MULTI BODY DYNAMICS (ME518)

ASSIGNMENT 1 REPORT

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Method of Using Euler's Method

Euler's Method on $\dot{x}$:

$$\frac{\dot{x}_{n+1} - \dot{x}_n}{\Delta t} = (1-\alpha)\ddot{x}_n + \alpha \ddot{x}_{n+1} \rightarrow (3)$$

but $$\ddot{x}_{n+1} = \frac{-k x_{n+1} - c \dot{x}_{n+1} + f(t)}{m}$$ from $\Rightarrow m\ddot{x} + c\dot{x} + kx = f(t)$

So $\rightarrow$ substituting this value of $\ddot{x}_{n+1} \rightarrow$

$$\dot{x}_{n+1} = \frac{1}{\text{denominator}}\left[\dot{x}_n + (1-\alpha)\Delta t\, \ddot{x}_n + \left(\frac{-\alpha k \Delta t}{m}\right)x_{n+1} + \left(\frac{\alpha f(t)\Delta t}{m}\right)\right] \rightarrow (1)$$

where denominator $= 1 + \dfrac{\alpha c \Delta t}{m}$

Euler's Method on x:

$$\frac{x_{n+1} - x_n}{\Delta t} = (1-\alpha)\dot{x}_n + \alpha \dot{x}_{n+1}$$

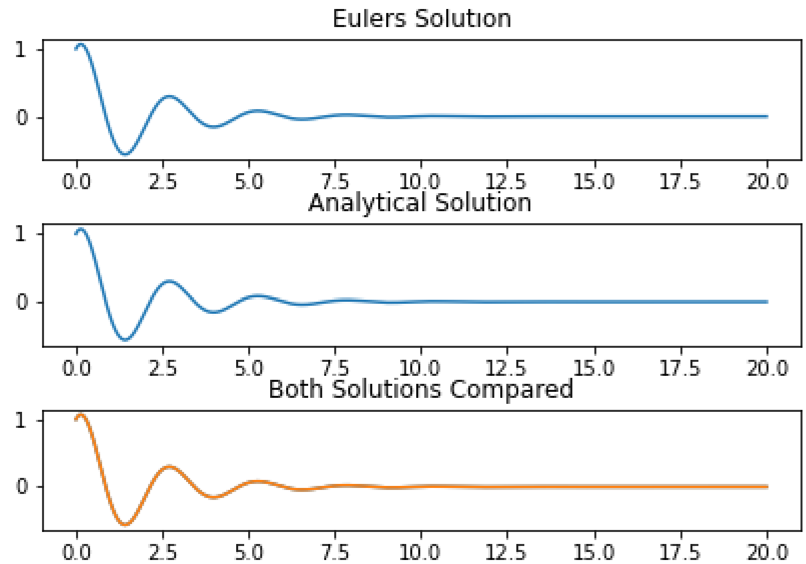
Substituting the value of $\dot{x}_{n+1}$ from (1)

$$x_{n+1} = \frac{1}{\text{denominator}}\left[x_n + (1-\alpha)\Delta t\, \dot{x}_n + \frac{\alpha \dot{x}_n \Delta t}{\text{denominator}} + \frac{\alpha(1-\alpha)\Delta t^2 \ddot{x}_n}{\text{denominator}} + \frac{\alpha^2 \Delta t^2 f(t)}{\text{denominator}}\right] \rightarrow (2)$$

where denominator $= 1 + \dfrac{\alpha^2 \Delta t^2 k}{(m)(\text{denominator})}$

(*) from (1), (2), & (3), we can iteratively find the functional values of $\ddot{x}_n$, $\dot{x}_n$, & x_n discretely.

Correctness of the Homogenous Case



Time Interval Specifics Chosen:

1. Ti = 0
2. Tf = 20
3. del_t = 0.01

Alpha = 0.5

Initial Conditions (non zero) Chosen:

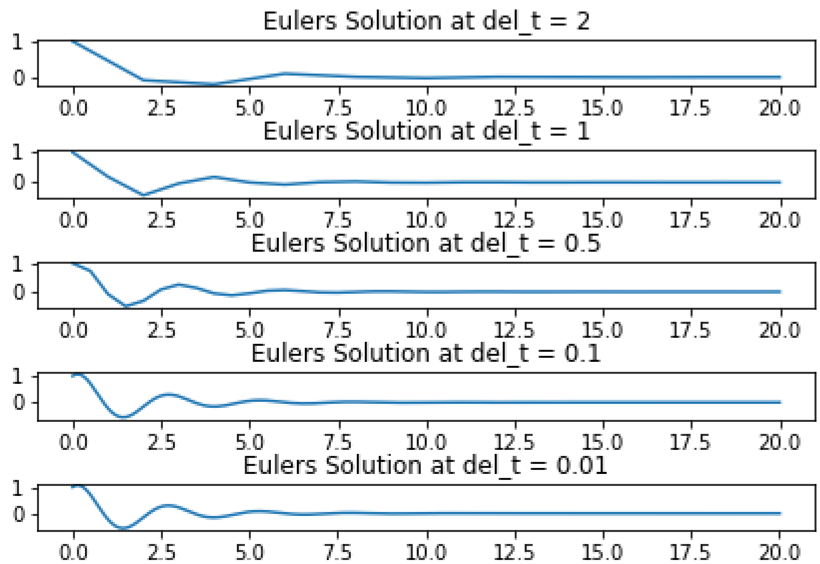
1. x(0) = 1
2. v(0) = 1

The initial values of k, m, & c have been chosen so that zeta < 1, thus indicating an underdamped case, clearly reflecting in all the plots.

Further, both the curves when plotted against each other look almost identical attributing to both the reliability of Euler's Method, as well as the small delta t size chosen (0.01).

Effect of Varying del_t (demonstrated on the homogenous case)

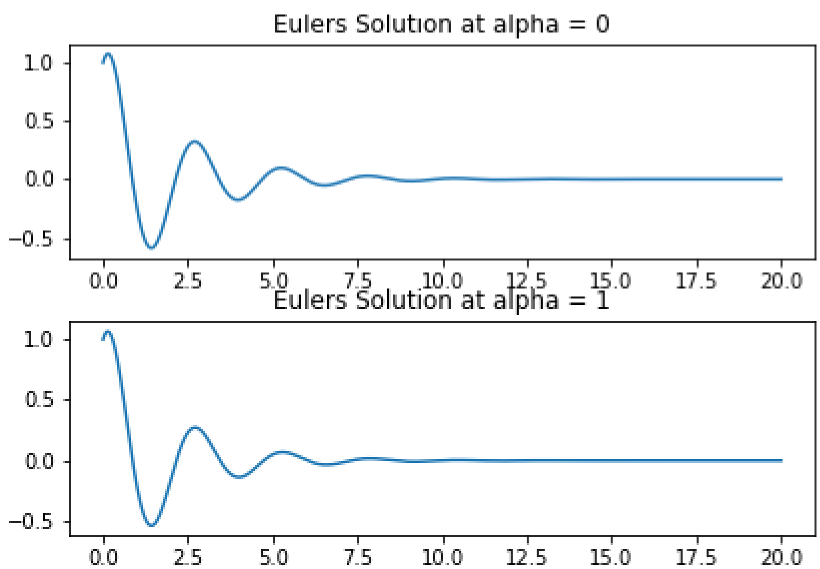
Under the same conditions for the analysis carried out for the homogenous case evaluated above, the value of del_t has been varied multiple times to get the plot displayed below, to show the effect of the choice of del_t on solution accuracy.



As expected, smaller delta t values give rise to more points to calculate from (though more computationally complex), thus leading to more accurate and smoother plots. This can be verified by observing the trend as we go from the plot with delta t = 2 all the way to the plot with delta t = 0.01.

Effect of Varying alpha (demonstrated on the homogenous case)

Again, the effect of varying alpha on the same set of conditions in the homogenous case was demonstrated to observe its effect on accuracy, stability, and computational complexity.



Though both plots look similar because of the range of variables (k, m, c, etc) chosen, there still are minute differences between the two curves.

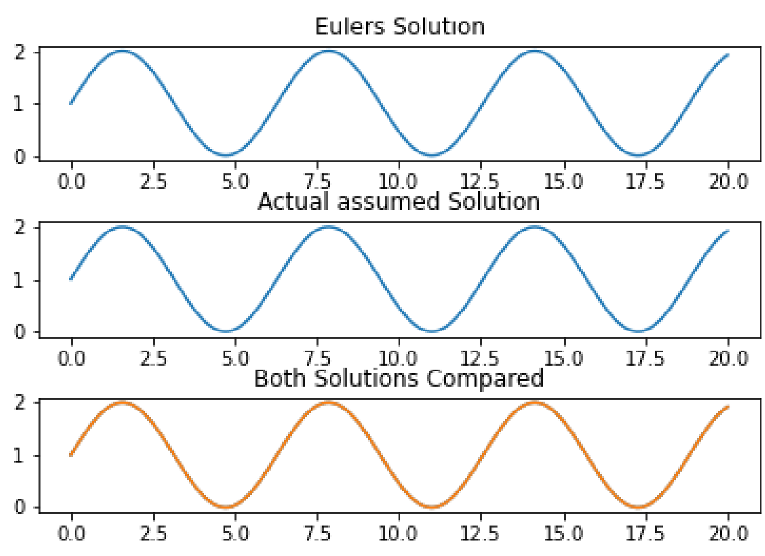
Both curves (thus both alpha values of 0 & 1) have similar accuracies. However, the initial curve drawn when comparing with the analytical solution was taken at alpha = 0.5, and had the highest accuracy of all (can also be seen visually as both the curves almost overlapped perfectly).

The stability for the curve where alpha = 1 was taken is slightly higher, which can be observed by just looking at the curve (especially in the horizontal line part, ie > 7.5 around). The stability definition used here is based off the standard formula of |Yn+1|/|Yn| < 1 signifying stability.

Though not completely visible from the plots, the general trend on varying alpha goes like:

1. The accuracy of the solution on increasing alpha from 0 to 1, first increases, reaches a peak, and then decreases (just a general trend). Thus, compared to alpha being 0 & 1 as shown here, the initial solution of alpha being 0.5 had higher accuracy.
2. The stability slowly and slightly increases on increasing alpha due to the increased presence of the Yn+1 term, as made clear by the |Yn+1|/|Yn| < 1 condition explained earlier.
3. The computational complexity of all cases except for alpha being 0 are almost similar (much higher than when alpha = 0) due to the presence of a non zero Yn+1 term on the right hand side of Euler's Equation.

Correctness of the Non-Homogenous Case



We initially assumed x(t) = 1 + Sin(wt) (under the same initial, time and alpha conditions as used in the homogenous case, with w=1), and substituting this value of x(t) in the spring mass differential equation, we got the value of f(t) we need to use as f(t) = Sin(wt)(k - mw^2) + cwCos(wt) + k. When we use this expression of f(t) back in Euler's Method, we expect the plot we get to be the initial sine curve of x(t) = 1 + Sin(wt) that we started out with.
x(t) = 1 + Sin(wt) with w=1 was specifically chosen as it satisfies both the initial conditions of x(0) = 1, as well as v(0) = 1.
As can be seen in the plot, the solution obtained is pretty accurate showing sine curve characteristics. The graph is pretty much exactly similar, mimicking the sine curve characteristics assumed initially, thus verifying that the Euler's Method coded works in case of non homogenous cases as well, with the assumption of x(t) = 1+ Sin(wt) for the purpose of demonstration.

Plotting Velocity and Acceleration Curves and Verifying Results

The analysis for velocity curves can be done just like position. In that case, in the function that does the euler method ('eulers_method()'), just return the 'x_dot_values' array and plot that to get the curve. Same goes for plotting acceleration, where we would then have to return and plot 'x_dot_dot_values' against time.