





## PROBLEM 2

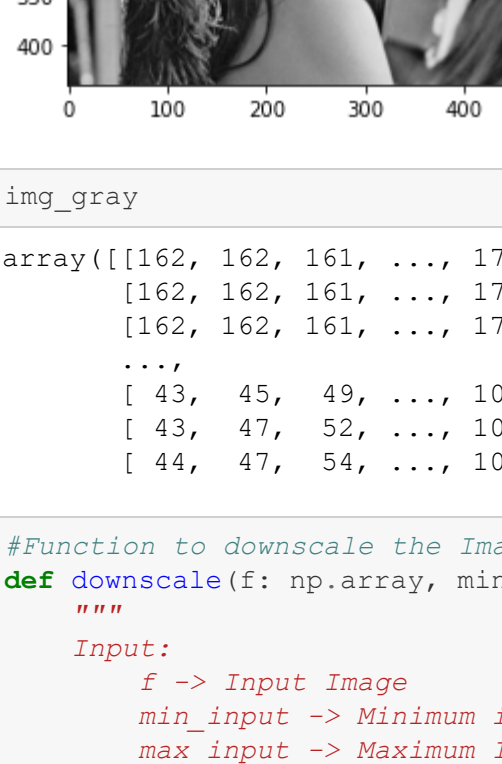
### Problem 2A

```
In [1]: import numpy as np
import pandas as pd
import cv2
import matplotlib.pyplot as plt
```

```
In [2]: img = cv2.imread('lena.png')
img_copy = img.copy()
```

```
In [3]: #Converting the Image into GrayScale
img_gray = cv2.cvtColor(img, cv2.COLOR_BGR2GRAY)
plt.imshow(img_gray,'gray')
plt.title('Gray Scaled Lena Image')
```

```
Out[3]: Text(0.5, 1.0, 'Gray Scaled Lena Image')
```



```
In [4]: img_gray
```

```
Out[4]: array([[162, 162, 161, ..., 172, 160, 131],
               [162, 162, 161, ..., 172, 160, 131],
               [162, 162, 161, ..., 171, 158, 130],
               ...,
               [ 43,  45,  49, ..., 104, 101,  97],
               [ 43,  47,  52, ..., 102, 104, 105],
               [ 44,  47,  54, ..., 101, 105, 108]], dtype=uint8)
```

```
In [5]: #Function to downscale the Image in range [0,1]
def downscale(f: np.array, min_input: int, max_input: int, min_output: int, max_output: int):
    """
    Input:
        f -> Input Image
        min_input -> Minimum input value
        max_input -> Maximum Input Value
        min_output -> Minimum output value
        max_output -> Maximum output Value
    Returns:
        Scaled Down Image -> np.array

    """
    # Similar implementation as Homework
    downscaled_image = (max_output - min_output) * (f / (max_input - min_input))
    return downscaled_image
```

```
In [6]: downscaled_img = downscale(img_gray,0,255,0,1)
dft_image
```

```
Out[6]: array([[0.63529412, 0.63529412, 0.63137255, ..., 0.6745098 , 0.62745098,
               0.51372549],
               [0.63529412, 0.63529412, 0.63137255, ..., 0.6745098 , 0.62745098,
               0.51372549],
               [0.63529412, 0.63529412, 0.63137255, ..., 0.67058824, 0.61960784,
               0.50980392],
               ...,
               [0.16862745, 0.17647059, 0.19215686, ..., 0.40784314, 0.39607843,
               0.38039216],
               [0.16862745, 0.18431373, 0.20392157, ..., 0.4          , 0.40784314,
               0.41176471],
               [0.17254902, 0.18431373, 0.21176471, ..., 0.39607843, 0.41176471,
               0.42352941]])
```

### FUNCTION TO IMPLEMENT DFT

```
In [7]: def DFT2(f: np.array) -> np.array:
    """
    Input:
        f - Input Downscaled Gray Image -> np.array
    Returns:
        DFT of Image -> np.array
    """
    #initializing the result with complex np matrix
    dfft = np.zeros(f.shape,dtype = complex)
    #Applying IDFFT to each row of the image
    for i in range(f.shape[0]):
        dfft[i] = np.fft.fft(f[i])
    #Getting the transpose of the output fft rows matrix
    dfft = np.transpose(dfft)
    #Applying IDFFT to the rows of the Transposed matrix i.e columns of original matrix

    for j in range(f.shape[1]):
        dfft[j] = np.fft.fft(dfft[j])
    #Transposing back to get the original matrix
    dfft = np.transpose(dfft)

    return dfft
```

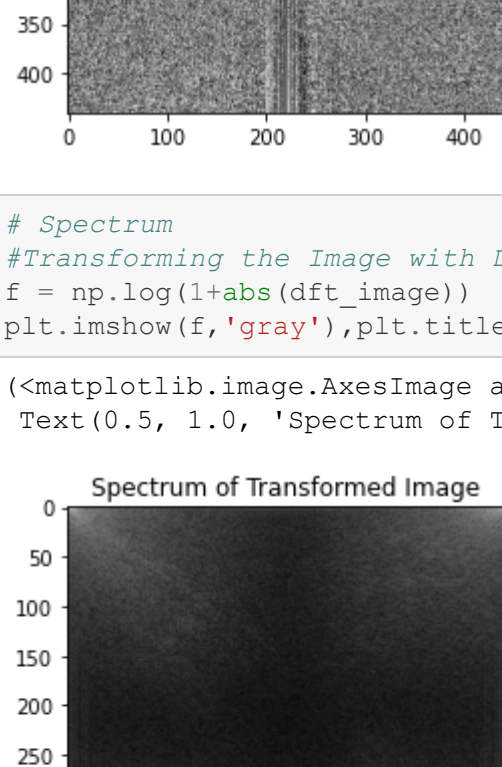
```
In [8]: dft_image = DFT2(downscaled_img)
dft_image
```

```
Out[8]: array([[ 9.41791569e+04 +0.j          , -1.32605209e+03+6928.25982131j,
               4.45443656e+03-3279.8744371j  , ...,
               -6.91412640e+02-1838.48708722j , 4.45443656e+03+3279.8744371j ,
               -1.32605209e+03-6928.25982131j],
               [-8.37846358e+01-3545.76432875j , -4.60583826e+03+4606.16030726j ,
               -3.20398724e+03 +217.75367606j , ...,
               1.89208259e+03+2351.68297678j , -1.02786238e+03+1296.80098955j,
               1.02025196e+02+2741.432367j  ],
               [-1.30229196e+03 -475.37875408j , -7.95709426e+02 ~387.1093878j ,
               1.37670303e+03-1211.96674252j , ...,
               1.02391886e+03+2684.36743816j , -2.09555155e+03 ~509.04882679j,
               2.37833148e+03 -182.71616886j],
               ...,
               [ 1.49437911e+03 ~627.39365192j , 2.13376241e+03+2046.81793011j,
               -1.52536297e+03 ~580.14634242j , ...,
               2.87895939e+03 +316.70305662j , 3.76828529e+02-1965.96854393j,
               -1.56015112e+02+1749.10229252j],
               [-1.30229196e+03 -475.37875408j , 2.37833148e+03 +182.71616886j,
               -2.09555155e+03 +509.04882679j , ...,
               -1.50090169e+03-2181.3107552j , 1.93767503e+03+1311.96674252j,
               -7.95709426e+02 +387.1093878j ],
               [-8.37846358e+01-3545.76432875j , 1.02025196e+02-2741.432367j ,
               -1.02786238e+03-1296.80098955j , ...,
               -1.48190250e+03+5509.4430389j , -3.20398724e+03 ~217.75367606j,
               -4.60583826e+03-4606.16030726j]])
```

### SPECTRUM AND PHASE ANGLE

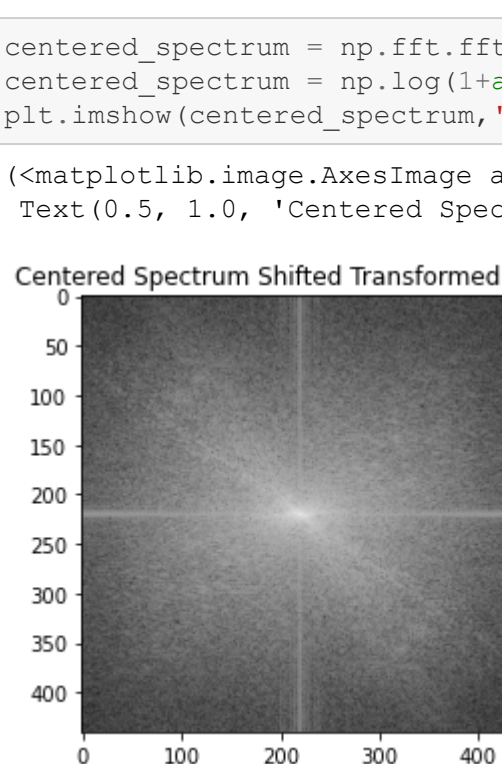
```
In [9]: #To obtain the Phase angle of the image
# Phase Angle
pa = np.angle(dft_image)
plt.imshow(pa,'gray'),plt.title('Phase Angle')
```

```
Out[9]: (<matplotlib.image.AxesImage at 0x15ae225f8e0>, Text(0.5, 1.0, 'Phase Angle'))
```



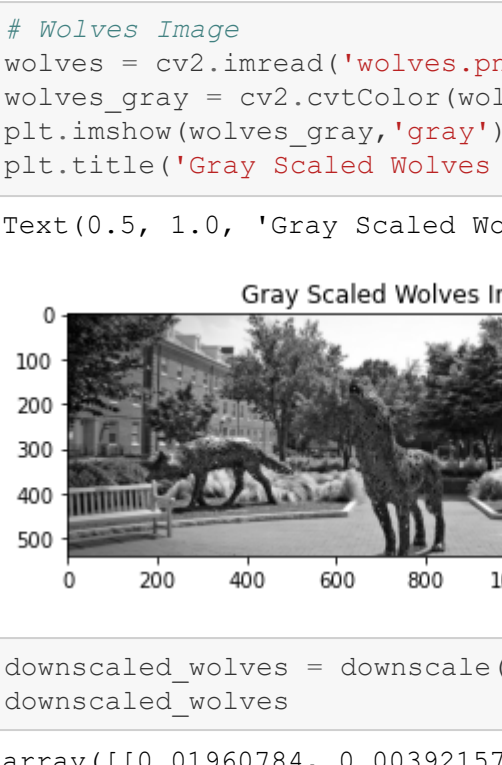
```
In [10]: # Shifted Phase Angle
pa_shifted = np.fft.fftshift(pa)
plt.imshow(pa_shifted,'gray'),plt.title('Centered Phase Angle')
```

```
Out[10]: (<matplotlib.image.AxesImage at 0x15ae244fdf0>,
Text(0.5, 1.0, 'Centered Phase Angle'))
```



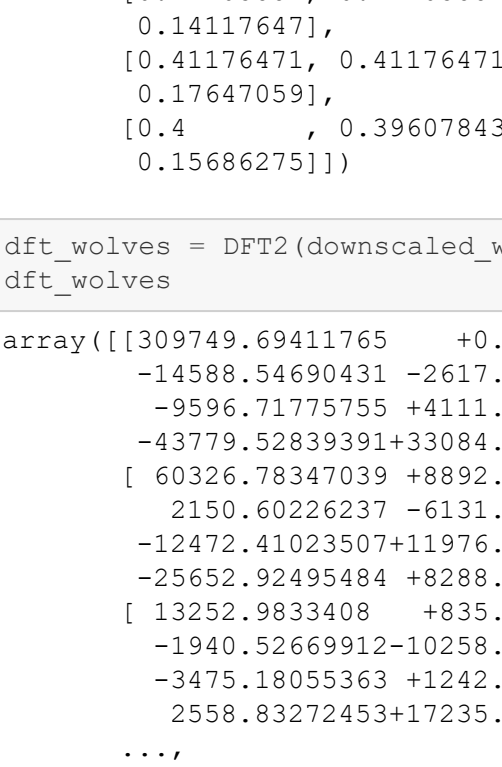
```
In [11]: # Spectrum
#Transforming the Image with DFT
f = np.log(1+abs(dft_image))
plt.imshow(f,'gray'),plt.title('Spectrum of Transformed Image')
```

```
Out[11]: (<matplotlib.image.AxesImage at 0x15ae27cef70>,
Text(0.5, 1.0, 'Spectrum of Transformed Image'))
```



```
In [12]: centered_spectrum = np.fft.fftshift(f)
centered_spectrum = np.log(1+abs(centered_spectrum))
plt.imshow(centered_spectrum,'gray'),plt.title('Centered Spectrum Shifted Transformed Image')
```

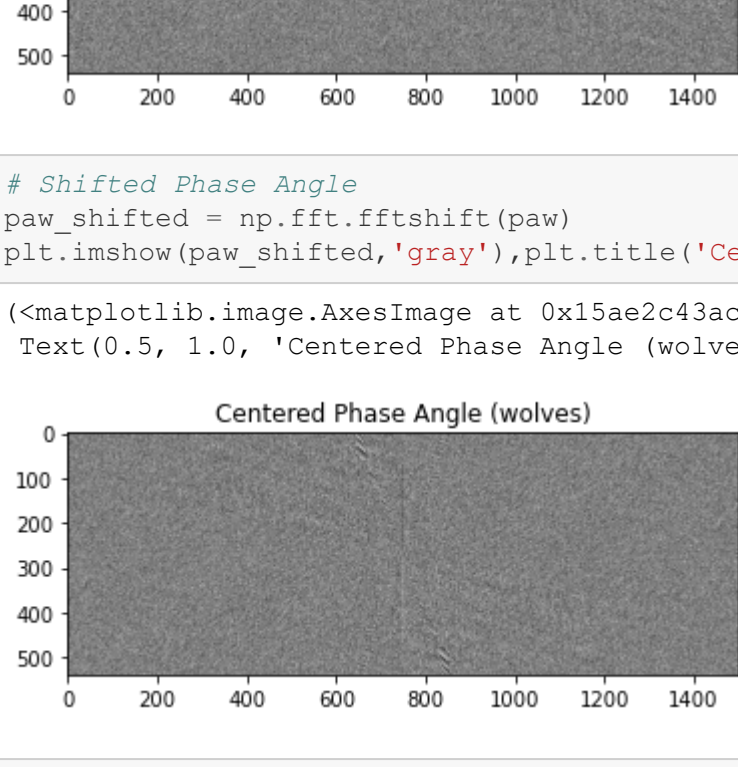
```
Out[12]: (<matplotlib.image.AxesImage at 0x15ae2b46130>,
Text(0.5, 1.0, 'Centered Spectrum Shifted Transformed Image'))
```



In the first images of spectrum and phase angle above, we can see the white spots at the corner of the image, they are low frequency components. We will take the low frequency components to the centre from corners so that the centre part will be lower frequency and others will be higher frequency components. For that, we will use "np.fft.fftshift(input\_image). HENCE, shifting is done"

```
In [13]: # Wolves Image
wolves = cv2.imread('wolves.png')
wolves_gray = cv2.cvtColor(wolves, cv2.COLOR_BGR2GRAY)
plt.imshow(wolves_gray,'gray')
plt.title('Gray Scaled Wolves Image')
```

```
Out[13]: Text(0.5, 1.0, 'Gray Scaled Wolves Image')
```



```
In [14]: downscaled_wolves = downscale(wolves_gray,0,255,0,1)
dft_wolves
```

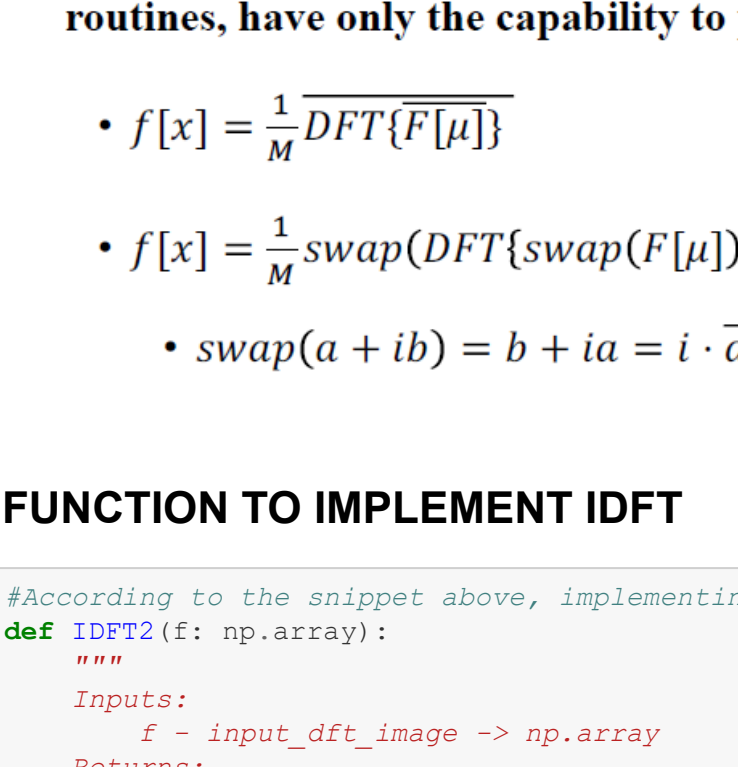
```
Out[14]: array([[0.01960784, 0.00392157, 0.03137255, ..., 0.07843137, 0.0745098 ,
               0.05490196],
               [0.00392157, 0.03921569, 0.01568627, ..., 0.07843137, 0.04705882,
               0.03137255],
               [0.01176471, 0.01960784, 0.01960784, ..., 0.01568627, 0.00784314,
               0.01960784],
               ...,
               [0.44705882, 0.44705882, 0.44705882, ..., 0.13333333, 0.1372549 ,
               0.14117647],
               [0.41176471, 0.41176471, 0.41176471, ..., 0.16470588, 0.17254902,
               0.17647059],
               [0.4          , 0.39607843, 0.39607843, ..., 0.14509804, 0.15294118,
               0.15686275]])
```

```
In [15]: dft_wolves = DFT2(downscaled_wolves)
dft_wolves
```

```
Out[15]: array([[309749.69411765 +0.j          , -43779.52839391-33084.28644852j,
               -14588.54690431 ~2617.35851031j , ...,
               -9596.71775755 +4111.57656599j , -14588.54690431 +2617.35851031j,
               -43779.52839391+33084.28644852j],
               [60326.78347039 +8892.10496815j , -34297.26423775 +5098.73972996j,
               2150.60226237 ~6131.17970098j , ...,
               -12472.41023507+11976.53044243j , -2621.14348949 ~8360.04910609j,
               -25652.92495484 +8288.91090502j],
               [ 13252.9833408 +835.68798497j , -10709.34052319+12325.54974752j,
               -1940.52669912-10258.88679056j , ...,
               -3475.18055363 +1242.605219j , -5917.53675298 ~7589.77996631j,
               2558.83272453+17235.68910706j],
               ...,
               [ 2586.05154279 +60.05017844j , 6939.14797489 ~3688.72255071j,
               -8783.21091252 +7432.63540663j , ...,
               7579.1367225 ~4271.37954036j , -2642.90862199 +2428.84450562j,
               -8995.27295498 ~6607.14483309j],
               [ 13252.9833408 +835.68798497j , 2558.83272453-17235.68910706j,
               -5917.53675298 +7589.77996631j , ...,
               6195.79996278-12121.99364773j , -1940.52669912+10258.88679056j,
               -10709.34052319-12325.54974752j],
               [ 60326.78347039 ~8892.10496815j , -25652.92495484 ~8288.91090502j,
               -2621.14348949 +8360.04910609j , ...,
               -2542.94239322-22574.44097422j , 2150.60226237 +6131.17970098j,
               -34297.26423775 ~5098.73972996j]])
```

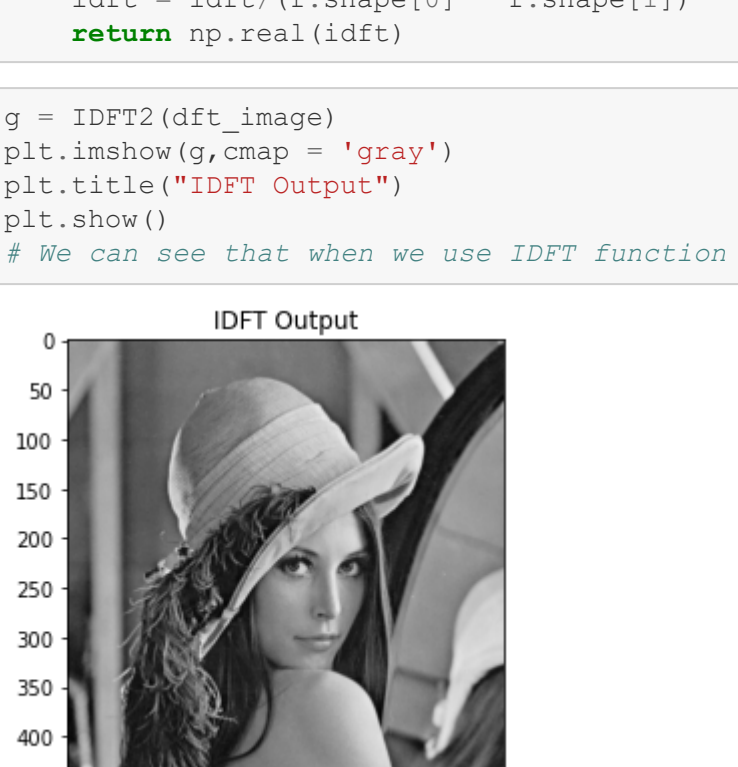
```
In [16]: paw = np.angle(dft_wolves)
paw
plt.imshow(paw,'gray'),plt.title('Phase Angle (wolves)')
```

```
Out[16]: (<matplotlib.image.AxesImage at 0x15ae2b4cd910>,
Text(0.5, 1.0, 'Phase Angle (wolves)'))
```



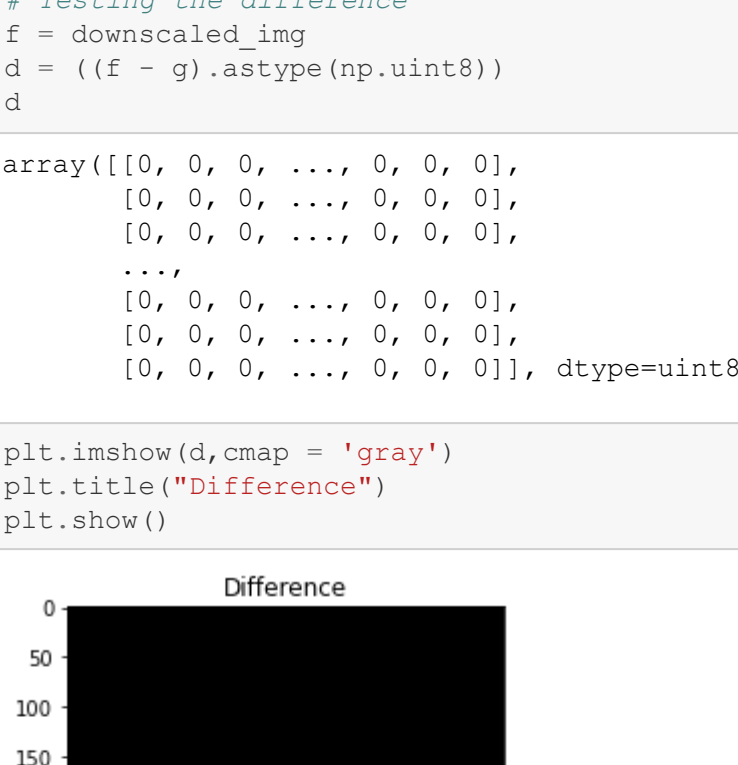
```
In [17]: # Shifted Phase Angle
paw_shifted = np.fft.fftshift(paw)
plt.imshow(paw_shifted,'gray'),plt.title('Centered Phase Angle (wolves)')
```

```
Out[17]: (<matplotlib.image.AxesImage at 0x15ae2b4ca0>,
Text(0.5, 1.0, 'Centered Phase Angle (wolves)'))
```



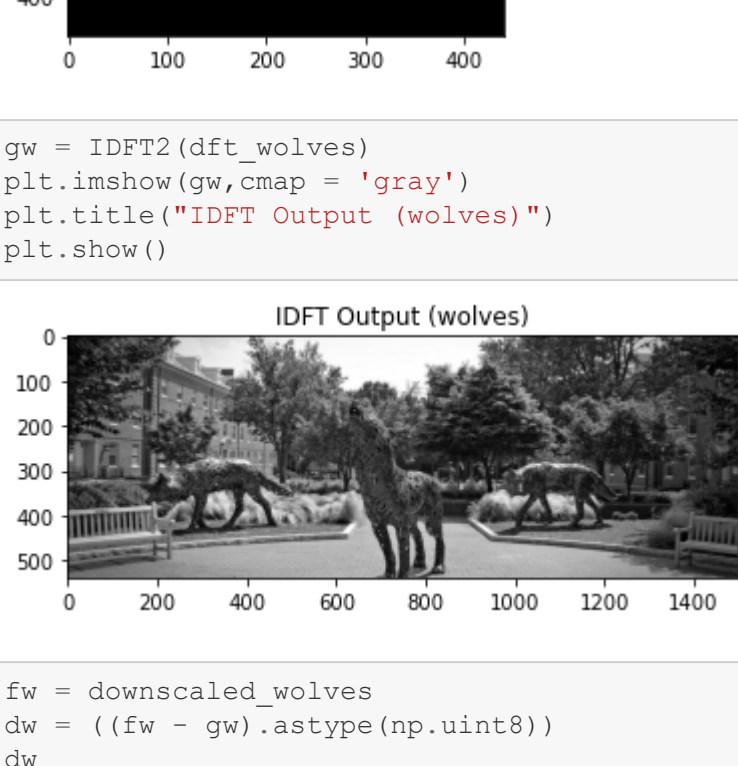
```
In [18]: # Spectrum
#Transforming the Image with DFT
fw = np.log(1+abs(dft_wolves))
plt.imshow(fw,'gray'),plt.title('Spectrum of Transformed Image (wolves)')
```

```
Out[18]: (<matplotlib.image.AxesImage at 0x15ae29cd00>,
Text(0.5, 1.0, 'Spectrum of Transformed Image (wolves)'))
```



```
In [19]: centered_spectrum_w = np.fft.fftshift(fw)
centered_spectrum_w = np.log(1+abs(centered_spectrum_w))
plt.imshow(centered_spectrum_w,'gray'),plt.title('Centered Spectrum Shifted Transformed Image (wolves)')
```

```
Out[19]: (<matplotlib.image.AxesImage at 0x15ae2c43ac0>,
Text(0.5, 1.0, 'Centered Spectrum Shifted Transformed Image (wolves)'))
```



### Problem 2B

#### IDFT Function

## 1-D DFT and Inverse DFT (IDFT)

• **DFT:**  $F[\mu] = \sum_{x=0}^{M-1} f[x]e^{-\frac{j2\pi\mu x}{M}}$ ,  $\mu = 0, 1, 2, \dots, M-1$

• **IDFT:**  $f[x] = \frac{1}{M} \sum_{\mu=0}^{M-1} F[\mu]e^{\frac{j2\pi\mu x}{M}}$ ,  $x = 0, 1, 2, \dots, M-1$

• **Express IDFT using DFT:** How to computer IDFT if available hardware, or software routines, have only the capability to perform the DFT?

•  $f[x] = \frac{1}{M} DFT\{F[\mu]\}$

•  $f[x] = \frac{1}{M} swap(DFT\{swap(F[\mu])\})$ ,

•  $swap(a + ib) = b + ia = i \cdot \overline{a + ib}$

### FUNCTION TO IMPLEMENT IDFT

```
In [20]: #According to the snippet above, implementing the same function
def IDFT2(f: np.array):
    """
    Inputs:
        f - input_dft_image -> np.array
    Returns:
        IDFT Image -> np.array
    """
    # Using the formula given in the lecture slides
    f_conjugate = np.imag(f) + complex('j') * np.real(f)
    idft_conjugate = DFT2(f_conjugate)
    idft = np.imag(idft_conjugate) * complex('j') * np.real(idft_conjugate)
    idft = idft/(f.shape[0] * f.shape[1])
    return np.real(idft)
```

```
In [21]: g = IDFT2(dft_image)
plt.imshow(g, cmap = 'gray')
plt.title("IDFT Output")
plt.show()
```



### VERIFICATION OF IDFT

```
In [22]: # Testing the difference
f = downscaled_img
d = ((f - g).astype(np.uint8))
d
```

```
Out[22]: array([[0, 0, 0, ..., 0, 0, 0],
               [0, 0, 0, ..., 0, 0, 0],
               [0, 0, 0, ..., 0, 0, 0],
               ...,
               [0, 0, 0, ..., 0, 0, 0],
               [0, 0, 0, ..., 0, 0, 0],
               [0, 0, 0, ..., 0, 0, 0],
               [0, 0, 0, ..., 0, 0, 0]], dtype=uint8)
```

```
In [23]: plt.imshow(d, cmap = 'gray')
plt.title("Difference (wolves)")
plt.show()
```



```
In [24]: gw = IDFT2(dft_wolves)
plt.imshow(gw, cmap = 'gray')
plt.title("IDFT Output (wolves)")
plt.show()
```



```
In [25]: fw = downscaled_wolves
dw = ((fw - gw).astype(np.uint8))
dw
```

```
Out[25]: array([[0, 0, 0, ..., 0, 0, 0],
               [0, 0, 0, ..., 0, 0, 0],
               [0, 0, 0, ..., 0, 0, 0],
               ...,
               [0, 0, 0, ..., 0, 0, 0],
               [0, 0, 0, ..., 0, 0, 0],
               [0, 0, 0, ..., 0, 0, 0],
               [0, 0, 0, ..., 0, 0, 0]], dtype=uint8)
```

```
In [27]: plt.imshow(dw, cmap = 'gray')
plt.title("Difference (wolves)")
plt.show()
```



```
In [ ]:
```