EE5600 Assignment 1

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***Abstract* - This document contains the solution to Lines and planes problem**

Problem:

Find the equation of the plane passing through the line of intersection of the planes

$$\begin{pmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1}\end{pmatrix}\cdot\mathbf{X} = 1 \text{ and}$$
$$\begin{pmatrix}\mathbf{2} & \mathbf{3} & \mathbf{-1}\end{pmatrix}\cdot\mathbf{X} = -4$$

and parallel to X-axis.

Solution:

For the above problem we try to find equation of plane ax + by + cz = d using point-normal approach. The process is as follows :

Step 1: Finding the normal vector of plane for finding coefficients of plane equation (a,b,c).

For this we utilize the concept of cross product of normal vectors of two planes.

Normal vector of plane 1: $\mathbf{N_{p1}} = \begin{pmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1}\end{pmatrix}$

Normal vector of plane 2: $\mathbf{N_{p2}} = \begin{pmatrix}\mathbf{2} & \mathbf{3} & \mathbf{-1}\end{pmatrix}$

Now cross-product of $\mathbf{N_{p1}}$ and $\mathbf{N_{p2}}$ yields

$$\mathbf{N_{p1}} \times \mathbf{N_{p2}} = \begin{pmatrix}\mathbf{-4} & \mathbf{3} & \mathbf{1}\end{pmatrix}$$

Call the resultant vector $\mathbf{P_{p3}}$

$$\mathbf{P_{p3}} = \begin{pmatrix}\mathbf{-4} & \mathbf{3} & \mathbf{1}\end{pmatrix}$$

This Vector is perpendicular to $\mathbf{N_{p1}}$ and $\mathbf{N_{p2}}$,hence it gives the direction of intersecting line which is present in required plane.

Now to find out normal vector we use the concept that dot product of orthogonal vectors gives 0.

Normal vector of required plane has form $\begin{pmatrix}0 & b & c\end{pmatrix}$ as plane is parallel to X-axis. Call the vector $\mathbf{N_{p3}}$.

Dot-product of $\mathbf{P_{p1}}$ and $\mathbf{N_{p3}}$ gives

$$\mathbf{P_{p3}} \cdot \mathbf{N_{p3}} = \mathbf{3b} + \mathbf{c} = 0$$
$$\implies \mathbf{c} = -\mathbf{3b}$$

Therefore the coefficients of plane equation $\begin{pmatrix}\mathbf{a} & \mathbf{b} & \mathbf{c}\end{pmatrix} = \begin{pmatrix}\mathbf{0} & \mathbf{1} & \mathbf{-3}\end{pmatrix}$

Step 2: Finding the point (x_0, y_0, z_0) which lies on required plane to complete the plane equation.

For finding a point lets substitute x=0 in plane equations of plane 1 and 2 and try to solve for y and z coordinates of point.

$$\mathbf{X} = \begin{pmatrix}\mathbf{x} & \mathbf{y} & \mathbf{z}\end{pmatrix} = \begin{pmatrix}\mathbf{0} & \mathbf{y_0} & \mathbf{z_0}\end{pmatrix}$$
$$\begin{pmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1}\end{pmatrix}\cdot\mathbf{X} = 1 \implies \mathbf{y_0} + \mathbf{z_0} = 1 \quad -(1)$$
$$\begin{pmatrix}\mathbf{2} & \mathbf{3} & \mathbf{-1}\end{pmatrix}\cdot\mathbf{X} = -4 \implies \mathbf{3y_0} + \mathbf{z_0} = -4 \, -(2)$$

Solving equations (1) and (2) we get

$$\mathbf{y_0} = -\mathbf{0.75}, \mathbf{z_0} = \mathbf{1.75}$$

Therefore point in plane

$$\begin{pmatrix}\mathbf{x_0} & \mathbf{y_0} & \mathbf{z_0}\end{pmatrix} = \begin{pmatrix}\mathbf{0} & \mathbf{-0.75} & \mathbf{1.75}\end{pmatrix}$$

Now coefficient $\mathbf{d} = \mathbf{a} \times \mathbf{x_0} + \mathbf{b} \times \mathbf{y_0} + \mathbf{c} \times \mathbf{z_0}$

$= -1\times0.75 - 3\times1.75 = -6$

The required plane equation is $\mathbf{y} - \mathbf{3z} = -6$

It can also be represented as $\begin{pmatrix}\mathbf{0} & \mathbf{1} & \mathbf{-3}\end{pmatrix}\cdot\mathbf{X} = -6$

Link for Python Code

code.py