S.Y.B.Sc. Computer Science

Semester I Examination

USCSMT-231 Groups and Coding Theory

Time: 2:00 Hours] **[Marks: 35**

Instructions for candidates:

1. All questions are compulsory.
2. Figures to right indicate full marks.
3. Non-programmable, single memory scientific calculator is allowed.

Q1) Attempt any five of the following. **[10]**

a) If a|b and a|c, then show that a|(b-c).

b) Find all generators of the group (Z6, +).

c) Write the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$ as a product of cycles.

d) Let G =(Z4, +) be a group and H = {0, 2} be a subgroup of G. Find all cosets of H in G.

e) Find the hamming distance between x and y, where x =1100010, y =1010001.

f) Prepare composition table of multiplication for Z_4-{0}.

g) State whether the following statement is True or False: "Z_6' is a group with respect to multiplication as operation." Justify your answer.

Q2) Attempt any three of the following. **[15]**

a) Prove that a cyclic group is abelian.

b) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$, $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$ be two permutations.

Compute $\sigma\tau\sigma^{-1}$.

c) Let e: $B^3 \rightarrow B^9$ be the encoding function defined as $e(x)=x^3$. Find e(101) and e(001). Decode the messages received y1=111111111 and y2=110110010. Find the number of errors which can be detected by this encoding function.

d) Check whether the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 5 & 4 & 7 & 6 \end{pmatrix}$ is even or odd. Justify.

e) Consider the unitary group U_{10} with multiplication as operation. What is the order of U_{10}? Can you find a subgroup of U_{10} which is of order 3? Justify your answer.

Q3) Attempt any one of the following. **[10]**

a) Find gcd of 4999 and 1109 and also find integers m, n such that gcd(4999,1109) = m 4999 + n1109.

b) Using RSA cryptosystem encrypt the message "IN". Take p=5, q=7 with e=11.

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