EE5600 Assignment 1

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***Abstract* - This document contains the solution to Lines and planes problem**

Problem:

Find the equation of the plane passing through the line of intersection of the planes

$$\begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix} \cdot \mathbf{X} = \mathbf{1},$$

$$\begin{pmatrix} \mathbf{2} & \mathbf{3} & \mathbf{-1} \end{pmatrix} \cdot \mathbf{X} = \mathbf{-4}$$

and parallel to X-axis.

Solution:

Equation of plane 1 :

$$\begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix} \cdot \mathbf{X} = \mathbf{1} \tag{1}$$

Equation of plane 2 :

$$\begin{pmatrix} \mathbf{2} & \mathbf{3} & \mathbf{-1} \end{pmatrix} \cdot \mathbf{X} = \mathbf{-4} \tag{2}$$

Let the equation of plane 3 which passes through line made by intersection of planes 1 and 2 and being parallel to X-axis :

$$\begin{pmatrix} \mathbf{0} & \mathbf{p} & \mathbf{q} \end{pmatrix} \cdot \mathbf{X} = \mathbf{c} \tag{3}$$

Now if three planes are passing through same line Then the Echelon matrix form obtained must be of form :

$$\left(\begin{array}{ccc|c} x & x & x & x \\ 0 & x & x & x \\ 0 & 0 & 0 & 0 \end{array}\right)$$

The augumented matrix from three planes :

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & -1 & -4 \\ 0 & p & q & c \end{array}\right) \tag{4}$$

Performing the following row operations on (4)

$$r_2 \rightarrow r_2 - 2 \cdot r_1 \tag{5}$$

$$r_3 \rightarrow r_3 - p \cdot r_2 \tag{6}$$

we end up with

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -3 & -6 \\ 0 & 0 & q + 3 \cdot p & c + 6 \cdot p \end{array}\right) \tag{7}$$

To convert this matrix to Echelon form for three planes passing through same line the last row must be made zeroes.

$$\implies \mathbf{q = -3p} \quad \& \quad \mathbf{c = -6p} \tag{8}$$

Therefore the required plane equation :

$$\begin{pmatrix} \mathbf{0} & \mathbf{p} & \mathbf{-3p} \end{pmatrix} \cdot \mathbf{X} = \mathbf{-6p} \tag{9}$$

Normalizing with **‘p’** our plane equation becomes,

$$\begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{-3} \end{pmatrix} \cdot \mathbf{X} = \mathbf{-6} \tag{10}$$

Link For Python Code

code.py