F.Y.B.Sc. (Computer Science) Semester - I
Regular Semester-End Examination
Session : Nov. 2022

Subject : Linear Algebra I **Subject Code : USCSMT-111**

Time : 2 Hrs. **Total Marks 35**

Instructions : **(1) All questions are compulsory.**
(2) Figures to the right indicate full marks.
(3) Use of single memory, non-programmable scientific calculator is allowed.

Q.1 Attempt any Five of the following. **10**

(a) Let $A = \begin{bmatrix} -1 & 2 \\ 5 & 4 \\ 2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$,

Compute the product matrix AB.

(b) Solve the following system of linear equations by graphical method.

$x + y = 2$
$x - y = 0$

$2x = 2$
$x = 1$
$y = 1$

(c) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$, defined as $T(x) = Ax$

and $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$ and $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$,

Find T (u).

(d) Let $A = \begin{bmatrix} -3 & -2 & 0 \\ 0 & 2 & -6 \\ 6 & 3 & 3 \end{bmatrix}$

Determine if the vector $u = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ is in null space of A.

(e) Verify that det (AB) = det (A) . det (B)

for the matrices $A = \begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ -1 & -3 \end{bmatrix}$.

(f) If $\overline{x} = (-3.\ 2)$ and $\overline{y} = (5, 4)$ then find the following :

(i) $\overline{x} + \overline{y}$

(ii) $5\overline{y}$

(g) Check whether, $u = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ is in linear combination of vectors $u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Q.2 Attempt any ~~Three~~ of the following. **15**

(a) Find the LU factorization of the matrix

$$A = \begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$$

(b) Solve the following system of linear equations by Gauss - elimination method.

$x_2 + 4x_3 = -5.$

$x_1 + 3x_2 + 5x_3 = -2.$

$3x_1 + 7x_2 + 7x_3 = 6.$

(c) Determine if the following verctors are linearly independent.

$$V_1 = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, V_3 = \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}.$$

(d) Find the standard matrix of T where, $T : IR^2 \to IR^2$ is horizontal shear transformation that leaves $\bar{e}_1$ unchanged and maps $\bar{e}_2$ into $\bar{e}_2 + 3\bar{e}_1$.

(e) Use Cramer's rule to compute the solution of the linear system given below :

$x_1 + x_2 = 3$

$-3x_1 + 2x_3 = 0$

$x_2 - x_3 = 2.$

Q.3 Attempt any One of the following. **10**

(a) Solve the following system of linear equations by LU factorization method.

$3x_1 - 7x_2 - 2x_3 + 2x_4 = -9$

$-3x_1 + 5x_2 + x_3 = 5$

$6x_1 - 4x_2 - 5x_4 = 7$

$-9x_1 + 5x_2 - 5x_3 + 14x_4 = 11$

(b) (i) Determine the value of h for which y is in the span $\{v_1, v_2, v_3\}$ where

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$$

(ii) Determine if the columns of the matrix A forms a linearly independent set.

Justify your answer, where $A = \begin{bmatrix} 1 & 10 & 0 \\ 2 & 5 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

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1-15

6x6

F.Y.B.Sc. (Computer Science)

USCSMT 121 Linear Algebra 2

Time: 2 Hours] **[Max marks: 35**

Instructions for candidates:

1. All questions are compulsory.
2. Figures to right indicate full marks.
3. Non-programmable, single memory scientific calculator is allowed.

Q1. Attempt any FIVE in the following. **[10 M]**

A. Let $w = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$. Determine whether w is in null space of A.

B. Let $T = \{ (a - 3b, b) | a, b\ \mathbb{R}\}$. Show that T is a subspace of vector space $\mathbb{R}^2$.

C. Find the vector $[\boldsymbol{x}]_{\boldsymbol{B}}$ determined by the given coordinate vector $\boldsymbol{x}$ and the given basis $\boldsymbol{B} = \{b_1, b_2\}$

$$b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}, \quad x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

D. Find the $\mathbb{B}$- matrix for the transformation $x \mapsto Ax$ when $\mathbb{B} = \{b_1, b_2\}$.

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

E. Let $x = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$ and $y = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$. Compute $\boldsymbol{x}.\boldsymbol{y}$ and $\mathbf{cos}\,\boldsymbol{\theta}$.

F. Find the eigenvalues for the following matrix A

$$A = \begin{bmatrix} 7 & 3 \\ 0 & -1 \end{bmatrix}$$

G. Determine if $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation.

$$T(x, y) = (x + y, 2y)$$

Q2. Attempt any THREE in the following- **[15 M]**

A. Prove that intersection of two subspaces is a subspace.

B. Find an eigenvector corresponding to the eigenvalue $\lambda = 3$ of the matrix A.

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

C. For what value of **k** will the vector w be in the subspace of $\mathbb{R}^3$ spanned by v_1, v_2, v_3 given below.

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -4 \\ 3 \\ \boldsymbol{k} \end{bmatrix}$$

D. Find the matrix of the quadratic form and classify the given quadratic form as positive definite, negative definite or, indefinite.

$$2x_1x_2 + 2x_2x_3 + 2x_1x_3$$

E. Check whether the vector y is the linear combination of u_1, u_2.

$$y = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}, \quad u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$$

Q3. Attempt any ONE in the following- **[10 M]**

A. Let $T: \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation whose standard matrix is given below. Find a basis for $\mathbb{R}^4$ with the property that $[T]_B$ is diagonal.

$$A = \begin{bmatrix} 15 & -66 & -44 & -33 \\ 0 & 13 & 21 & -15 \\ 1 & -15 & -21 & 12 \\ 2 & -18 & -22 & 8 \end{bmatrix}$$

B. i) Diagonalize the following matrix if possible. [5]

$$M = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

ii) Let $v_1 = \begin{bmatrix} 1 \\ -6 \\ 2 \\ 0 \\ 12 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -3 \\ 2 \\ 7 \\ -3 \\ -9 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 9 \\ -8 \\ -11 \\ 4 \\ 7 \end{bmatrix}$ [5]

Compute the following

i. $\|v_1\|, \|v_2\|, \|v_3\|$

ii. $\text{dist}(v_1, v_2), \text{dist}(v_2, v_3)$

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