

Proposition:

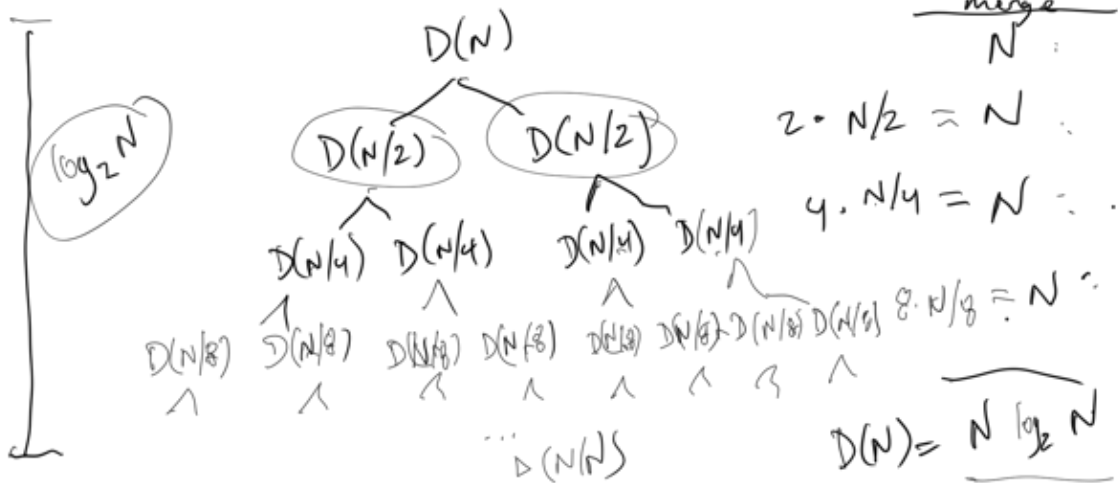
$$\underline{C(N)} \leq \underset{\substack{\uparrow \\ \text{left half}}}{C(N/2)} + \underset{\substack{\uparrow \\ \text{right half}}}{C(N/2)} + \underset{\substack{\text{merge} \\ \sim N/2 - \\ \sim N}}{N}$$

$D(N)$

Proposition: If $D(N)$ satisfies $D(N) = 2 \cdot D(N/2) + N$ for $N > 1$ with $D(1) = 0$, the $D(N) = N \log_2 N$

Total # Comparisons
(less (aux[i], aux[j])

Proof by picture [assuming N is a power of 2]
extra cost for merge
 N :



Proposition: If $D(N)$ satisfies $D(N) = 2 \cdot D(N/2) + N$ for $N \geq 1$
 with $D(1) = 0$ then $D(N) = N \cdot \log_2 N$

Proof by telescoping [assuming N is a power of 2]

for $N \geq 1$: $\frac{D(N)}{N} = \frac{2 \cdot D(N/2)}{N/2} + \frac{N}{N}$

$$= \frac{D(N/4)}{N/4} + 1 + 1$$

$$= \frac{D(N/8)}{N/8} + 1 + 1 + 1$$

$$= \frac{D(N/2^k)}{N/2^k} + \underbrace{1 + \dots + 1}_{\log_2 N}$$

$$D(N) = N \cdot \log_2 N$$

$$\Theta(N \cdot \log_2 N)$$