Bit Manipulation

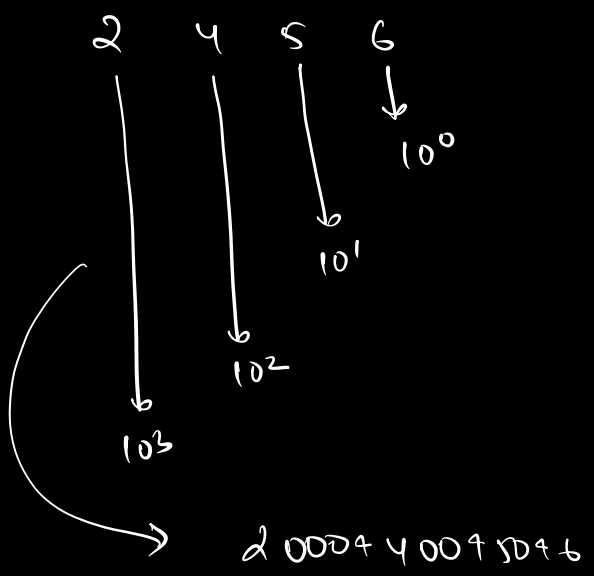
Number System :-

$734 \Rightarrow 700 + 30 + 4$

$\Rightarrow 7 \times 100 + 3 \times 10 + 4 \times 1$

$\Rightarrow 7 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$.

$6594 \Rightarrow 6 \times 10^3 + 5 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$.



This is decimal no. system

base $\Rightarrow$ 10.

digits $\Rightarrow$ [0-9]

Other no. system :-

octal, hexadecimal, ternary, binary

Octal no. system

base ⇒ 8

digits ⇒ [0 - 7].

ex ⇒ $(1\ 3\ 2)_8$ ⟶ 1 (×8^2) 3 (×8^1) 2 (×8^0)

$= 1\times8^2 + 3\times8^1 + 2\times8^0$

$\Rightarrow 64 + 24 + 2$

$\Rightarrow 90$

ex ⇒ $(1\ 2\ 5)_8$ ⇒ 1 (×8^2) 2 (×8^1) 5 (×8^0)

$= 1\times8^2 + 2\times8^1 + 5\times8^0$

$\Rightarrow 64 + 16 + 5$

$\Rightarrow 85$

⇒ Incorrect octal

100011 ✓ 6745 ✓ 6785 ✗ 2222 ✓

⇒ ternary ⇒ base power ⇒ 3.

digits ⇒ [0 - 2].

eg ⇒ $(0\ 2\ 1\ 0\ 1)_3$ ⇒ $0\ \ 2\ \ 1\ \ 0\ \ 1$

$\downarrow\ \ \downarrow\ \ \downarrow\ \ \downarrow\ \ \downarrow$

$3^4\ \ 3^3\ \ 3^2\ \ 3^1\ \ 3^0$

⇒ $0 \times 3^4 + 2 \times 3^3 + 1 \times 3^2 + 0 \times 3^1 + 1 \times 3^0$

⇒ $0 + 54 + 9 + 0 + 1$

⇒ $\underline{\underline{64}}$.

⇒ Binary Number System :-

base ⇒ 2

digits ⇒ [0 - 1].

< B → D >

$(1\ 0\ 1\ 1\ 0)_2$ ⇒

$1\ \ 0\ \ 1\ \ 1\ \ 0$

$\downarrow\ \ \downarrow\ \ \ \ \downarrow\ \ \downarrow\ \ \downarrow$

$2^4\ \ 2^3\ \ \ \ 2^1\ \ 2^0$

2^2

⇒ $1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

⇒ $16 + 0 + 4 + 2 + 0$

⇒ $\underline{\underline{22}}$.

$(0\ 1\ 1\ 0\ 1\ 0)_2 \Rightarrow$

0	1	1	0	1	0
↓	↓	↓	↓	↓	↓
2^5	2^4	2^3	2^2	2^1	2^0

$\Rightarrow 0\times2^5 + 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$

$\Rightarrow 0 + 16 + 8 + 0 + 2 + 0$

$\Rightarrow \underline{\underline{26}}$

< D → B >

eg $\Rightarrow$ 28 $\Rightarrow$

2 | 28	→ 0
2 | 14	→ 0
2 | 7	→ 1
2 | 3	→ 1
2 | 1	→ 1
0	

$(1\ 1\ 1\ 0\ 0)_2$

1	1	1	0	0
↓	↓	↓	↓	↓
2^4	2^3	2^2	2^1	2^0

$\Rightarrow 2^4 + 2^3 + 2^2$

$\Rightarrow 16 + 8 + 4$

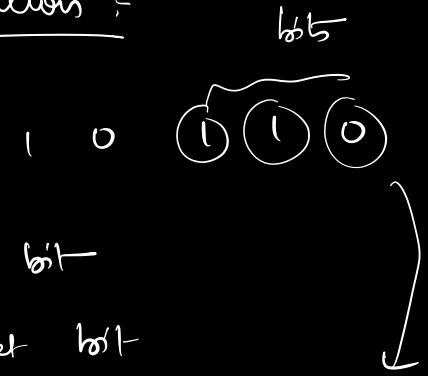
$\Rightarrow \underline{\underline{28}}$

eg $\Rightarrow$ 25 $\Rightarrow$

2 | 25	→ 1
2 | 12	→ 0
2 | 6	→ 0
2 | 3	→ 1
2 | 1	→ 1
0	

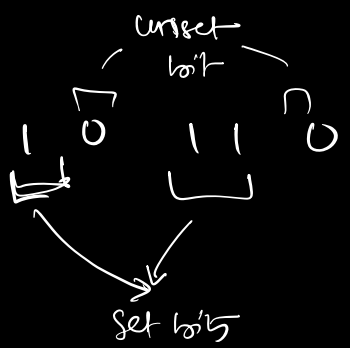
$\Rightarrow (1\ 1\ 0\ 0\ 1)_2$

Naming convention :

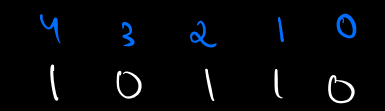


1 : Set bit

0 : Unset bit



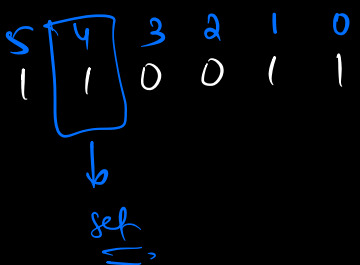
bit position :



0 to N

right to left

ex :)



: Addition [decimal]

```
  1  1  1
  3  4  6  9
+ 2  8  4  7
-------------
  6  3  0  6
```

```
  0  1  1  1
     2  4  5  8
     3  7  6  4
-------------
  =  6  2  2  2
```

Sum => 8+4 = 12

V = 2 → 12%10

C = 1 → 12/10.

Sum => 6+5+1 = 12

V = 2 → 12%10

C = 1 → 12/10.

Sum = 7+4+1 = 12

V = sum%10 = 12%10 = 2

C = sum/10 = 12/10 = 1

Sum = 3+2+1 = 6

V = sum%10 = 6%10 = 6

C = sum/10 = 6/10 = 0

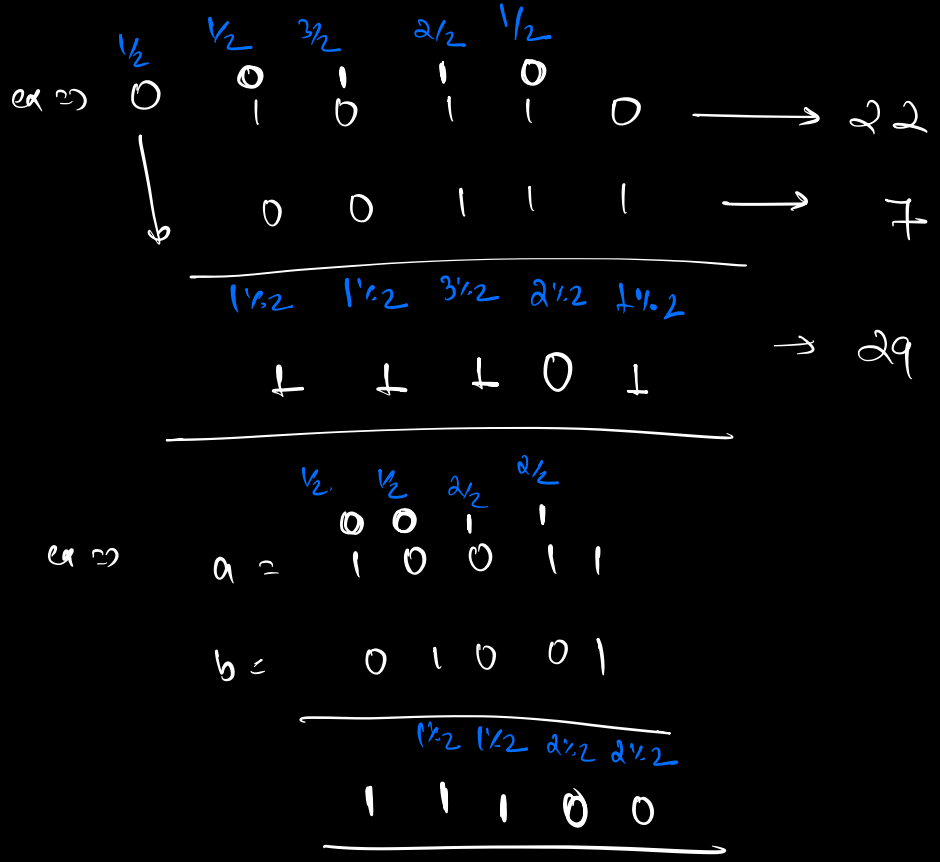
at every step. V = sum%10.

C = sum/10.

: add binary nos.

V => sum % 2

C => sum / 2.



:- We use in binary :-

8 bit representation

10 :- 0 0 0 0 1 0 1 0

-10 :- 1 0 0 0 1 0 1 0

↳ leftmost bit can be used as signed bit.

10 :- 0 0 0 0 1 0 1 0

-4 :- 1 0 0 0 0 1 0 0

1 0 0 0 1 1 1 0

-ve

14

-14

10 + (-4) = -14

0 :- 0 0 0 0 0 0 0 0

-0 :- 1 0 0 0 0 0 0 0

$a \Rightarrow -a$ [2's complement of a].

↓

1's comp + 1

↓

flip [0 ⇄ 1]

10 :- 0 0 0 0 1 0 1 0

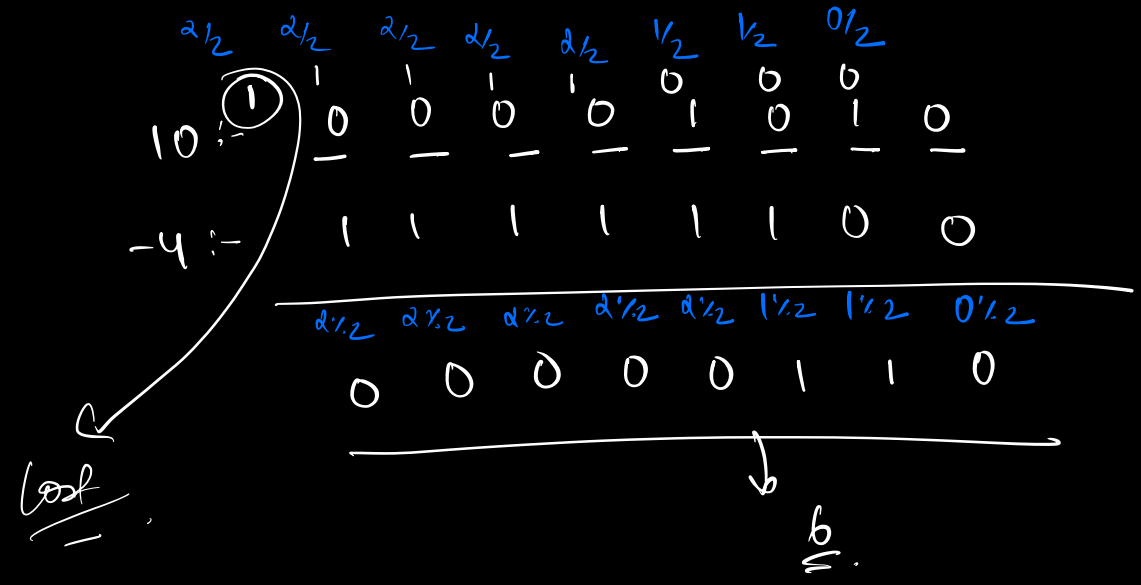
-4 :-

4 :- 0 0 0 0 0 1 0 0

1's ⇒ 1 1 1 1 1 0 1 1

+1 ⇒ 0 0 0 0 0 0 0 1

1 1 1 1 1 1 0 0

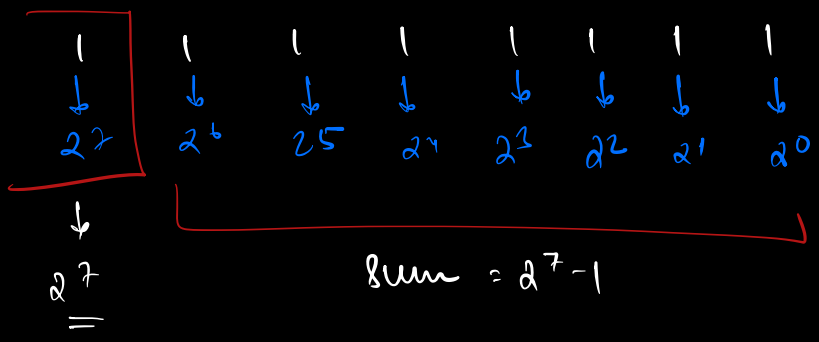


10 + (-4) = 6

LHS = RHS

Proved

8 bits ⇒



Sum $= 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$

⇒ Sum of GP $\quad a = 2^0 = 1$

$r = 2$

$N = 7$

$$\text{Sum} = \frac{1 * (2^7 - 1)}{(2-1)} = 2^7 - 1$$

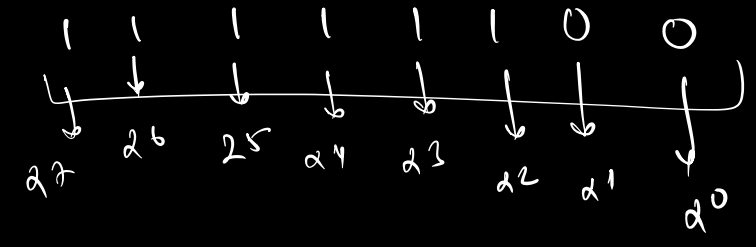
Left most bit $>$ all Other bits combined

(2^7) $\qquad$ $(2^7 - 1)$

MOST SIGNIFICANT BIT

(MSB)

-4 :-



$\Rightarrow 128 + 64 + 32 + 16 + 8 + 4$

$\Rightarrow 192 + 48 + 12$

$\Rightarrow 252 \neq -4$

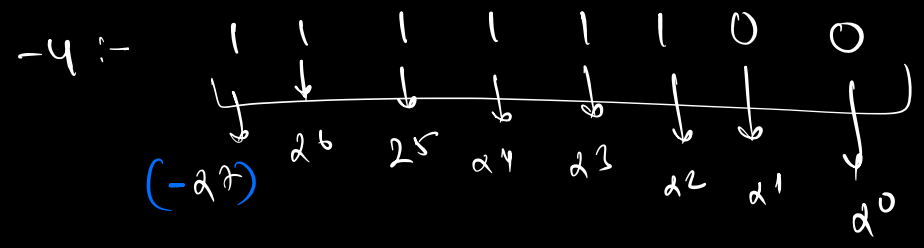
Note

MSB $\Rightarrow$ base power $\Rightarrow$ $-ve$

8 bit $\Rightarrow$ $\overset{\uparrow}{\underset{\text{MSB.}}{-}}$ — — — — — — —

16 bit $\Rightarrow$ $\overset{15}{\overset{\uparrow}{\underset{\text{MSB.}}{-}}}$ — — — — — — — — — — — — — — — $\overset{2\ 1\ 0}{}$

-4 :-



$-2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2$

$\Rightarrow -128 + 64 + 32 + 16 + 8 + 4 \Rightarrow \boxed{-4}$

10 ⇒ 0 0 0 0 1 0 1 0

-10 ⇒ 1 1 1 1 0 1 0 1

1's 10 ⇒ 0 0 0 0 0 0 0 1

+1

(-10) 1 1 1 1 0 1 1 0

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

⇒ $-128+64+32+16+0+4+2+0$

⇒ $-128+96+22$

⇒ $-128+118 = -10$

eg ⇒ 8 bit ⇒ 1 0 0 1 0 0 1 0

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

⇒ $-2^7+0+0+2^4+0+0+2^1+0$

⇒ $-128+16+2$

⇒ $-128+18$ ⇒ -110

8 bit ⇒ 0 0 0 1 1 1 1 1

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

$\Rightarrow 0 + 0 + 0 + 16 + 8 + 4 + 2 + 1$

$\Rightarrow 30 + 1 \Rightarrow \underline{31}$.

4 bit no. $\Rightarrow$ 1 0 1 1

$-2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$\Rightarrow -2^3 + 0 + 2^1 + 2^0$

$\Rightarrow -8 + 2 + 1 \Rightarrow \underline{-5}$.

8 bit $\Rightarrow$ 0 0 0 0 1 0 1 1

$-2^7 \quad 2^6 \quad 2^5 \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$\Rightarrow 0 + 0 + 0 + 0 + 8 + 0 + 2 + 1$

$\Rightarrow \underline{11}$.

N bit

N-1 ... 3 2 1 0

— — — — — — — — — — —

$-(2^{N-1}) \quad \ldots \quad 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$

Range

bits	minm	maxm
2 $[-2, 1]$	$\underset{-2^1}{1}\ \underset{2^0}{0} \Rightarrow -2$	$\underset{-2^1}{0}\ \underset{2^0}{1} \Rightarrow 1$
3 $[-4, 3]$	$\underset{-2^2}{1}\ \underset{2^1}{0}\ \underset{2^0}{0} \rightarrow -4$	$\underset{-2^2}{0}\ \underset{2^1}{1}\ \underset{2^0}{1} \rightarrow 3$
4 $[-8, 7]$	$\underset{-2^3}{1}\ \underset{2^2}{0}\ \underset{2^1}{0}\ \underset{2^0}{0} \Rightarrow -8$	$\underset{-2^3}{0}\ \underset{2^2}{1}\ \underset{2^1}{1}\ \underset{2^0}{1} \Rightarrow 7$
5 $[-16, 15]$	$\underset{-2^4}{1}\ \underset{2^3}{0}\ \underset{2^2}{0}\ \underset{2^1}{0}\ \underset{2^0}{0} \Rightarrow -16$	$\underset{-2^4}{0}\ \underset{2^3}{1}\ \underset{2^2}{1}\ \underset{2^1}{1}\ \underset{2^0}{1} \Rightarrow 15$

N bits

$$[-2^{N-1}, 2^{N-1}-1]$$

$$N = 5 \Rightarrow [-2^{5-1}, 2^{5-1}-1]$$

$$\Rightarrow [-2^4, 2^4-1]$$

$$\Rightarrow [-16, 15]$$

Range for N bits $\Rightarrow [-2^{N-1}, 2^{N-1}-1]$.

Datatypes

byte ⇒ 8 bits ⇒ $[-2^7, 2^7 - 1]$

short ⇒ 16 bits ⇒ $[-2^{15}, 2^{15} - 1]$

int ⇒ 32 bits ⇒ $[-2^{31}, 2^{31} - 1]$

long ⇒ 64 bits ⇒ $[-2^{63}, 2^{63} - 1]$

⇒ constraints ⇒ $1 \leq arr[i] \leq 10^9$

$2^{10} = 1024 \approx 1000 \Rightarrow 10^3$

$2^{10} \approx 10^3$

cubing both sides

$(2^{10})^3 \approx (10^3)^3$

$\Rightarrow 2^{30} \approx 10^9$

$\Rightarrow 2 * 2^{30} \approx 2 * 10^9$

$\Rightarrow 2^{31} \approx 2 * 10^9$] int range

$2^{10} \leq 10^3$

power 6 on both sides

$(2^{10})^6 \leq (10^3)^6$

$\Rightarrow 2^{60} \leq 10^{18}$

$\Rightarrow 8 \times 2^{60} \leq 8 \times 10^{18}$

$\Rightarrow 2^{63} \leq 8 \times 10^{18}$] long ans

Doubts

8 bits: 1 0 0 1 0 0 1 1

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

$\Rightarrow -2^7 + 0 + 0 + 2^4 + 0 + 0 + 2^1 + 2^0$

$\Rightarrow -128 + 16 + 2 + 1$

$\Rightarrow -128 + 19$

$\Rightarrow -109$

1 0 1 1

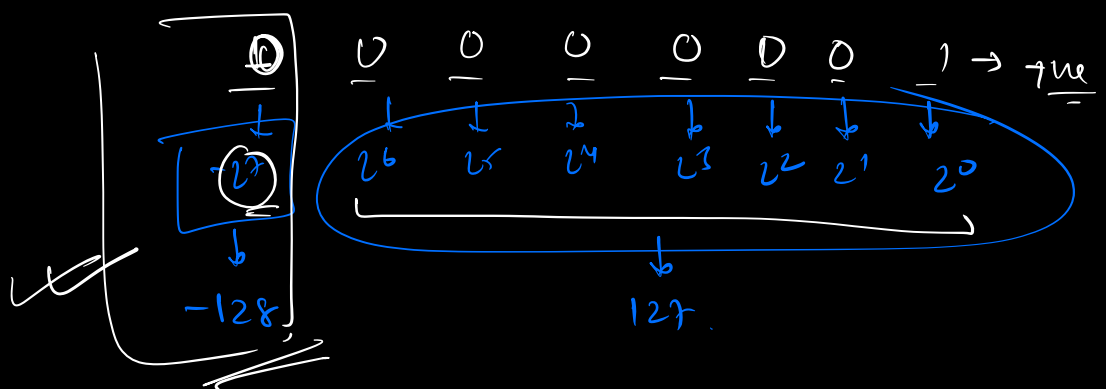
8 bit ⇒ 1 _ _ _ _ _ _ _ → -ve

0 _ _ _ _ _ _ _ _ → +ve

1011 →

0 0 0 0 1 0 1 1

0 0 0 0 0 0 1 0 1 1



64 → 1 0 0 0 0 0 0

(6 5 4 3 2 1 0)

7 bit

4 bit

$2^{31}-1$

int n = 2^{32}