35}{1.15^8} + \frac{292132.37}{1.15^9} + \frac{3318424.28}{1.15^{10}} = ₹10,13,632$$

Similarly, by using formula of constantly growing

$$PV = \frac{A}{i-g}\left[1 - \left(\frac{1+g}{1+i}\right)^n\right]$$

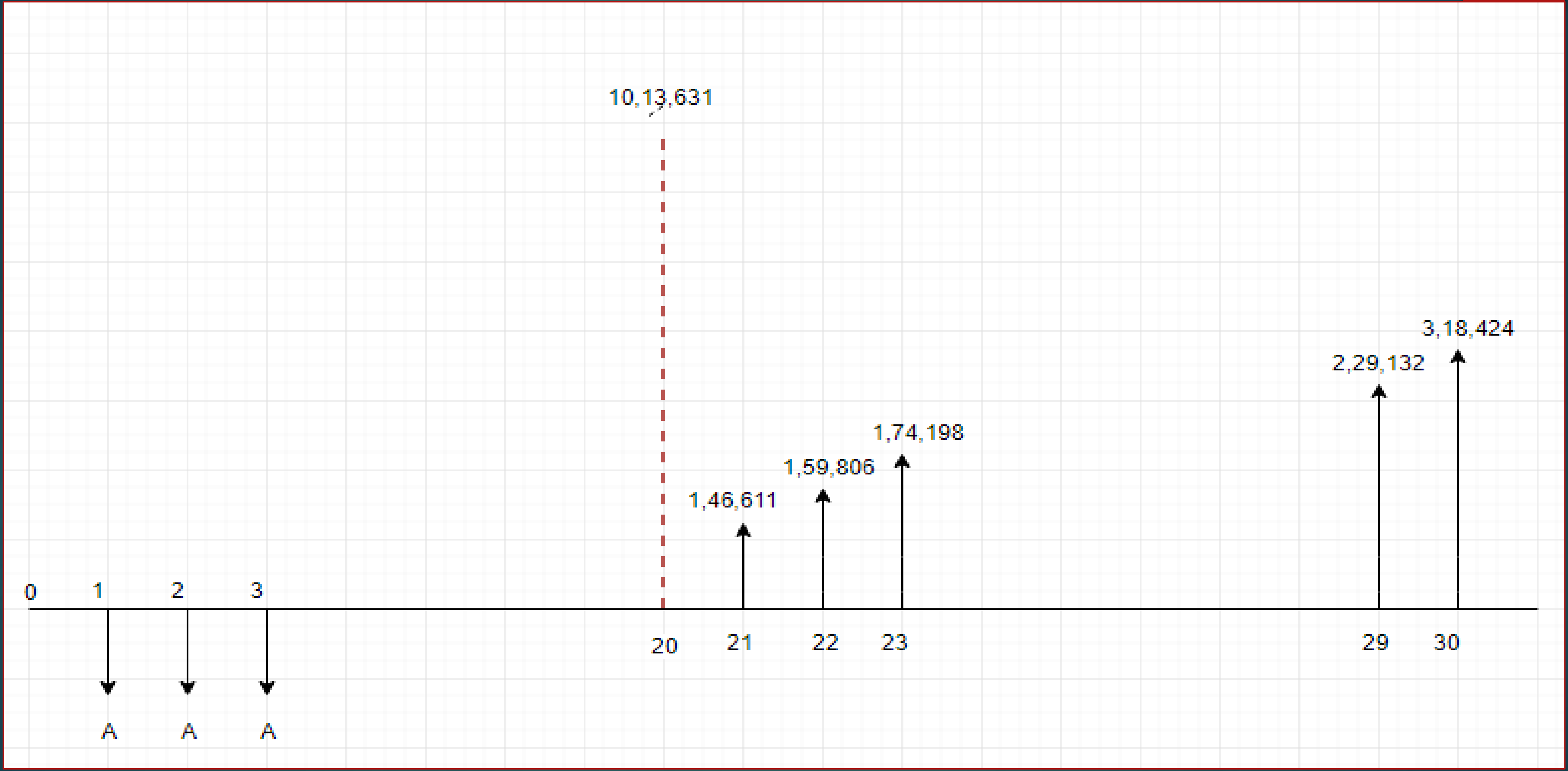
- $PV_{20} = \frac{146611.39}{0.15-0.09}\left[1 - (\frac{1.09}{1.15})^{10}\right]$ = ₹10,13,632

Now consider PV_{20} = ₹10,13,632 as FV_{20} from now and calculate the A using SFF

$A = FV\ [\frac{i}{(1+i)^n-1}]$

$= 10,13,632\ [\frac{0.15}{1.15^{20}-1}] = 1013632 \times 00976 =$ ₹ 9894.5

Time Line of the Investment



Hence the person has to invest ₹ 9894.5 at the end of every year starting at the age of 40 till he reaches the age of 60 in order to withdraw a ₹ 24000 equivalent money value of rupee at the subsequent years for 10 years.

Thank You