Fine-Grained Buy-Many Mechanisms are Not Much Better than Bundling

Sepehr Assadi[1] Vikram Kher[2] George Li[3] Ariel Schvartzman[4]

[1]Rutgers University

[2]University of Southern California

[3]University of Maryland, College Park

[4]Google Research

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Single Item, Single Buyer Auctions

- Single item and single buyer.
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- Buyer will purchase the item if they value it at least its posted price.

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Question

How should the seller price the item to maximize expected revenue?

Example of Single Item Auction

Historically, people are willing to pay either \$2, \$4, or \$6 for a cup of coffee.

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P(coffee) $= 4$	$2/3$	$8/3$
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Remark

For this example, the optimal price is \$4. Myerson [1981] proved this general "algorithm" is optimal. Namely, the optimal selling price is $p^* = argmax_p(p \cdot (1 - F(p))$, where F is the CDF of $\mathcal{D}$.

What about selling more than one item?

Two natural generalizations of Myerson's idea to a multi-parameter space. Note, we allow $\mathcal{D}$ to be arbitrarily correlated. We additionally assume that buyers can only purchase at most one menu option (buy-one model).

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Theorem [BCKW 2015, HN 2017]

There exist distributions over two items such that the optimal revenue is unbounded, but the revenue from item pricing/bundling is finite.

Multi-item auctions are hard!

- Computational Barriers
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- Structural Barriers
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- Mathematical Barriers
  - Simple mechanisms can't approximate optimal mechanisms ([BCKW 2015, HN 2017].

Example of Unintuitive Optimal mechanism

	Coffee	Scone
Bob	2	0
Sally	0	4
Alice	4	6

Optimal Mechanism:

- Price of Coffee: $2
- Price of Scone: $4
- Price of Coffee and Scone: $7.99

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Remark

Optimal Mechanism achieves expected revenue $(2 + 4 + 7.99)/3 = \$4.66$. But, crucially, this mechanism relies on a super-additive pricing scheme. The seller is taking advantage of the fact that the buyer can only interact with the mechanism once! If the buyer could purchase multiple menu items then the expected revenue drops to \$4!

Buy-Many Mechanisms

- A mechanism is buy-many incentive-compatible (IC) if the buyer prefers to purchase one menu option from the mechanism as opposed to any multi-set of allocations [BCKW 2015].

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Theorem [BCKW 2015, CTT 2019]

For arbitrary distributions with arbitrary buyer valuations, item pricing gets within $O(\log n)$ of the revenue of the optimal buy-many IC mechanism.

The Previous State of Affairs

Buy-one mechanisms: Sellers have too much power to extract revenue!
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Buy-many mechanisms: Buyers have too much power!

Question

Is there a good middle ground between the classical buy-one model and newer buy-many model? More specifically, to what extent does the number of menu option deviations that the seller must guard against dictate whether simple mechanisms can approximate optimal ones.

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- Setting $k = 1$ returns notion of buy-one IC.
- Setting $k = \infty$ returns the notion of buy-many IC.

Benchmarks of Interests

For a given distribution $\mathcal{D}$, let

- $\mathrm{BRev}(\mathcal{D})$ be the revenue of the mechanism which sells the grand bundle for its optimal price.
- $\mathrm{Buy}^k\mathrm{Rev}(\mathcal{D})$ be the revenue of the optimal buy-k incentive-compatible mechanism.
- $\mathrm{Buy}^k\mathrm{Rev}(\mathcal{D}, \mathcal{M})$ be the revenue of (not necessarily buy-k incentive-compatible) mechanism $\mathcal{M}$ when the buyer is allowed to buy up to k menu entries from $\mathcal{M}$.

Main Results

Theorem 1 [AKLS 2022]

For buyers with additive valuations and allowing arbitrary n-dimensional distributions, $BRev(D)O(n^2) \geq Buy^n Rev(D)$

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Theorem 2 [AKLS 2022]

For buyers with general monotone valuations and allowing arbitrary n-dimensional distributions, $BRev(\mathcal{D})O(2^n n^2) \geq Buy^n Rev(D)$

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Theorem 3 [AKLS 2022]

For buyers with additive valuations, if $k \leq n^{1/2-\epsilon}$ for some $\epsilon > 0$, there exists a n-dimensional distribution $\mathcal{D}$ such that $Buy^k Rev(D) \geq BRev(\mathcal{D}) exp(\Omega(n^{\epsilon}))$

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Preliminaries

- Buyer is utility-maximizing and draws their valuation x from a known, possibly correlated n-dimensional distribution $\mathcal{D}$, with support set $\mathcal{X} = \textsc{supp}(\mathcal{D})$

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- Valuations are additive (i.e., for any subset of items $S \subseteq [n]$, $x(S) = \sum_{i \in S} x_i$)
- A mechanism $\mathcal{M} = (p, q)$ is then defined by a pair of functions:
  - Payment Function: $p : \mathcal{X} \to \mathcal{R}_{\geq 0}$
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  - Payment Function: $p : \mathcal{X} \to \mathcal{R}_{\geq 0}$
  - Allocation Function: $q : \mathcal{X} \to [0, 1]^n$
- Given a possibly randomized allocation of items $\vec{q} \in [0, 1]^n$, we use $x(\vec{q}) - p(x)$ to denote the buyer's expected utility.

Purchasing Multiple Allocations

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- We let $\vec{\mathrm{Lot}}(\Lambda) \in [0,1]^n$ denote the expected allocation that results from being allocated every $\vec{q}_i \in \Lambda$ independently and at once.
  - In particular, for all $j \in [n]$, $\vec{\mathrm{Lot}}(\Lambda)_j = 1 - \prod_{i=1}(1 - q_{ij})$.

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Buy-k IC Definition

$\mathcal{M}$ is buy-k incentive compatible if for every valuation $x \in \mathcal{X}$ there exists some $(\vec{q}, p) \in \mathcal{M}$ such that $x(\vec{q}) - p(x) \geq x(\vec{\mathrm{Lot}}(\Lambda)) - \sum_{i=1} p(\vec{q}_i)$ for any possible multi-set of menu options Λ of size at most k.

Menu Gap: An Intermediary Measure

The definitions below are generalizations of ones proposed in [HN 2017].

Gap Definition

Let $X = \{\vec{x}_i\}_{i=1}^N \in \mathbb{R}^n_{\geq 0}$, $Q = \{\vec{q}_i\}_{i=0}^N \in [0,1]^n$ be sequences of vectors with $\vec{q}_0 = \vec{0}^n$. Then,

$$\mathrm{gap}_i^k(X, Q) = \min_{j_1, j_2, \ldots, j_k < i} \vec{x}_i \cdot (\vec{q}_i - \vec{\mathrm{Lot}}(\vec{q}_{j_1}, \vec{q}_{j_2}, \ldots, \vec{q}_{j_k}))$$

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- Think of X as holding a sequence of possible valuations vectors and Q holding their corresponding allocations
- $\mathrm{gap}_i^k(X, Q)$ is the largest price a seller can post on menu entry $(\vec{q}_i, p_i)$ such that x_i will prefer it any subset of the previous allocations for free.

Menu Gap: An Intermediary Measure (cont.)

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MenuGap Definition

$$\mathrm{MenuGap}^k(X, Q) = \sum_{i=1}^{N} \mathrm{gap}_i^k(X, Q) / ||\vec{x}_i||_1$$

Proof Strategy for Theorem 1

Theorem 1 [AKLS 2022]

For buyers with additive valuations and allowing arbitrary n-dimensional distributions, $BRev(D)O(n^2) \geq Buy^n Rev(D)$

Proof Strategy

1. First, show that there exists an X, Q such that $\mathrm{MenuGap}^k(X, Q)$ upperbounds the revenue gap between $\mathrm{Buy}^k\mathrm{Rev}(\mathcal{D}, \mathcal{M})$ and $\mathrm{BRev}(\mathcal{D})$, up to a $O(k)$ factor.
2. Then, show that for $k = n$, $n \geq \mathrm{MenuGap}^n(X, Q)$ for all X, Q.

Lemma 1

Lemma 1

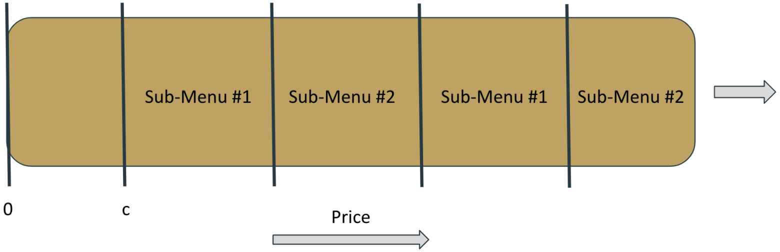
For any distribution $\mathcal{D}$, additive valuation over n items and any buy-k incentive compatible mechanism $\mathcal{M}$ for $\mathcal{D}$, there exists a sequence of valuations $X = \{x_i\}_{i=1} \subseteq \mathcal{X}$, and a sequence of allocations $Q = \{\vec{q}_i\}_{i=0} \subseteq \mathcal{M}$ (starting with $\vec{q}_0 = (0, \ldots, 0)$) such that

$$\mathrm{MenuGap}^k(X, Q) \geq \frac{\mathrm{Buy}^k\mathrm{Rev}(\mathcal{D}, \mathcal{M})}{8k \cdot \mathrm{BRev}(\mathcal{D})}.$$

Proof Outline for Lemma 1

Step One

1. Take $\mathcal{M}$ and create a sub-menu $\mathcal{M}'$ with the following properties:
   - All prices are at least c.
   - All prices belong to the set of intervals $\bigcup_{i=0}^{\infty}[c \cdot (2k)^{2i+a}, c \cdot (2k)^{2i+a+1})$ for an $a \in \{0, 1\}$.
   - $\mathrm{Buy}^k\mathrm{Rev}(\mathcal{D}, \mathcal{M}') \geq \frac{\mathrm{Buy}^k\mathrm{Rev}(\mathcal{D},\mathcal{M}) - c}{2}$.



Proof Outline for Lemma 1 (cont.)

Step Two

1. Let $\mathcal{M}'$ be the menu which satisfies step one. We will use this menu to find our X, Q:
   - Let $\mathcal{B}_j \subseteq \mathcal{M}$ be the sub-menu that has all menu entries priced in $[c \cdot (2k)^{2j+a}, c \cdot (2k)^{2j+a+1})$ for the same $a \in \{0, 1\}$.
   - Let $\vec{x}_j$ be the valuation on $\mathcal{B}_j$ such that $||\vec{x}_j||_1 \leq (1+\delta)||\vec{x}||_1 \quad \forall \vec{x} \in \mathcal{B}_j$.
   - Then,

$$\Pr(x \in \mathcal{B}_j) \leq \frac{\mathrm{BRev}(\mathcal{D})(1+\delta)}{||\vec{x}_j||_1}$$

.

Selecting our choice of X, Q

Let (X, Q) be defined by these $\vec{x}_j$ and their corresponding $\vec{q}_j$.

Proof Outline for Lemma 1 (cont.)

Step Three

We can now lowerbound $\mathrm{gap}_i^k(X, Q)/||\vec{x}_i||_1$. Recall,

$$\mathrm{gap}_i^k(X, Q) = \min_{j_1, j_2, \ldots, j_k < i} \vec{x}_i \cdot (\vec{q}_i - \vec{\mathrm{Lot}}(\vec{q}_{j_1}, \vec{q}_{j_2}, \ldots, \vec{q}_{j_k}))$$

.

1. Since $\mathcal{M}$ was buy-k incentive compatible, we have that for each x_j and any previous set of k options $\vec{q}_{j_1}, \ldots, \vec{q}_{j_k}$,

$$x_j(\vec{q}_j) - p_j \geq x_j(\vec{\mathrm{Lot}}(\vec{q}_{j_1}, \ldots, \vec{q}_{j_k})) - \sum_{i=1}^{k} p_{j_i}.$$

2. By rearrangment, we have that

$$\frac{\mathrm{gap}_j^k(X, Q)}{||\vec{x}_j||_1} \geq \frac{p_j - \sum_{i=1}^{k} p_{j_i}}{||\vec{x}_j||_1}.$$

Proof for Lemma 1 (cont.)

Step Three (cont.)

Continuing from before

1. Since for $j_i < j$, $p_j \geq 2k \cdot p_{j_i}$. Additionally, recall that

$$\Pr(x \in \mathcal{B}_j) \leq \frac{\mathrm{BRev}(\mathcal{D})(1+\delta)}{||\vec{x}_j||_1}.$$

2. It follows that

$$\frac{\mathrm{gap}_j^k(X,Q)}{||\vec{x}_j||_1} \geq \frac{p_j}{2 \cdot ||\vec{x}_j||_1} \geq \frac{\Pr(x \in \mathcal{B}_j) p_j}{2 \cdot \mathrm{BRev}(\mathcal{D})(1+\delta)}.$$

Proof for Lemma 1

Lemma 1

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$$\mathrm{MenuGap}^k(X, Q) \geq \frac{\mathrm{Buy}^k\mathrm{Rev}(\mathcal{D}, \mathcal{M})}{8k \cdot \mathrm{BRev}(\mathcal{D})}.$$

Proof.

1. Take as a fact that $\sum_j \Pr(x \in \mathcal{B}_j) p_j \geq \frac{(\mathrm{Buy}^k\mathrm{Rev}(\mathcal{D},\mathcal{M}) - c)}{4k}$.
2. Then, $\mathrm{MenuGap}^k(X, Q) = \sum_j \frac{\mathrm{gap}_j^k(X,Q)}{||\vec{x}_j||_1} \geq \sum_j \frac{\Pr(x \in \mathcal{B}_j) p_j}{2 \cdot \mathrm{BRev}(\mathcal{D})(1+\delta)} \geq \frac{(\mathrm{Buy}^k\mathrm{Rev}(\mathcal{D},\mathcal{M}) - c)}{8k \cdot \mathrm{BRev}(\mathcal{D})(1+\delta)}$.

Lemma 2

For all sequences X, Q, where $X = \{\vec{x}_i\}_{i=1}^N \in \mathbb{R}^n_{\geq 0}$, $Q = \{\vec{q}_i\}_{i=0}^N \in [0,1]^n$ with $\vec{q}_0 = \vec{0}^n$. and when buyer's have additive valuations, it holds that $\mathrm{MenuGap}^n(X, Q) \leq n$.

Proof.

Proof omitted. The (vague) intuition:

- Break down the sum of $\mathrm{gap}_j^k(X, Q)$ into summing across n coordinates of a vector.
- Show that the sum across each individual coordinate telescopes to 1.
- Since there are n coordinates, $\mathrm{MenuGap}^n(X, Q) \leq n$.
- Note, setting both sequences (X, Q) equal to the standard basis of $\mathbb{R}^n$ makes this inequality tight.

$\square$

Proof for Theorem 1

Theorem 1 [AKLS 2022]

For buyers with additive valuations and allowing arbitrary distributions, $BRev(D)O(n^2) \geq Buy^n Rev(D)$

Proof.

Follows directly from applying Lemma 1 and Lemma 2. $\square$

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- Multi-item auctions are exhibit suprisingly unintuitive features. In particular, simple mechanisms cannot approximate optimal ones.
- In the buy-many world, many of these bad properties (like revenue inapproximability) disappear.
- Our work shows that when $k = n$, the optimal buy-k IC mechanism can be well-approximated by bundling.

Future Directions

- What happens to the revenue gap between optimal and simple mechanisms when $\sqrt{n} < k < n$?

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- Do there exist distributions $\mathcal{D}$ such that $\mathrm{Buy}^k\mathrm{Rev}(\mathcal{D}) > \mathrm{Buy}^{k+1}\mathrm{Rev}(\mathcal{D})$?

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- What happens to the revenue gap between optimal and simple mechanisms when $\sqrt{n} < k < n$?
- Do there exist distributions $\mathcal{D}$ such that $\mathrm{Buy}^k\mathrm{Rev}(\mathcal{D}) > \mathrm{Buy}^{k+1}\mathrm{Rev}(\mathcal{D})$?
- Not obvious how to efficiently test whether a mechanism is buy-many or buy-k IC for some k.

Thank you! Questions?

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