EE5600 Assignment 1

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***Abstract* - This document contains the solution to Lines and planes problem**

Problem:

Find the equation of the plane passing through the line of intersection of the planes

$$\begin{pmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1}\end{pmatrix}\cdot\mathbf{X} = 1 \text{ and}$$
$$\begin{pmatrix}\mathbf{2} & \mathbf{3} & \mathbf{-1}\end{pmatrix}\cdot\mathbf{X} = \text{-}4$$

and parallel to X-axis.

Solution:

Equation of plane 1 : $\begin{pmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1}\end{pmatrix}\cdot\mathbf{X} = 1$
Equation of plane 2 : $\begin{pmatrix}\mathbf{2} & \mathbf{3} & \mathbf{-1}\end{pmatrix}\cdot\mathbf{X} = \text{-}4$
Let the equation of plane 3 which passes through line made by intersection of planes 1 and 2 and being parallel to X-axis : $\begin{pmatrix}\mathbf{0} & \mathbf{p} & \mathbf{q}\end{pmatrix}\cdot\mathbf{X} = \text{c}$

Now if three planes are passing through same line Then the Echelon matrix form obtained must be of form :

$$\left(\begin{array}{ccc|c} x & x & x & x \\ 0 & x & x & x \\ 0 & 0 & 0 & 0 \end{array}\right)$$

The augumented matrix from three planes :

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & -1 & -4 \\ 0 & p & q & c \end{array}\right)$$

Performing row operations $r_2 \rightarrow r_2 - 2 \cdot r_1$ & $r_3 \rightarrow r_3 - p \cdot r_2$, we end up with

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -3 & -6 \\ 0 & 0 & q+3\cdot p & c+6\cdot p \end{array}\right)$$

To convert this matrix to Echelon form for three planes passing through same line the last row must be made zeroes.

$$\implies q = -3 \cdot p \quad \& \quad c = -6 \cdot p$$

Therefore the required plane equation :

$$\begin{pmatrix}\mathbf{0} & \mathbf{p} & \mathbf{-3p}\end{pmatrix}\cdot\mathbf{X} = -6p$$

Normalizing the above plane equation with p we get

$$\begin{pmatrix}\mathbf{0} & \mathbf{1} & \mathbf{-3}\end{pmatrix}\cdot\mathbf{X} = -6$$