Ex. 1.5.3

Rotate point P[4 3] in counter clockwise direction through $\angle 45°$ about origin.

☑ **Soln. :**

– As this is rotation about origin, we use standard rotation matrix. Rotation is counter clock wise through $\angle\theta = 45°$

$$\therefore \text{Rotation matrix} = [T]$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(45) & \sin(45) \\ \sin(45) & \cos(45) \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{bmatrix}$$

$$\therefore \text{Transformed point } [P^*] = [P][T]$$

$$= [4 \quad 3]\begin{bmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{bmatrix}$$

$$= [0.707 \quad 4.949]$$

▶ **Ex. 1.5.4** **SPPU - Oct. 2010, 1 Mark**

Write the transformation matrix for rotation about the origin through an angle $(-25)°$.

☑ **Soln. :**

Matrix of rotation about origin by an angle θ is [T]

$$[T] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\text{Here } \theta = (-25)$$

$$[T] = \begin{bmatrix} \cos(-25) & \sin(-25) \\ -\sin(-25) & \cos(-25) \end{bmatrix}$$

$$= \begin{bmatrix} \cos 25 & -\sin 25 \\ \sin 25 & \cos 25 \end{bmatrix}$$

▶ **Ex. 1.5.6 SPPU - April 2011, 1 Mark**

What is determinant of the inverse of any pure rotation matrix ?

☑ **Soln. :**

The transformation matrix for inverse rotation about origin is

$$[T] = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(\theta) & \cos(-\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore \det[T] = \cos^2\theta + \sin^2\theta = 1$$

▶ **Ex. 1.5.7 SPPU - April 2006, 1 Mark**

Determine whether the transformation matrix,

$$[T] = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1\sqrt{2} \end{bmatrix}$$ represents a reflection.

Justify !

☑ **Soln. :**

Here

$$\det[T] = \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{-1}{2} - \frac{1}{2} = -1$$

i.e. $\det[T] = -1$

Hence [T] is a pure reflection.

$\therefore$ Given matrix [T] represents a pure rotation.

Ex. 1.5.9

Show that a 2D reflection through the x-axis, followed by a 2D reflection through the line $y = -x$, is equivalent to a pure rotation about origin.

☑ **Soln. :**

Let $[T_1]$ = The transformation matrix for reflection through x-axis.

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$[T_2]$ = The transformation matrix for reflection through the line $y = -x = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Now combined transformation matrix is,

$$[T] = [T_1] \cdot [T_2] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Now, $\det [T] = \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = 0 + 1 = 1$

i.e. $\det [T] = 1 \therefore$ [T] represents a pure rotation

Ex. 1.5.10

Ex. 1.5.13 **SPPU - Oct. 2014, 1 Mark**

What is the effect of the transformation matrix $[T] = \begin{bmatrix} 1 & -5 \\ 0 & 1 \end{bmatrix}$ on a two dimensional object ?

Soln. :

The transformation matrix is,

$$[T] = \begin{bmatrix} 1 & -5 \\ 0 & 1 \end{bmatrix}$$

This will produce shearing in y-direction by – 5 units.

Ex. 1.5.14 **SPPU - Oct. 2014, 1 Mark**

Write the transformation matrix for scaling x and z-co-ordinates by factors $\frac{1}{2}$ and 3 respectively.

Soln. :

The transformation matrix for scaling in X and Z-co-ordinates by factor $\frac{1}{2}$ and 3 respective is

$$[T] = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Ex. 1.5.15 **SPPU - April 2015, 2 Mark**

Write the transformation matrix for shearing in x and y directions by – 2 and 5 units respectively. Apply it on the point P [3, 4]

Soln. :

The transformation matrix for shearing in x and y directions by – 2 and 5 units respectively = [T]

$$= \begin{bmatrix} 1 & 5 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x_2^* & y_2^* \end{bmatrix}$$

Ex. 1.6.1

Transformed line AB using $[T] = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

where $A = [2 \;\; 5]$ $\quad B = [7 \;\; 10]$

☑ **Soln. :**

Transformed line $A^* B^* = [L^*]$

$$[L^*] = [L]\,[T] = \begin{bmatrix} A \\ B \end{bmatrix} [T]$$

$$= \begin{bmatrix} 2 & 5 \\ 7 & 10 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 10 \\ 14 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} A^* \\ B^* \end{bmatrix}$$

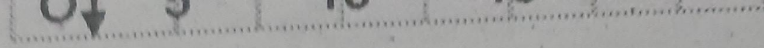

Fig. P. 1.6.1

▶ **Ex. 1.6.2** **SPPU - April 2013, 1 Mark**

If we apply the 2×2 transformation matrix onto the points A(1, 0) and B(0, 1); then they are transformed to the points A* (4, 1) and B* (– 2, 2) respectively. What is the transformation matrix ?

☑ **Soln. :**

Let $[T] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be 2×2 transformation matrix

As $\begin{bmatrix} A^* \\ B^* \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} [T]$

$$\begin{bmatrix} 4 & 1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\therefore a = 4, \quad b = 1,$

$c = -2, \quad d = 2$

The matrix $[T] = \begin{bmatrix} 4 & 1 \\ -2 & 2 \end{bmatrix}$

Ex. 1.7.1

Show that the midpoint is transformed to the midpoint where, $A = \begin{bmatrix} 0 & \frac{5}{2} \end{bmatrix}$,

$B = \begin{bmatrix} \frac{5}{3} & 0 \end{bmatrix}$, $[T] = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

☑ **Soln. :**

Consider line segment AB

Where, $A = \begin{bmatrix} 0 & \frac{5}{2} \end{bmatrix}, B = \begin{bmatrix} \frac{5}{3} & 0 \end{bmatrix}$

$$[T] = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$M = \text{Midpoint of AB} = \begin{bmatrix} \dfrac{0+\frac{5}{3}}{2} & \dfrac{\frac{5}{2}+0}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{6} & \frac{5}{4} \end{bmatrix}$$

We transform this point,

$$\therefore \quad [M]\,[T] = \begin{bmatrix} \frac{5}{6} & \frac{5}{4} \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$[M]\,[T] = \begin{bmatrix} \frac{5}{3} & \frac{15}{4} \end{bmatrix}$$

Consider $A^* = A\,[T]$ ...(1)

$$= \begin{bmatrix} 0 & \frac{5}{2} \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{15}{2} \end{bmatrix}$$

$$B^* = B\,[T] = \begin{bmatrix} \frac{5}{3} & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{10}{3} & 0 \end{bmatrix}$$

The midpoint of transformed line segment A^* B^*

$$= \begin{bmatrix} \dfrac{0+\frac{10}{3}}{2} & \dfrac{\frac{15}{2}+0}{2} \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & \frac{15}{4} \end{bmatrix} \quad ...(2)$$

From Equations (1) and (2) $[M^*] = [M]\,[T]$

∴ The midpoint [M] is transformed to the midpoint [M*].

Ex. 1.7.4

If A = [4 9] B = [3 2] line AB is uniformly scaled by 3 units. What is midpoint of transformed line ?

☑ **Soln. :**

Uniform scaling by a factor 3.

$$\therefore \quad [T] = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$A = [4 \quad 9], \qquad B = [3 \quad 2]$$

$$\therefore \text{ Midpoint of AB } = M = \begin{bmatrix} \frac{7}{2} & \frac{11}{2} \end{bmatrix}$$

Using Theorem 2, under any 2 × 2 transformatio matrix the midpoint is transformed to the midpoint.

We transform midpoint,

$$[M^*] = M\,[T] = \begin{bmatrix} \frac{7}{2} & \frac{11}{2} \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{21}{2} & \frac{33}{2} \end{bmatrix}$$

Ex. 1.7.7

If a line segment AB is transformed under $[T] = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ where $A \equiv [15 \ \ 7]$ and $B = [1 \ \ 3]$. Find midpoint of the transformed line segment A*B*.

Soln. :

Let
$$A = [x_1 \ \ y_1] = [15 \ \ 7]$$
$$B = [x_2 \ \ y_2] = [1 \ \ 3]$$

Let M be the midpoint of segment AB

$$[M] = \begin{bmatrix} \frac{x_1 + x_2}{2} & \frac{y_1 + y_2}{2} \end{bmatrix} = \begin{bmatrix} \frac{15+1}{2} & \frac{7+3}{2} \end{bmatrix}$$
$$= [8 \ \ 5]$$

Given transformation matrix,

$$[T] = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

We know that midpoint of a line segment transforms to midpoint under 2×2 transformation matrix.

$\therefore$ Midpoint of A*B*

$$[M^*] = [M]\,[T]$$
$$= [8 \ \ 5] \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} = [34 \ -3]$$

Ex. 1.7.8

Show that under transformed $[T] = \begin{bmatrix} -1 & 0 \\ 5 & 2 \end{bmatrix}$ point [P] which divides AB in the ratio 2 : 3 is transformed to point [P^*] which divides A^*B^* in the ratio 2 : 3, use $A = [-1 \ \ 2]$, $B = [3 \ \ 1]$.

☑ **Soln. :**

Consider 2 × 2 transformed matrix $[T] = \begin{bmatrix} -1 & 0 \\ 5 & 2 \end{bmatrix}$

Using section formula,

$$[P] = \begin{bmatrix} \frac{2.3 + 3(-1)}{2+3} & \frac{2(1)+3(2)}{2+3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{5} & \frac{8}{5} \end{bmatrix}$$

We transform this point [P]

$$[P][T] = \begin{bmatrix} \frac{3}{5} & \frac{8}{5} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{37}{5} & \frac{16}{5} \end{bmatrix} \qquad \ldots(1)$$

Now, we transform line [A] [B] to get line A^*B^*

$$\begin{bmatrix} A^* \\ B^* \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} [T]$$

$$= \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 11 & 4 \\ 2 & 2 \end{bmatrix}$$

Let point [P^*] divides $A^* B^*$ in the ratio 2 : 3

$$[P^*] = \begin{bmatrix} \frac{2(2)+3(11)}{2+3} & \frac{2(2)+3(4)}{2+3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{37}{5} & \frac{16}{5} \end{bmatrix} \qquad \ldots(2)$$

$$(1) = (2)$$

$$[P^*] = [P]\,[T]$$

☑ **Soln. :**

The midpoint of

The transfor

factor 2,

$[T] = [$ 2 …

Under any 2 ×

transformed to the

$[M^*] = [$m …

$= [$ 1/2 …

$[M^*] = [1$ …

1.8 Slop…

UQ. If the lin… to the li… transform… then prov…

Under any 2 … is not preserved. B… find slope of transf…

Theorem

If line AB wi…

The position vector of the points A and B are [2 3] and [– 4 7] respectively. The straight line AB is transformed to the straight line A′B′ using the transformation matrix $[T] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find the slope of A′B′.

☑ **Soln. :**

Given points are

$$A = [x_1 \;\; y_1] = [2 \;\; 3]$$

$$B = [x_2 \;\; y_2] = [-4 \;\; 7]$$

∴ Slope of line A B,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{-4-2} = \frac{4}{-6} = \frac{-2}{3}$$

i.e. $$m = \frac{-2}{3}$$

Given transformation matrix $= \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

i.e. $a = 1,\ b = 2,\ c = 3,\ d = 4$

We have slope of the transformed line A′B′ is,

$$m^* = \frac{b + dm}{a + cm} = \frac{2 + (4)\left(\frac{-2}{3}\right)}{1 + (3)\left(\frac{-2}{3}\right)} = \frac{\frac{(6-8)}{3}}{(1-2)} = \frac{-2}{-3}$$

$$\therefore m^* = \frac{2}{3}$$

Ex. 1.8.5

If the line $y = 5x - 2$ is transformed under $[T] = \begin{bmatrix} 15 & -1 \\ 7 & 2 \end{bmatrix}$ find the slope of transformed line.

☑ **Soln. :** Given line is, $y = 5x - 2$

$\therefore$ Slope $m = 5$ ...(Comparing with $y = mx + c$)

Let $[T] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 15 & -1 \\ 7 & 2 \end{bmatrix}$

$\therefore a = 15, \quad b = -1, \quad c = 7, \quad d = 2.$

Then we have if a line having slope m is transformed under $[T] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then slope of transformed line m^* is given by,

$$m^* = \frac{b + dm}{a + cm} = \frac{(-1) + (2)(5)}{15 + (7)(5)} = \frac{9}{50}$$

$$m^* = \frac{9}{50}$$

Ex. 1.8.6

If a line segment AB is transformed to segment A^*B^* under $[T] = \begin{bmatrix} 3 & -1 \\ 7 & 8 \end{bmatrix}$. Find slope of segment A^*B^* where $A^* = [-5 \quad 1]$ $B^* = [2 \quad 3]$.

☑ **Soln. :**

Given that transformed points,

$$A^* = [-5 \quad 1]$$

$$B^* = [2 \quad 3]$$

$\therefore$ Slope of transformed segment A^*B^* is,

$$m^* = \frac{3-1}{2+5} = \frac{2}{7}$$

Let $[T] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 7 & 8 \end{bmatrix}$

$\therefore$ $a = 3, \quad b = -1, \quad c = 7, \quad d = 8$

We have if a line having slope m is transformed under $[T] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then slope of the transformed line m^* is given by,

$$m^* = \frac{b + dm}{a + cm}$$

$$\frac{2}{7} = \frac{(-1) + (8)\,m}{(3) + 7\,m} = \frac{8m - 1}{7m + 3}$$

$$\therefore \quad 14m + 6 = 56m - 7$$

$$7 + 6 = 56m - 14m$$

$$13 = 42m$$

$$\therefore \; m = \frac{13}{42}$$

Ex. 1.8.7 SPPU - March 2014, 1 Mark

If we apply a 2×2 transformation matrix $[T] = \begin{bmatrix} 2 & 4 \\ -4 & 3 \end{bmatrix}$ on the line $X + 2Y - 5 = 0$ then find the slope of the resulting line.

Soln. :

The transformation matrix,

$$[T] = \begin{bmatrix} 2 & 4 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\therefore$ $a = 2, \quad b = 4, \quad c = -4, \quad d = 3$

The line L is $\quad x + 2y - 5 = 0$

$$2y = -x + 5$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

Ex. 1

If we app… onto the … line.

Soln

2×2 tra…

a = …

Line L …

$\therefore$

Slope o…

Ex.

If the … transfor…

[T] = …

Sol

The lin…

Ex. 1.8.8 SPPU - April 2013, 1 Mark

If we apply a 2 × 2 transformation matrix $[T] = \begin{bmatrix} 2 & 1 \\ -4 & 2 \end{bmatrix}$ onto the line $3x - 2y - 8 = 0$, then find slope of the resulting line.

Soln. :

$$2 \times 2 \text{ transformation matrix } [T] = \begin{bmatrix} 2 & 1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a = 2, \quad b = 1, \quad c = -4, \quad d = 2$$

Line L : $3x - 2y - 8 = 0$

$$2y = 3x - 8$$

$$y = \frac{3}{2}x - 4 \qquad (y = mx + k)$$

$$\therefore \text{ Slope of line L is } m = \frac{3}{2}$$

Slope of transformed line [L*]=[L] [T] is

$$m^* = \frac{b + dm}{a + cm} = \frac{1 + 2\left(\frac{3}{2}\right)}{2 - 4\left(\frac{3}{2}\right)} = \frac{4}{-4}$$

$$m^* = -1$$

Ex. 1.8.9 SPPU - March 2016, 2 Marks

▶ Ex. 1.9.4 SPPU - April 2005, 5 Marks

If a 2×2 transformation matrix $[T] = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}$ is used to transform the line passing through two points $A = \begin{bmatrix} 3 & \frac{-1}{2} \end{bmatrix}$ and $B = [0 \;\; 1]$. Find the equation of resulting line.

☑ Soln. :

Given,

$$[T] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix} \Rightarrow a = 1, b = 3, c = -2, d = 2$$

Also $A = [x_1, y_1] = \begin{bmatrix} 3 & \frac{-1}{2} \end{bmatrix}$

$B = [x_2 \;\; y_2] = [0 \;\; 1]$

$\therefore$ m = slope of line AB

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - \left(\frac{-1}{2}\right)}{0 - 3} = \frac{(2+1)}{-3 \cdot (2)} = \frac{-1}{2}$$

Hence the equation of the line passing through A and B;

$$y - y_1 = m(x - x_1)$$

$$y - \left(\frac{-1}{2}\right) = \left(\frac{-1}{2}\right)(x - 3)$$

$$y = \left(\frac{-1}{2}\right)(x - 3) + \left(\frac{-1}{2}\right) = \left(\frac{-1}{2}\right)(x - 3 + 1)$$

$$= \left(\frac{-1}{2}\right)(x - 2)$$

$$y = \left(\frac{-1}{2}\right)x + 1 \qquad \text{[compare with } y = mx + k\text{]}$$

$$m = \frac{-1}{2}, \; k = 1$$