% modulus $\Rightarrow$ remainder

$n \% a \Rightarrow$ remainder when n is divided by a

$10 \% 4 \Rightarrow 2$

$13 \% 5 \Rightarrow 3 \longrightarrow$

Divisor, Dividend

$$5 \overline{) 13} (2 \rightarrow \text{Quotient}$$

$$\begin{array}{r} 10 \\ \hline \times\ 3 \end{array} \rightarrow \text{remainder}$$

Dividend = Quotient × divisor + remainder
(D) (q) (d) (r)

$\Rightarrow D = qd + r$

$\Rightarrow \boxed{r = D - \underbrace{qd}}$

largest multiple of divisor less than or equal to dividend

Quiz 1 $\Rightarrow$ 150 % 11

$$\begin{array}{r} 11 \overline{) 150} (13 \\ 11 \\ \hline \times\ 40 \\ 33 \\ \hline \times\ 7 \end{array}$$

$\underbrace{\text{largest multiple}}$ of $11 <= 150$

$11 \times 13 = 143 \leftarrow$

$11 \times 14 = 154 \not< 150$

$r = 150 - 143 = 7$

or, $150 \% 11 = 7$

Quiz 2 ⇒ 100 % 7

$$7\overline{)100}(14$$
$$\begin{array}{r} 7 \\ \hline \times 30 \\ 28 \\ \hline \times 2 \end{array}$$

$7 \times 14 = 98$

$100 - 98 = 2$

or $100 \% 7 = 2$.

Quiz 3 ⇒ largest no. ⇒ $-43 \quad -42 \quad -76 \quad -35$

$-76 < -43 < -42 < -35$

Quiz-4 ⇒ $-40 \% 7$

r = Dividend − largest multiple of divisor <= Dividend

$= -40 - [-35]$

-5

$= -40 - (-42)$

$= -40 + 42$

$= 2.$

-7
-14
-21
-28
-35
-40
-42
-49

∴ remainder can never be −ve.

Quiz 5 $\Rightarrow$ $-60 \% 9$

$$\begin{array}{r} 9\,)\,-60\,(\,-7 \\ \underline{-(-63)} \\ 3 \end{array}$$

-60

-63

-72

-81

-90

-99

$\vdots$

or, remainder $= -60 - [$largest multiple of $9 <=$ dividend$]$

$= -60 - (-63)$

$= 3$ ans

$$\boxed{a \% m \Rightarrow [0, m-1]}$$

: C | C++ | Java | Js   Python

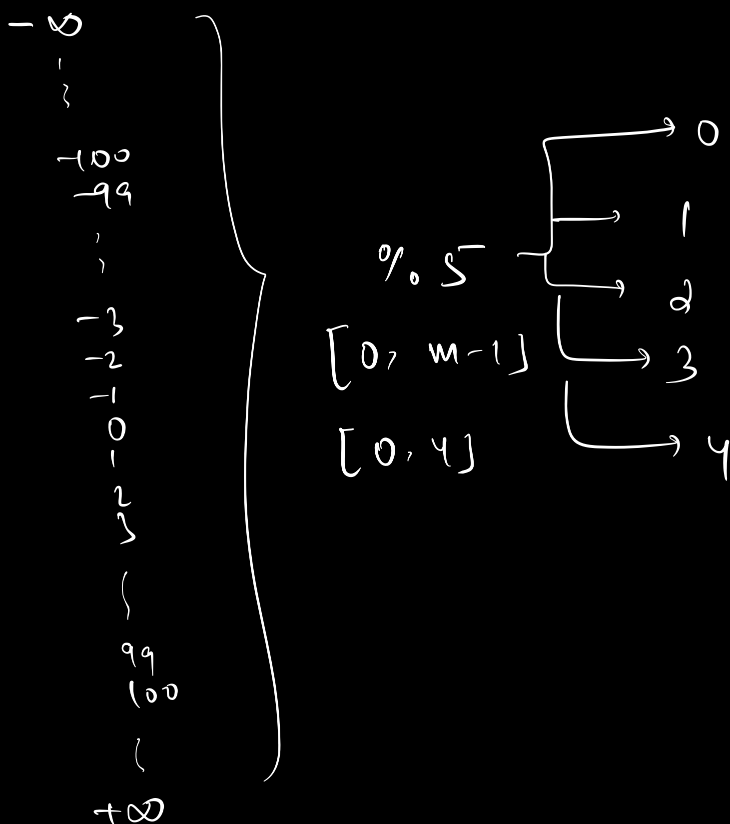
$-40 \% 7 = -5 + 7 \longrightarrow 2$

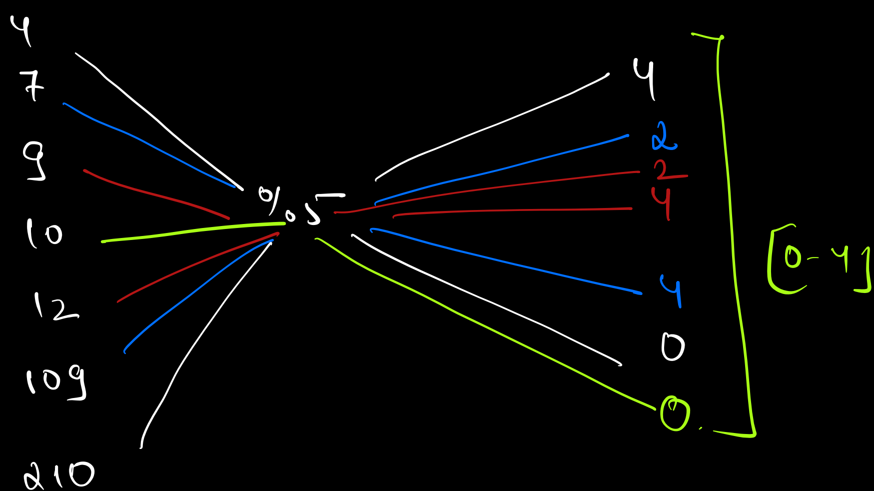
$-60 \% 9 = -6 + 9 \longrightarrow 3$

$-30 \% 4 = -2 + 4 \longrightarrow 2$

for -ve no. (%) in C | C++ | Java | Js.

$$\boxed{a \% m \Rightarrow a \% m + m}$$





$$\boxed{a \% m \Rightarrow [0, m-1]}$$

very helpful for :- 1) Hashmap | Hashtable | Hashset | Dictionary | Map

2) Consistent Hashing

3) Encryption

Modular Arithmetic :- (+) (x) (÷) (−)

advanced classes

$$\boxed{(a+b)\%M = (a\%M + b\%M)\%M}$$

Modular Additive Rule

ex 2) $a=6$ $b=8$ $M=10$

$(a+b)\%M$

$=(6+8)\%M$

$=14\%10$

$=4$

$a\%M + b\%M$

$=6\%10 + 8\%10$

$=6+8$

$=14\%10$

$=\underline{4}$

$$\boxed{(a\times b)\%M = (a\%M \times b\%M)\%M}$$

Modular Multiplication Rule

$a=6$ $b=8$ $M=10$

$(a\times b)\%M$

$=(6\times 8)\%10 = 48\%10$

$=8$

$a=6$ $b=8$ $M=10$

$(a\%M \times b\%M)\%10$

$=(6\%10 \times 8\%10)\%10$

$=(6\times 8)\%10 = 8$

: ans, return ans % 10^9+7

Q1. Implement a power func. which returns $a^n \% p$.

ie power(a, n, p) → $a^n \% p$

ex ⇒ a = 2, n = 5, p = 7 ⟶ $2^5 \% 7$

⇒ 32 % 7 = 4

a = 3, n = 4, p = 6 → $3^4 \% 6$

⇒ 81 % 6 ⇒ 3

```
int power(a, n, p) {
    ans = 1
    for(i = 1; i <= N; i++) {
        ans = (ans * a) % p
    }
    return ans
}
```

a = 10

N = 40

$(10)^{40}$

int → 10^9

long → 10^{18}

strings

BigInt

$(a \times b) \% M = (a \% M \times b \% M) \% M$

$a^n \% p \Rightarrow [0 \quad p-1]$

```
int power(a, n, p) {
    ans = 1
    for(i=1; i<=N; i++){
        ans = (ans%p × a%p) % p
    }
    return ans
}
```

$(2, 8, 10)$

$\downarrow$

6

ans =

ans Before	an a	i
1	2	1
2	4	2
4	8	3
8	6	4
6	2	5
2	4	6
4	8	7
8	6	8

$\boxed{\begin{array}{l} TC \Rightarrow O(N) \\ SC \Rightarrow O(1) \end{array}}$

$a = 2$
$n = 8$
$p = 10$

$2^8 \% 10$
$= 256 \% 10 = 6$

$a^n \% p \rightarrow a \mid N \leftarrow$ no matter value of aN

$$ans = (\underbrace{\underbrace{ans \% p}_{p-1} \times \underbrace{a \% p}_{p-1}}_{})\% p$$

$$\underbrace{(10^9+4) \times (10^9+4)}_{\text{overflow for int range}}$$

$p = 10^9+5$

$p = 0 <= p <= 10^4$

$0 <= p <= 2908$

: **Divisibility rules:-**

⇒ rules that helps us to check whether a no. is divisible by a specific

x divisible by 3 → $x \% 3 == 0$ then divisible

$x \% 3 \; != 0$ not divisible

: **Rule of 3** : Sum of all digits should be divisible by 3

ex ⇒ 4563 → $18 \% 3 == 0$ ✓

↓

$4563 \% 3 == 0$.

$(4563) \% 3 \longrightarrow (4+5+6+3) \% 3$

↓

$(4000 + 500 + 60 + 3) \% 3$ $\left[(a+b) \% M = (a \% M + b \% M) \% M \right]$

↓

$(4000 \% 3 + 500 \% 3 + 60 \% 3 + 3 \% 3) \% 3$

$= \left[(4 \times 10^3) \% 3 + (5 \times 10^2) \% 3 + (6 \times 10) \% 3 + 3 \% 3 \right] \% 3$

$(a \times b) \% M = (a \% M \times b \% M) \% M$

$$\Big[(4\%3 \times 10^3\%3)\%3 + (5\%3 \times 10^2\%3)\%3 + (6\%3 \times 10\%3)\%3 + 3\%3 \Big]\%3$$

1 1 1

$10^3\%3 = 1$

$10^2\%3 = 1$

$10\%3 = 1$

$10^n\%3 = 1$

$$\Rightarrow \big[(4\%3)\%3 + (5\%3)\%3 + (6\%3)\%3 + 3\%3\big]\%3$$

$$\Rightarrow \big[4\%3 + 5\%3 + 6\%3 + 3\%3\big]\%3$$

$$\big[(a+b)\%M = (a\%M + b\%M)\%M\big]$$

$(a\%b)\%b$

$$\Rightarrow [4+5+6+3]\%3 = (4563)\%3$$

Proved

$(41\%10)\%10$

$= 1\%10$

$= 1$

$41\%10$

$= 1$

$(a\%b)\%b$

$= a\%b$

rule of 4: No. formed by the last two digits should be divisible by 4.

$13824 \rightarrow 24\%4 == 0$

$\downarrow$

$13824\%4 == 0$

$441221 \rightarrow 21$ XX

$\downarrow$

$441221\%4 \;!= 0$

$(3484)\%4 \longrightarrow (84)\%4$

$\downarrow$

$(3000 + 400 + 80 + 4)\%4$ $[(a+b)\%M = [a\%M + b\%M]\%M]$

$= (3000\%4 + 400\%4 + 80\%4 + 4\%4)\%4$

$= [(3 \times 10^3)\%4 + (4 \times 100)\%4 + (8 \times 10)\%4 + 4\%4]\%4$

$[(a \times b)\%M = (a\%M \times b\%M)\%M]$

$= [(3\%4 \times \underbrace{10^3\%4}_{0})\%4 + (4\%4 \times \underbrace{100\%4}_{0})\%4 + (8\%4 \times 10\%4)\%4 + 4\%4]\%4$

0 0

$= [(8\%4 \times 10\%4)\%4 + 4\%4]\%4$

$= [(8 \times 10)\%4 + 4\%4]\%4$

$100\%4 = 0$

$10^3\%4 = 0$

$10^4\%4 = 0$

$10\%4 = 2$

$= [80\%4 + 4\%4]\%4$

$= 84\%4 \longrightarrow (3484)\%4$

Proved

HW Prove divisibility rule for

5, 8, 9

Google

Q2. Given a no. A in the form of an array of size N, and a no. P, calculate A % P.

ex ⇒ A ⇒ | 1 | 2 | 3 | 4 | 4 | → 12344

P ⇒ 4

$A\%P \Rightarrow 12344\%4 \Rightarrow 0.$

Cons: $1 \le P \le 10^9$

$0 \le arr[i] \le 9$

$1 \le N \le 10^5$

long ⇒ 10^{18}

$\Rightarrow \underbrace{1000000000000000000}_{19}$

long ⇒ 10^{18}

1 + 18 0's

= 19

— — — — — — — — — — — — —

— — — — — — — — — — — — — — — —

— — — — — — — — — — — — — —

— — — — — — — — — — — — — —

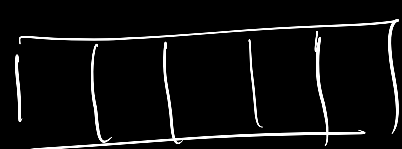
max value → $10^{10^6}-1$ → $10^{1000000}-1$

— — — — → 9999 → 10000 - 1

10^4-1

— — — — — → 10^5-1

10^5 → 999 . . . 10^5



→ 5

now ↳ 9 9 9 9 9

100

999999 ——— 100 bi

$10^5 \Rightarrow 9999\_ \quad 10^5$ times

SQM

	0	1	2	3	4	5	6	
array $\Rightarrow$	3	8	4	3	6	8	9	$\Rightarrow$ 3843689

$3\times10^6 + 8\times10^5 + 4\times10^4 + 3\times1000 + 6\times100 + 8\times10 + 9$

$$\left((3\times10^6) + (8\times10^5) + (4\times10^4) + (3\times10^3) + (6\times10^2) + 8\times10 + 9\right)\%p$$

$$\Rightarrow \left[(3\times10^6)\%p + (8\times10^5)\%p + (4\times10^4)\%p + (3\times10^3)\%p + (6\times10^2)\%p + (8\times10)\%p + 9\%p\right]\%p$$

(additive rule)

0-9

$(0-9)\%p$

multiplicative rule

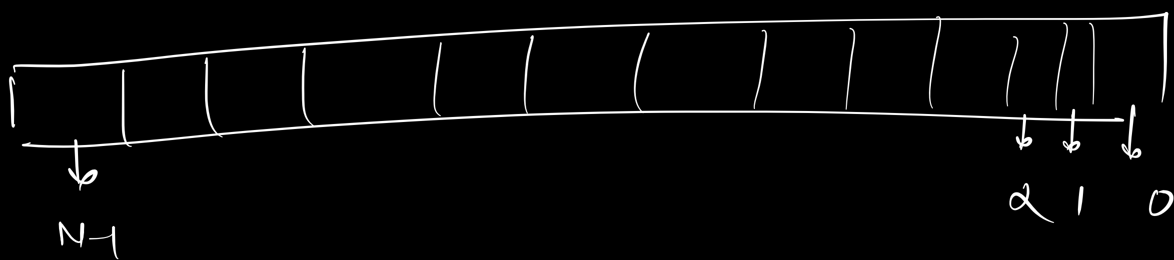
$$\left[(3\%p \times 10^6\%p)\%p + (8\%p \times 10^5\%p)\%p + (4\%p \times 10^4\%p)\%p - - - - \right]\%p$$

10^7

10^{70}

10^{700}

10^{105}



0	1	2	3	4	5	—	—	N-1
a_0	a_1	a_2	a_3	a_4	a_5	—	—	a_{n-1}
N-1							1	0

$$(a_0 \times 10^{N-1}) + (a_1 \times 10^{N-2}) + (a_2 \times 10^{N-3})\%p - - - -$$

$$(a_0\%p \times 10^{N-1}\%p)\%p \qquad (a_1\%p \times power(10, N-2, p))\%p$$

arr[i], $i=1$ $i=1,\ N-i-1$

$$(10)^{N-1}\%p \rightarrow a^x\%p$$

$$\left(a_0 \% P \times \text{power}(10, \underbrace{N-1}, P) \right) \% P$$

$\downarrow$

arr[i] $\quad i=0, \quad N-i-1$

$i=0$

pseudo

ans = 0

for(i = 0; i < N; i++) {

N $\quad$ ans = ans + (A[i] % p X power(10, N-i-1, p)) % p $\quad$ O(N)

$\hookrightarrow$ an = [ans % p + ($\longrightarrow$) % p] % p.

return ans % p

}

$$\boxed{\begin{aligned} TC &\Rightarrow O(N \times N) = O(N^2) \\ SC &= O(1) \end{aligned}}$$

$\downarrow$

max = 10^5

$N^2 = 10^{10}$ (Bruteforce)

Try to optimise

$\downarrow$

O(N log N)

O(N)

$0 \rightarrow N-1$

$1 \rightarrow N-2$

$2 \rightarrow N-3$

$\vdots$

$i \rightarrow N-i-1$

```
for(i=0; i<N; i++){
    ans = ans + (A[i]%p x power(10, N-i-1, p))%p
}
return ans%p
}
```

$((3\%13) \times pw(10,5,13))\%13$

$(6\%13) \times pw(10,4,13)\%13$

0	1	2	3	4	5
3	6	8	9	2	9

13

```
13) 368929 ( 28379
    26
    ---
     108
     104
     ---
       49
       39
       ---
       102
```

$\begin{array}{r} 91 \\ \hline 9119 \\ 117 \\ \hline 2 \end{array}$

$10^5 \rightarrow 999\ldots$ 10^5 times

$10^5 - 1$

$10^4 - 1 = 9999$

$10^5 - 1 = 99999$

10^{N-1}

10^{100}

10^{1}

0

N_2 $N-1$

$N = 10^5$

$10^{10^5 - 1}$