Permutations - 1

Thursday, 15 July 2021 7:12 PM

n-box, r-items (non identical), print all possible arrangement

$n = 3$

$r = 2 \rightarrow \{1, 2\}$

_ _ _

Level and options:

level - items

option → boxes

→ 1 2 0

→ 1 0 2

→ 2 1 0

→ 2 0 1

→ 0 1 2

→ 0 2 1

arrangement is same but permuted.

$${}^{n}P_{r} = \frac{n!}{(n-r)!} = (n-0)(n-1)(n-2)\cdots(n-(r-1))$$

$n = 4, r = 2,$ ${}^{4}P_{2} = \frac{4!}{2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1} = 12$

level- items-

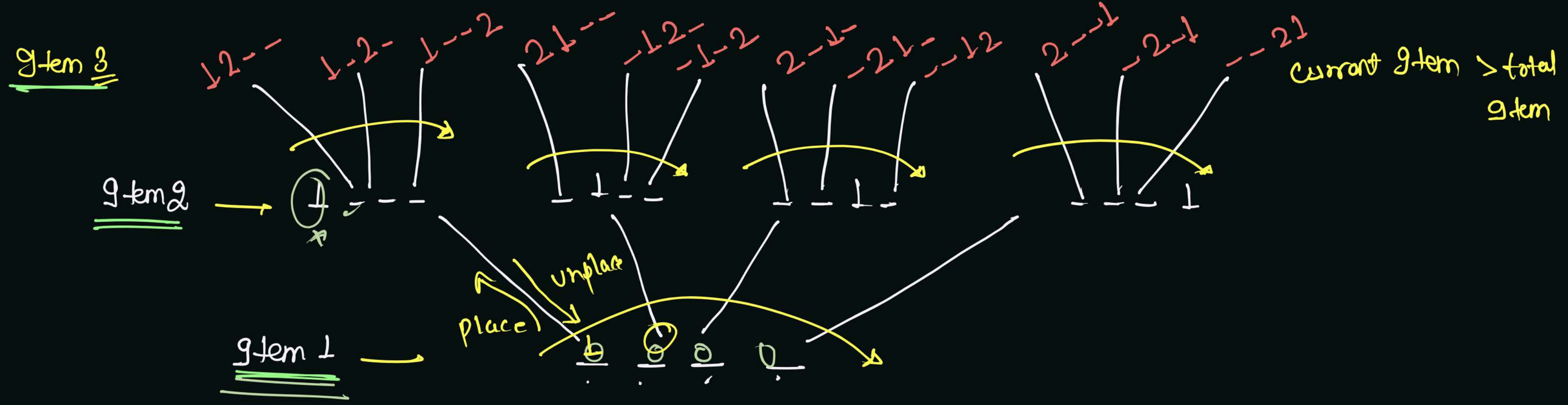
option- boxes

n=4, r=2.

non-identical object
all possibilities } → permutation

1200
1020
1002
2100
0120
0102

2010
0210
0012
2001
0201
0021



Combinations - 1

Thursday, 15 July 2021 7:13 PM

n = boxes, r identical items, print all possibilities to arrange r items in n boxes.

n = boxes = 4, r = 2 (items)

Level - places

options - choice of items.

$${}^{n}P_r = {}^{4}P_2 = \frac{4!}{2!} = \frac{4\times3\times2!}{2!} = 12$$

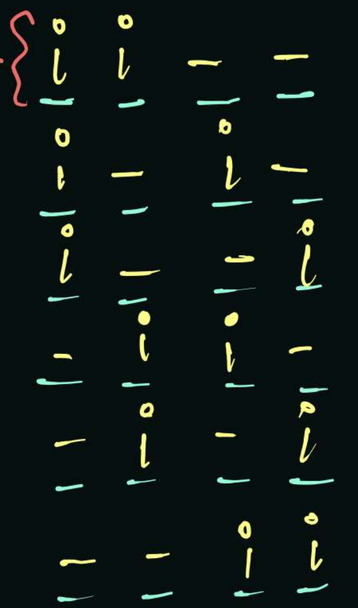
$${}^{n}C_r = \frac{n!}{(n-r)!\,r!} \Rightarrow {}^{4}C_2 = \frac{4!}{2!\,2!} = \frac{\overset{2}{\cancel{4}}\times3\times\cancel{2!}}{\cancel{2!}\times\cancel{2!}} = 6$$

Permutations

1 2 0 0	2 1 0 0
1 0 2 0	2 0 1 0
1 0 0 2	2 0 0 1
0 1 2 0	0 2 1 0
0 1 0 2	0 2 0 1
0 0 1 2	0 0 2 1

Permuted

Combinations



$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n.$$

${}^nC_r$ = possible no. of way to place r identical items in n boxes.

$n = 4$:

$$2^4 = {}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$$

Level → box

options → Choice of item Yes or No

boxes

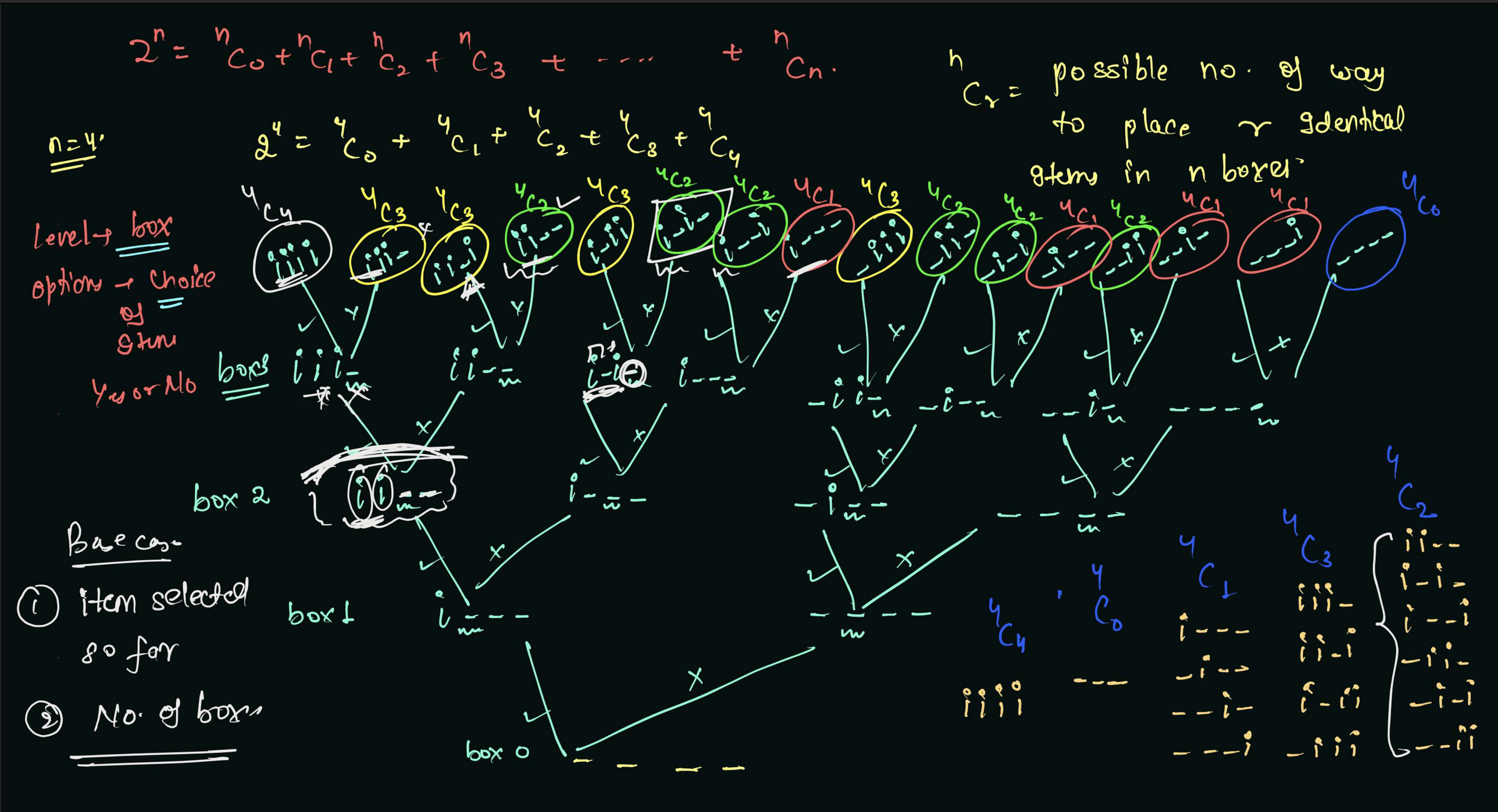
box 2

box 1

box 0

Base case:

1. item selected so far
2. No. of boxes



Permutations - 2

Thursday, 15 July 2021 7:13 PM

n - box and r non identical items.

print all possible arrangement

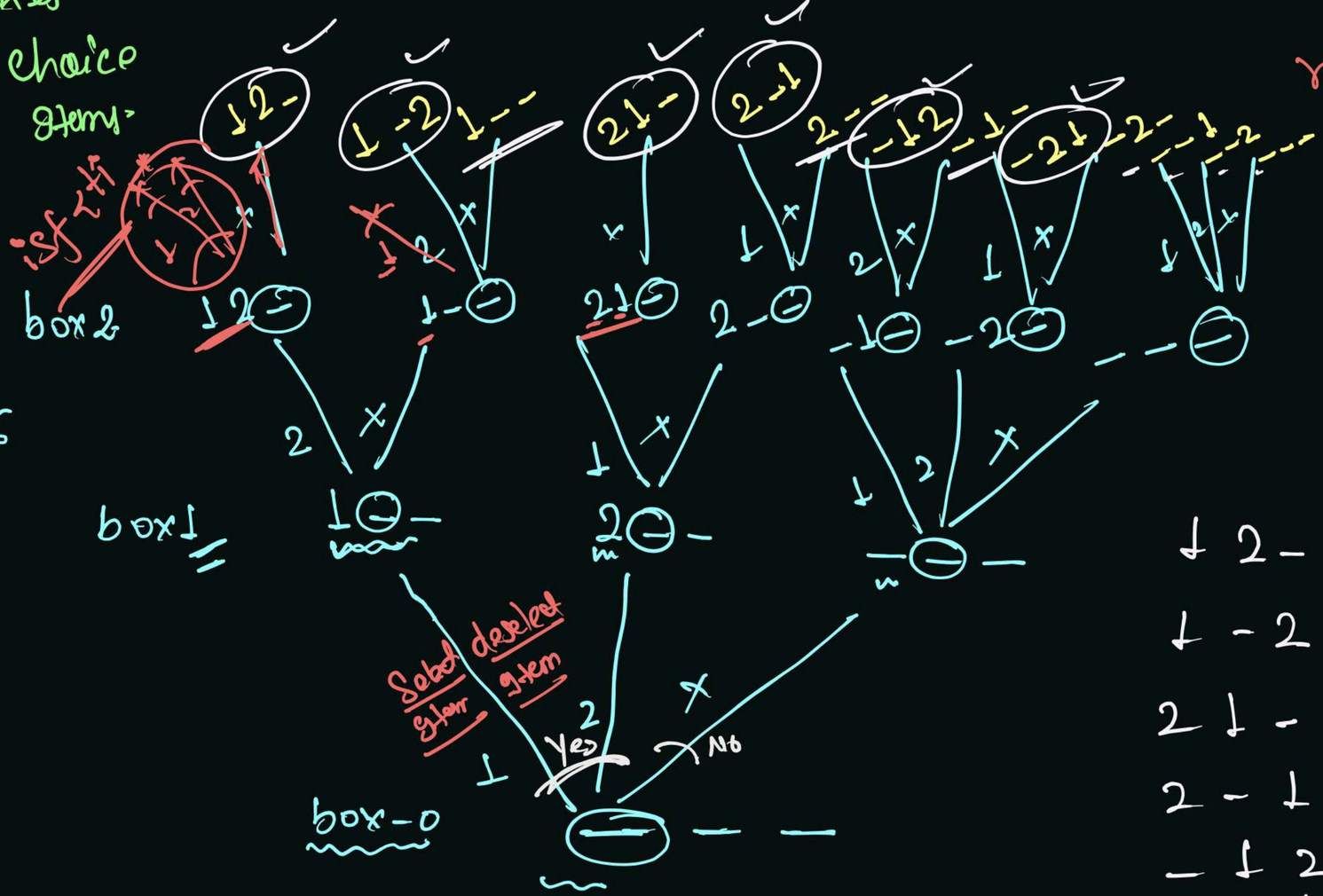
$^nP_2$

level → boxes

options → choice of items

n = 3

r = 2



$^nP_r = \frac{n!}{0!} = 6$

$^nC_r = \frac{n!}{(n-r)!\,r!}$

$^nP_r = \frac{n!}{(n-r)!}$

relation b/w Perm. and Comb.

$^nP_r = {}^nC_r \times r!$

Permute all combinations to achieve permutations.

1 2 -

1 - 2

2 1 -

2 - 1

- 1 2

- 2 1

$^3P_2 = \frac{3!}{1!} = 3 \times 2 \times 1 = 6$

Combinations - 2

Thursday, 15 July 2021 7:13 PM

n- boxes, r. Identical items, print all possibilities to arrange them

Permutation

boxes = 3 , item = 2

$${}^{n}P_{2} = (n-0)(n-1)(n-2)(n-3)\cdots\cdots\cdots(n-(r-1))$$

$${}^{3}P_{2} = \frac{3!}{1!} = 6$$

1 2 0 ⟷ 2 1 0

1 0 2 ⟷ 2 0 1

0 1 2 ⟷ 0 2 1

Combination

boxes = 3 item = 2

$${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!} = \frac{{}^{n}P_{r}}{r!}$$

$${}^{3}C_{2} = \frac{3!}{2!\,1!} = \frac{3\times 2!}{2!\times 1} = 3$$

i i -

i - i

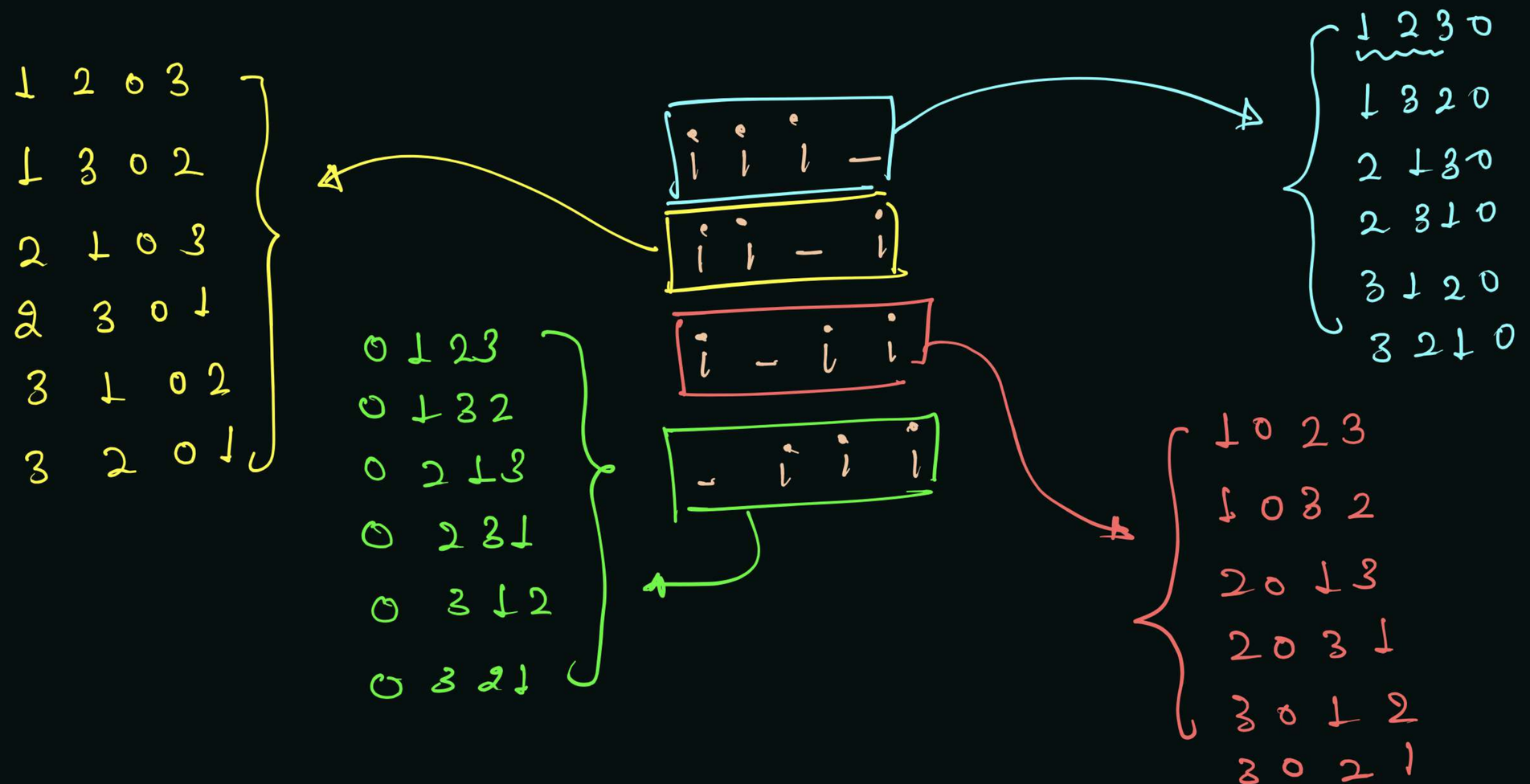
- i i

4 boxes, 3 items → {1, 2, 3}

4boxes, 3 items → {i, i, i}

$$^4P_3 = \frac{4!}{1!} = 4 \times 3 \times 2 = 24$$

$$^4C_3 = \frac{4!}{1!\,3!} = 4$$



24 permutations from 4 combination

let us figure out combination form permutation →

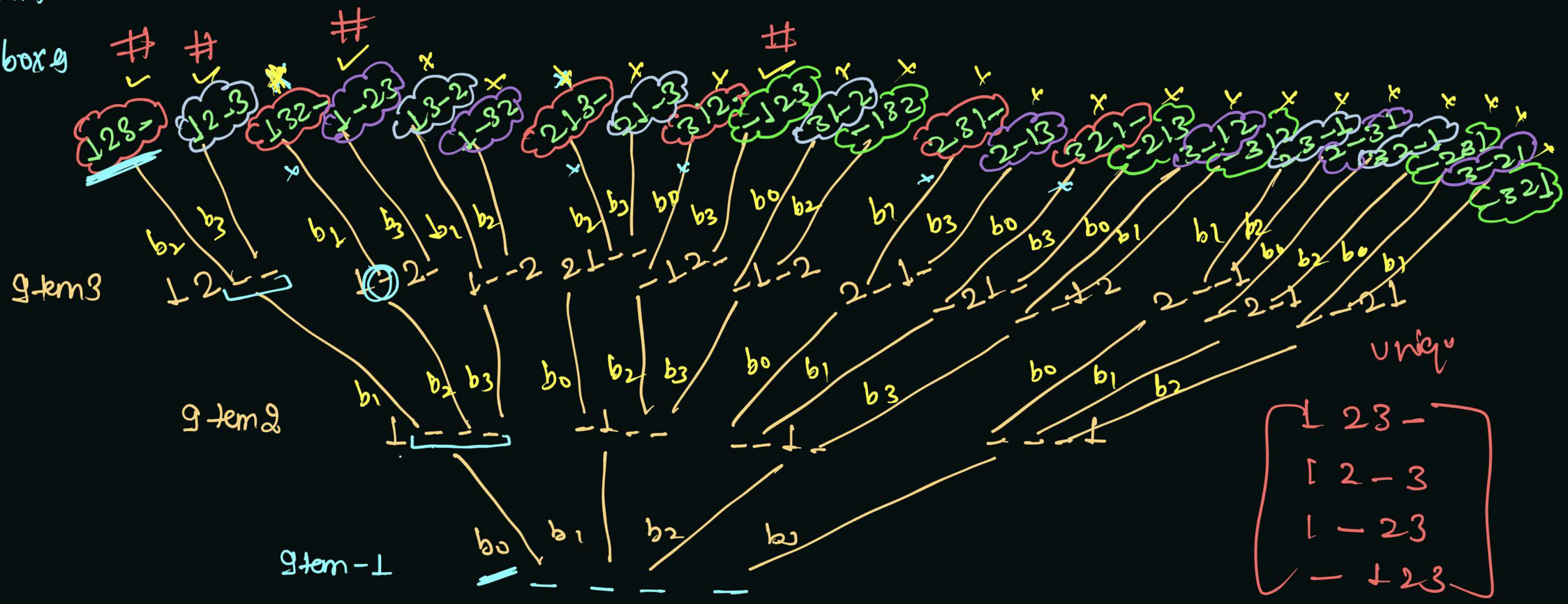
$^4P_3 = \frac{4!}{1!} = 24.$

Permutation-

Make calls in sorted order Index. →

level → 3 items.

options → boxes



$$^{4}C_{3} = \frac{4!}{3!\,1!} = \frac{4\times 3!}{3!} = 4.$$

box – for placing.

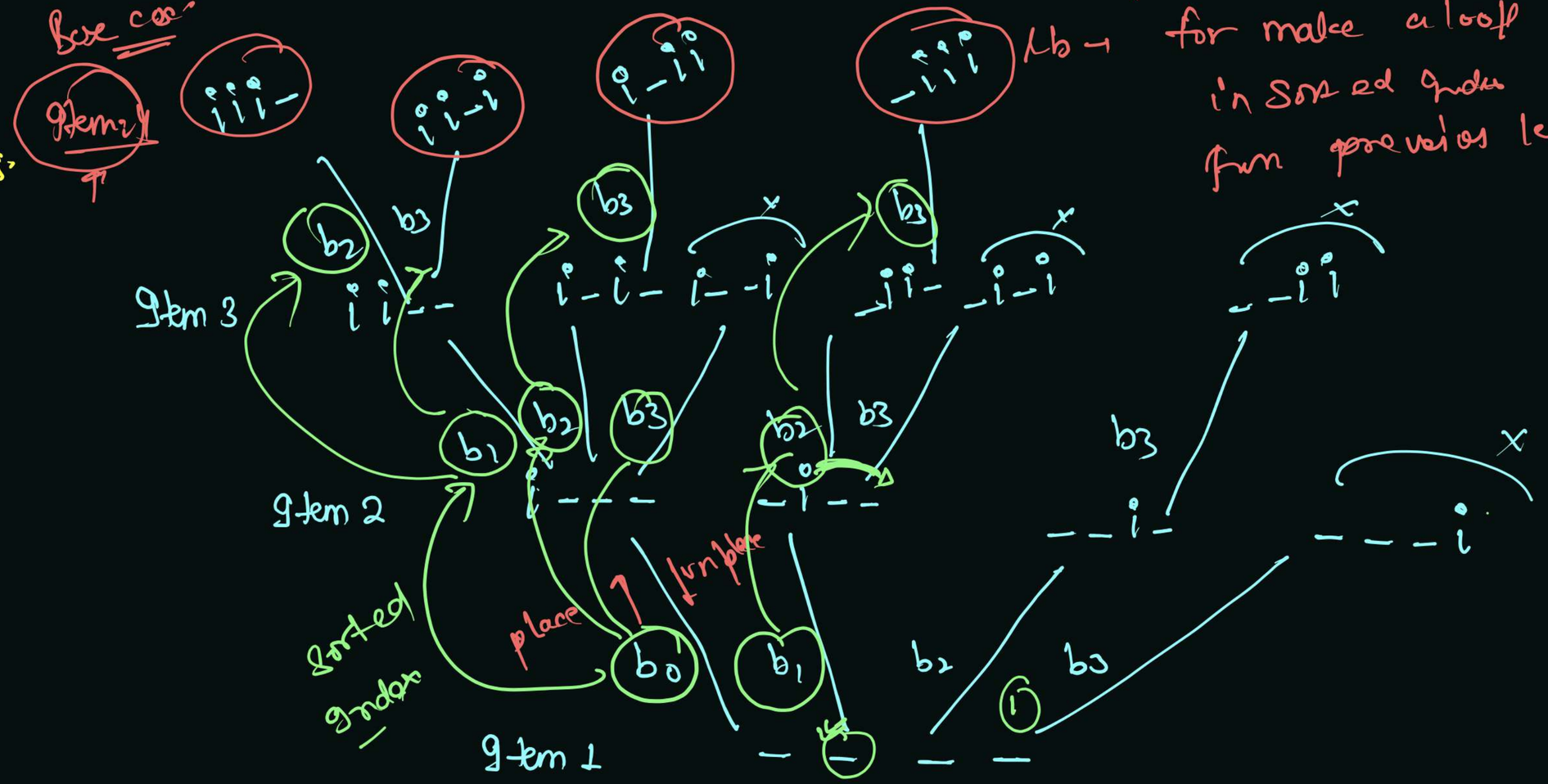
ci → for level

ti → for base case.

lb → for make a loop in sorted order from previous level.

Level → 3 items.

Options → boxes.



i i i –

i i – i

i – i i

– i i i