

①  
a) Consider a vector  $y \in R(AB)$ ; We can say that

$$\Rightarrow y = (AB)x$$

$$\Rightarrow y = A(Bx)$$

$$\Rightarrow y = Ax' \quad \text{--- (1)} \quad \cdot [x' = Bx \text{ is also a vector}]$$

from ① we can see that  $R(AB) \subseteq R(A)$

$$\text{So, } \boxed{\dim(R(AB)) \leq \dim(R(A))}$$
$$\text{or } \boxed{\text{Rank}(AB) \leq \text{Rank}(A)} \quad \left[ \begin{array}{l} \text{dim(Range)} \\ = \text{Rank} \end{array} \right]$$

② As  $B$  is non-singular, inverse of  $B$  exists.

~~We can~~

Following the same procedure used in ①, we can

$$\text{say that } \Rightarrow \dim[R(ABB^{-1})] \leq \dim[R(AB)] \quad \text{--- (1)}$$

$$\text{Also from ①, } \dim[R(AB)] \leq \dim[R(A)]$$

(combine ① & ②)

$$\Rightarrow \dim[R(ABB^{-1})] \leq \dim[R(AB)] \leq \dim[R(A)]$$

$$\Rightarrow \dim[R(A)] \leq \dim[R(AB)] \leq \dim[R(A)]$$

$$\Rightarrow \dim[R(AB)] = \dim[R(A)]$$

(1c) from part (a) & (d)

$$\dim[N(AB)] \geq \dim[N(A)] \quad - (1)$$

$$\dim[N(AB)] \geq \dim[N(B)] \quad - (2)$$

Let  $n_a, n_b$  &  $n_{ab}$  are nullity of  $A, B$  &  $AB$  respectively

$$N(A) = \{x_1, x_2, \dots, x_{n_b}\} \Rightarrow \text{basis of nullspace of } A$$

We can extend this to basis of  $N(AB)$ .

$$N(AB) = \{x_1, x_2, \dots, x_{n_b}, x_{n_b+1}, \dots, x_{n_{ab}}\}$$

$$\Rightarrow C_1 x_1 + C_2 x_2 + \dots + C_{n_b} x_{n_b} + \dots + C_{n_{ab}} x_{n_{ab}} = 0 \quad - (1)$$

$$\Rightarrow \underbrace{C_1 B x_1 + C_2 B x_2 + \dots + C_{n_b} B x_{n_b}}_{=0} + \dots + C_{n_{ab}} B x_{n_{ab}} = 0$$

$$\Rightarrow B (C_{n_b+1} x_{n_b+1} + \dots + C_{n_{ab}} x_{n_{ab}}) = 0$$

where  $C$  is collection of coefficients; a matrix of  $1$  for which  $\sum C_i x_i = 0$

$$(d) \dim [R(A)] + \dim [N(A)] = n$$

proof:  $\dim [R(A)] = r$  (assumption)  $r < n$  — (1)

$\Rightarrow$  When we convert  $A$  in row-reduced echelon form.

~~Now~~ There will be  $r$  non-zero rows  $\swarrow$  = Rank of  $A$   
column  
also

There will be  $n-r$  ~~non-zero~~ zero ~~rows~~  $\swarrow$  in row  
column

reduced echelon form of  $A$

which contributes to  $Ax=0$

$$= \dim (N(A))$$

$$n-r = \dim (N(A)) \quad - (2)$$

from (1) & (2)

$$\dim [R(A)] + \dim (N(A)) = r + (n-r) = n$$

$\therefore$  Hence proved

(1e) proof:-

from part (c) :

$$\begin{aligned} \Rightarrow \dim[N(AB)] &\leq \dim[N(A)] + \dim[N(B)] \\ \Rightarrow \text{p/- } \text{rank}(AB) &\leq n - \text{rank}(A) + \cancel{n} - \text{rank}(B) \quad [AB \in \mathbb{R}^{n \times n}] \\ \Rightarrow \text{rank}(AB) &\geq \text{rank}(A) + \text{rank}(B) - n \quad \text{--- (1)} \end{aligned}$$

also from part (A)

$$\text{rank}(AB) \leq \text{rank}(A)$$

$$\text{rank}(AB) \leq \text{rank}(B)$$

Combining above two eq<sup>ns</sup>,

$$\text{rank}(AB) \leq \min\{\text{rank}(A), \text{rank}(B)\} \quad \text{--- (2)}$$

from (1) & (2) we can say that

$$\boxed{\text{rank}(A) + \text{rank}(B) - n \leq \text{rank}(AB) \leq \min\{\text{rank}(A), \text{rank}(B)\}}$$

$\therefore$  Hence proved.

(1f)

$$\text{rank}(u) = 1 \quad [\text{rank of non-zero vector} = 1]$$

lets  $u = A$  &  $u^T = B$  ; then from part (a)

$$\text{rank}(AB) \leq \text{rank}(A)$$

$$\text{rank}(uu^T) \leq \text{rank}(u)$$

$$\text{rank}(uu^T) \leq 1$$

$$\boxed{\text{rank}(uu^T) = 1}$$

[rank = 0 ; only iff all elements are zero]

(19) Let  $A \in \mathbb{R}^{m \times n}$

Let column rank of  $A = r$  — (1)

then there will exist a basis  $(B) = \{b_1, b_2, \dots, b_r\}$   
where  $b_i \in \mathbb{R}^n$

spans the column space of  $A$

Now, we know that we can represent column of  $A$   
as linear combination of column of  $B$

$$\text{if } A = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \dots & a_n \\ | & | & & | \end{bmatrix} \text{ then}$$

$$a_1 = c_{11}b_1 + c_{21}b_2 + \dots + c_{r1}b_r$$

$$a_2 = c_{12}b_1 + c_{22}b_2 + \dots + c_{r2}b_r$$

"  
"

$$a_n = c_{1n}b_1 + c_{2n}b_2 + \dots + c_{rn}b_r$$

where  $c_{ij}$  are coefficients

We can write above equations as

$$A = BC$$

where  $C \in \mathbb{R}^{r \times n}$  collection  
of coefficients

from class notes, we can represent

$$A = BC = b_1 c_1 + b_2 c_2 + \dots + b_r c_r$$

~~Now let~~ We have  $y$  rows in  $C$ , so

where  $c_i$  is  $i$ th  
row of  $C$   
&  $b_j$  is column  
of  $B$

$A$  has maximum of  $y$  independent rows

so row rank = independent rows.

$$\text{row rank} \leq y$$

$$\text{row rank} \leq \text{column rank} - (a)$$

If we ~~do~~ follow same procedure for rank  
we will get  $A = DR$  where  $R$  is basis

$$\Rightarrow \text{Column Rank} \leq \text{Row Rank} - (2)$$

from (1) & (2)

$$\boxed{\text{Column Rank} = \text{Row Rank}}$$

$$\begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_z \end{bmatrix}$$

$z \rightarrow \text{Row rank}$

$E$  is collection  
of coefficients  
 $E \in \mathbb{R}^{m \times z}$



Problem-2

Solution:-

(a) Let  $D$  is a vector,  $D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_8 \\ d_9 \\ d_{10} \end{bmatrix}$ ;  $d \in \mathbb{R}^{10}$

and

$C$  is also a vector,  $C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_8 \\ c_9 \\ c_{10} \end{bmatrix}$ ;  $c \in \mathbb{R}^{10}$

Let  $F$  is a matrix, whose entries are  $f_j(i)$ , where  $i \Rightarrow$  row number and  $j \Rightarrow$  column number.  $f \in \mathbb{R}^{i \times j}$

for  $d \in \mathbb{R}^{10}$ , there always exists a set of real coefficients  $c \in \mathbb{R}^{10}$ , such that

$$\Rightarrow \sum_{j=1}^{10} c_j f_j(i) = d_i \quad \text{for } i \in \{1, 2, \dots, 9, 10\}$$

[Given in question]

$$\Rightarrow FC = D$$

It means  $\text{range}(F) = \mathbb{R}^{10}$

This implies that  $\text{null}(F) = \{0\}$

[Theorem 1.3,  
The fethen Bau,  
If a matrix is full  
rank or  $\text{range}(F) = \text{dim}$ ,  
then,  $\text{null}(F) = \{0\}$ ]

As discussed in class, that if a matrix  $A \in \mathbb{R}^{m \times n}$  is full rank, then

$$\Rightarrow Ax_1 = y_1; Ax_2 = y_2 \quad \text{[a]}$$

$$\Rightarrow x_1 \neq x_2, \text{ then } y_1 \neq y_2$$

So  $d \in \mathbb{R}^{10}$  maps  $c \in \mathbb{R}^{10}$  uniquely.

(6)

Given :-  $AD = C$  ; where  $A \in \mathbb{R}^{10 \times 10}$

from previous part,  $FC = D$ ,

we can write  $AD = C$  as  $AFC = C$ ,

$\Rightarrow$  As  $F$  is full rank, we can say that  $A$  is inverse of  $F$ , also  $\boxed{A^{-1} = F}$

So  $i, j$ th entry of  $A^{-1} = f_j(i)$ .



$$(3) (a) \Rightarrow Q = (I - S)^{-1} (I + S)$$

Taking transpose on both sides

$$Q^T = \left[ (I - S)^{-1} (I + S) \right]^T$$

$$= (I + S)^T \left( (I - S)^{-1} \right)^T$$

$$= (I + S)^T \left( (I - S)^T \right)^{-1}$$

$$= (I^T + S^T) (I^T - S^T)^{-1}$$

$$= (I - S) (I + S)^{-1} \quad \text{--- (1)}$$

~~$$= Q$$~~

$$\begin{aligned} (A^T)^{-1} &= (A^{-1})^T \\ (A+B)^T &= A^T + B^T \\ &\text{(for skew-symmetric} \\ &\text{matrix } S^T = -S) \end{aligned}$$

$$\Rightarrow Q Q^T = (I - S)^{-1} (I + S) (I - S) (I + S)^{-1} \quad \text{(from 1)}$$

$$= (I - S)^{-1} (I^2 - S^2) (I + S)^{-1}$$

$$= \underbrace{(I - S)^{-1} (I - S)}_I \underbrace{(I + S) (I + S)^{-1}}_I$$

$$= I$$

$\therefore$  hence proved.

$$(3) (b) \text{ Let's assume } u^T A u = 0$$

$$\Rightarrow u^T (Q D Q^T) u = 0$$

[given]

$$\Rightarrow u^T Q D Q^T u = 0$$

$$\text{Let } u^T Q = y_{1 \times m} \text{ (row vector)} \Rightarrow (u^T Q)^T = Q^T u^T = Q^T u = y^T$$

$$\Rightarrow y D y^T = 0$$

If  $u$  is not zero vector then  $y$  is not zero vector; So above

$\Rightarrow$  equation implies that  $D = 0$

$$D = 0 \Rightarrow Q D Q^T = 0 \Rightarrow \boxed{A = 0}$$

hence proved.

(3c)

$$u^T S u = [u_1, u_2, \dots, u_m] \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} & \dots & \delta_{1m} \\ \delta_{21} & \delta_{22} & \delta_{23} & \dots & \delta_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \delta_{m1} & \delta_{m2} & \delta_{m3} & \dots & \delta_{mm} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$$= \begin{bmatrix} u_1 \delta_{11} + u_2 \delta_{21} + \dots + u_m \delta_{m1} \\ u_1 \delta_{12} + u_2 \delta_{22} + \dots + u_m \delta_{m2} \\ \vdots \\ u_1 \delta_{1m} + u_2 \delta_{2m} + \dots + u_m \delta_{mm} \end{bmatrix}$$

$\delta_{ij}$  = element of  $S$

$$= \begin{bmatrix} u_1 \delta_{11} & u_2 \delta_{12} & \dots & u_m \delta_{1m} \\ u_1 \delta_{21} & u_2 \delta_{22} & \dots & u_m \delta_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ u_1 \delta_{m1} & u_2 \delta_{m2} & \dots & u_m \delta_{mm} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{j=1}^m u_j \sum_{i=1}^m u_i \delta_{ij} \end{bmatrix}$$

We know if  $S$  is skew-symmetric

$$\boxed{\delta_{ij} = -\delta_{ji}}$$

Then all terms will cancel out

$= 0$

$$\begin{bmatrix} j=1; i=2 \\ u_1 u_2 \delta_{12} \\ \& \text{for } j=2; i=1 \\ u_2 u_1 \delta_{21} = -u_2 u_1 \delta_{12} \\ \text{also } \delta_{ii} = 0 \end{bmatrix}$$

This happens only if  $S$  is skew-symmetric

hence proved.

Problem 4:  $x \in \mathbb{R}^m$ ;  $A \in \mathbb{R}^{m \times n}$

(a) Assumption:  $|x_j| = \max_{1 \leq i \leq m} |x_i|$

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_j \\ \vdots \\ x_m \end{bmatrix}$$

Then  $\Rightarrow \|x\|_\infty = |x_j|$  — (1)

Also  $\Rightarrow \|x\|_2 = \sqrt{\sum_{i=1}^m |x_i|^2}$  — (2)

$$\Rightarrow \sqrt{\sum_{i=1}^m |x_i|^2} \geq |x_j|$$

$\Rightarrow$  from (1) and (2)

$$\Rightarrow \|x\|_2 \geq \|x\|_\infty \text{ hence proved}$$

The equality holds, if  $x = \alpha e_j$

where  $\alpha \in \mathbb{R}$  and

$$e_j = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \rightarrow j\text{th entry}$$

(b) Assumption:  $|x_j| = \max_{1 \leq i \leq m} |x_i|$ ;  $x = \begin{bmatrix} x_1 \\ \vdots \\ x_j \\ \vdots \\ x_m \end{bmatrix}$

$$\Rightarrow \sqrt{\sum_{i=1}^m |x_j|^2} \geq \sqrt{\sum_{i=1}^m |x_i|^2} \quad \left[ |x_j| \text{ is max of all of the entries} \right]$$

$$\Rightarrow |x_j| \sqrt{\sum_{i=1}^m 1} \geq \|x\|_2$$

$$\Rightarrow \sqrt{m} |x_j| \geq \|x\|_2$$

$$\Rightarrow \sqrt{m} \|x\|_\infty \geq \|x\|_2 \quad \left[ \|x\|_\infty = \max_{1 \leq i \leq m} |x_i| \right]$$

proved

if  $x = \alpha \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ ; where  $\alpha \in \mathbb{R}$ , then the equality holds.

(c)

As we know that

$$\|A\|_p^{(m,n)} = \max_{\substack{x \in \mathbb{R}^n \\ x \neq \{0\}}} \frac{\|Ax\|_p^{(m)}}{\|x\|_p^{(n)}}$$

$$\|A\|_\infty^{(m,n)} = \max \frac{\|Ax\|_\infty^{(m)}}{\|x\|_\infty^{(n)}} \quad - (1)$$

$$\|A\|_2^{(m,n)} = \max \frac{\|Ax\|_2^{(m)}}{\|x\|_2^{(n)}} \quad - (2)$$

from previous part (a), (b) and (1), (2)  
We can say that

$$\Rightarrow \frac{\|Ax\|_\infty^{(m)}}{\sqrt{n} \|x\|_\infty^{(n)}} \leq \frac{\|Ax\|_2^{(m)}}{\|x\|_2^{(n)}}$$

$$\Rightarrow \frac{\|Ax\|_\infty^{(m)}}{\|x\|_\infty^{(n)}} \leq \sqrt{n} \left[ \frac{\|Ax\|_2^{(m)}}{\|x\|_2^{(n)}} \right]$$

Take maximum on both sides

$$\Rightarrow \boxed{\|A\|_\infty \leq \sqrt{n} (\|A\|_2)} \quad \left[ (1) \quad (2) \right]$$

proved

Equality holds for  $x = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$

d) from previous a, b & c parts.

$$\begin{array}{l} x \in \mathbb{R}^n \\ x \neq \{0\} \end{array}$$

~~$$\frac{\|Ax\|_2}{\sqrt{m} \|x\|_2} \leq \frac{\|Ax\|_\infty}{\|x\|_\infty}$$~~

$$\Rightarrow \frac{\|Ax\|_2}{\sqrt{m} \|x\|_2} \leq \frac{\|Ax\|_\infty}{\|x\|_\infty}$$

$$\Rightarrow \frac{\|Ax\|_2}{\|x\|_2} \leq \sqrt{m} \frac{\|Ax\|_\infty}{\|x\|_\infty}$$

~~$\Rightarrow$~~  Take maximum of ratios on both sides.

$$\Rightarrow \boxed{\|A\|_2 \leq \sqrt{m} \|A\|_\infty} \text{ proved}$$

Equality holds for  $x = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$

② Frobenius Norm:

$$A \in \mathbb{R}^{m \times n}; \|A\|_F = \left( \sum_{i=1}^m \sum_{j=1}^n |a_{ij}|^2 \right)^{1/2}$$

It is a 2-norm ~~vector~~ of a matrix when viewed as a non-dimensional vector

So, this can also be written in terms of individual rows or columns.

$$\Rightarrow \|A\|_F = \left( \sum_{j=1}^n \|a_j\|_2^2 \right)^{1/2}$$

$$\Rightarrow \boxed{\|A\|_F = \sqrt{\text{tr}(A^T A)}} \quad \text{proved}$$

$$\begin{aligned} \|x\|_2 &= \left( \sum_{i=1}^m |x_i|^2 \right)^{1/2} \quad \text{--- (1)} \\ &= \sqrt{x^T x} \end{aligned}$$

③ From Holder's inequality:- ( $p=2; q=2$ )

$$|x^T y| \leq \|x\|_2 \|y\|_2 \quad \text{--- (1)}$$

$$\text{Let } y = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{m \times 1} \text{ \& } x \in \mathbb{R}^m$$

$$\text{then } x^T y = [x_1, x_2, \dots, x_m] \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$\begin{aligned} &= [x_1 + x_2 + x_3 + \dots + x_m] \\ &= \|x\|_1 \quad \text{--- (3)} \end{aligned}$$

$$\|y\|_2 = \sqrt{\underbrace{1^2 + 1^2 + 1^2 + \dots + 1^2}_m} = \sqrt{m} \quad \text{--- (4)}$$

③ & ④ in ①

$$\boxed{\|x\|_1 \leq \|x\|_2 \sqrt{m}} \quad \text{--- (5)}$$



from (5);

$$\|A\|_1 = \max_{\substack{x \in \mathbb{R}^n \\ x \neq 0}} \frac{\|Ax\|_1}{\|x\|_1} \leq \max \frac{\sqrt{n} \|Ax\|_2}{\|x\|_2} \quad \left[ \begin{array}{l} \|Ax\|_1 \\ \text{---} \\ \|x\|_1 = \|x\|_2 \end{array} \right]$$
$$\|A\|_1 \leq \sqrt{n} \|A\|_2 \quad - (6)$$

Now

$$\|A\|_2 = \max_{\substack{x \in \mathbb{R}^n \\ x \neq 0}} \frac{\|Ax\|_2}{\|x\|_2} \leq \max_{\substack{x \in \mathbb{R}^n \\ x \neq 0}} \frac{\|Ax\|_1}{\frac{\|x\|_1}{\sqrt{n}}} \quad \left[ \frac{\|x\|_1}{\sqrt{n}} = \|x\|_2 \right]$$
$$\|A\|_2 \leq \sqrt{n} \|A\|_1 \quad - (7)$$

Combining (6) & (7)

$$\boxed{\frac{1}{\sqrt{n}} \|A\|_1 \leq \|A\|_2 \leq \sqrt{n} \|A\|_1}$$

Equality holds if  $x = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$

$$(4g) \quad \|A\|_2 \leq \sqrt{\|A\|_1, \|A\|_\infty}$$

proof:- from holder's inequality

$$\Rightarrow |x^T y| \leq \|x\|_p \|y\|_q$$

$$\text{Put } p=1 \text{ \& } q=\infty$$

$$\Rightarrow \text{let } x = Ax \text{ \& } y = Ax, \text{ unit vector}$$

$$|(Ax)^T Ax| \leq \|Ax\|_1 \|Ax\|_\infty \quad - (1)$$

We know that from Frobenius norm

$$\|A\|_2 = \sqrt{\max_{\substack{\|x\|=1 \\ x \neq 0}} (Ax)^T (Ax)}$$

Put above in (1)

$$(\|A\|_2)^2 \leq \|A\|_1 \|A\|_\infty$$

$$\|A\|_2 \leq \sqrt{\|A\|_1 \|A\|_\infty}$$

equality holds if  $x = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$

## 5.Solutions:-

### Matlab Code:-

```
clc
clear all

%Creating a random matrix of size (100*2)
A = randn(100,2);
max_norm_of_Ax = double.empty(6,0); %Creating an empty vector to store maximum
value of norms of A
j=0;

for p = [1 2 3 4 5 inf]
    j=j+1;
    norm_of_Ax = double.empty(1000,0); %Creating an empty vector to store norms of
A.
    for i = 1:1000
        temp = randn(2,1);%random 2x1 vector
        x = (temp/norm(temp));%normalizing random vector
        Ax= A*x;%matrix multiplication
        norm_of_Ax(i) = (norm(Ax, p));% function to calculate p-norm of Ax
    end
    max_norm_of_Ax(j) = max(norm_of_Ax);% Storing maximum value of p-norm of Ax
    fprintf("%d norm of Ax\t %.8f\n",p,max_norm_of_Ax(j));
end

norm_A = double.empty(3,0);%Creating an empty vector to store norms of A
i=0;
for p = [1 2 inf]
    i=i+1;
    norm_A(i) = vpa(norm(A,p));%Storing p-norm of A
    fprintf("%d norm of A\t\t %.8f\n",p,norm_A(i));
end
```

### Output:-

For 1000 iterations

Editor - C:\Users\vikra\Documents\MATLAB\numerical methods\ass1la.m

```

1 clear all
2
3 %Creating a random matrix of size (100*2)
4 A = randn(100,2);
5 max_norm_of_Ax = double.empty(6,0); %Creating an empty vector to store maximum value of norms of A
6 j=0;
7
8 for p = [1 2 3 4 5 inf]
9     j=j+1;
10    norm_of_Ax = double.empty(1000,0); %Creating an empty vector to store norms of A.
11    for i = 1:1000
12        temp = randn(2,1); %random 2x1 vector
13        x = (temp/norm(temp)); %normalizing random vector
14        Ax = A*x; %matrix multiplication
15        norm_of_Ax(i) = (norm(Ax, p)); % function to calculate p-norm of Ax
16    end
17    max_norm_of_Ax(j) = max(norm_of_Ax); % Storing maximum value of p-norm of Ax
18    fprintf('%d norm of Ax\t %.8f\n',p,max_norm_of_Ax(j));
19 end
20
21
22

```

Workspace

Name	Value
A	100x2 double
Ax	100x1 double
i	3
j	6
max_norm_of_Ax	[79.558 10.067 5.675 4.488 4.001 3.260]
norm_A	[79.417 10.067]
norm_of_Ax	1x1000 double
p	Inf
temp	[-0.447 0.893]
x	2x1 double

Command Window

```

1 norm of Ax      79.55824570
2 norm of Ax      10.06706107
3 norm of Ax      5.67502252
4 norm of Ax      4.48829252
5 norm of Ax      4.00162061
Inf norm of Ax    3.2603665
1 norm of A       79.41768755
2 norm of A       10.06706293
Inf norm of A     4.16808253
fx >>

```

## For 10000 iterations

Editor - C:\Users\vikra\Documents\MATLAB\numerical methods\ass1la.m

```

1 clc
2 clear all
3
4 %Creating a random matrix of size (100*2)
5 A = randn(100,2);
6 max_norm_of_Ax = double.empty(6,0); %Creating an empty vector to store maximum value of norms of A
7 j=0;
8
9 for p = [1 2 3 4 5 inf]
10    j=j+1;
11    norm_of_Ax = double.empty(10000,0); %Creating an empty vector to store norms of A.
12    for i = 1:10000
13        temp = randn(2,1); %random 2x1 vector
14        x = (temp/norm(temp)); %normalizing random vector
15        Ax = A*x; %matrix multiplication
16        norm_of_Ax(i) = (norm(Ax, p)); % function to calculate p-norm of Ax
17    end
18    max_norm_of_Ax(j) = max(norm_of_Ax); % Storing maximum value of p-norm of Ax
19    fprintf('%d norm of Ax\t %.8f\n',p,max_norm_of_Ax(j));
20 end
21
22 norm_A = double.empty(3,0); %Creating an empty vector to store norms of A
23 i=0;
24 for p = [1 2 inf]
25     i=i+1;
26     norm_A(i) = vpa(norm(A,p)); %Storing p-norm of A
27     fprintf('%d norm of A\t %.8f\n',p,norm_A(i));
28 end
29
30

```

Workspace

Name	Value
A	100x2 double
Ax	100x1 double
i	3
j	6
max_norm_of_Ax	[90.496 11.374 6.137 4.718 4.144 3.357]
norm_A	[90.263 11.374]
norm_of_Ax	1x10000 double
p	Inf
temp	[-0.447 0.893]
x	2x1 double

Command Window

```

1 norm of Ax      90.49609066
2 norm of Ax      11.37407890
3 norm of Ax      6.13782997
4 norm of Ax      4.71844533
5 norm of Ax      4.14475040
Inf norm of Ax    3.35715481
1 norm of A       90.26361301
2 norm of A       11.37407891
Inf norm of A     4.46537624
fx >>

```

## For 1000000 iterations

Editor - C:\Users\vikra\Documents\MATLAB\numerical methods\ass1la.m

assign1la.m modified\_newton3a.m ass1la.m untitled.m

```
1 clc
2 clear all
3
4 %Creating a random matrix of size (100*2)
5 A = randn(100,2);
6 max_norm_of_Ax = double.empty(6,0); %Creating an empty vector to store maximum value of norms of A
7 j=0;
8
9 for p = [1 2 3 4 5 inf]
10     j=j+1;
11     norm_of_Ax = double.empty(1000000,0); %Creating an empty vector to store norms of A.
12     for i = 1:1000000
13         temp = randn(2,1); %random 2x1 vector
14         x = (temp/norm(temp)); %normalizing random vector
15         Ax= A*x; %matrix multiplication
16         norm_of_Ax(i) = (norm(Ax, p)); % function to calculate p-norm of Ax
17     end
18     max_norm_of_Ax(j) = max(norm_of_Ax); % Storing maximum value of p-norm of Ax
19     fprintf("%d norm of Ax\t %.8f\n",p,max_norm_of_Ax(j));
20 end
21
22 norm_A = double.empty(3,0); %Creating an empty vector to store norms of A
23 i=0;
24 for p = [1 2 inf]
25     i=i+1;
26     norm_A(i) = vpa(norm(A,p)); %Storing p-norm of A
27     fprintf("%d norm of A\t %.8f\n",p,norm_A(i));
28 end
29
30
```

Command Window

```
1 norm of Ax      84.13953457
2 norm of Ax      10.45116935
3 norm of Ax      5.55154753
4 norm of Ax      4.14940056
5 norm of Ax      3.55026312
Inf norm of Ax    2.86386105
1 norm of A       78.10282400
2 norm of A       10.45116935
Inf norm of A     3.81733453
fx >>
```