transformation matrix.

$$[T_r] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_1 & -y_1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1.2 & -5.9 & 1 \end{bmatrix}$$

followed by

(ii) Rotation about origin through, angle $\theta = 49°$ using transformation matrix.

$$[R_\theta] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.656 & 0.7547 & 0 \\ -0.7547 & 0.656 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ followed by,}$$

(iii) Inverse translation using transformation matrix

$$[T_r]^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_1 & y_1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1.2 & 5.9 & 1 \end{bmatrix}$$

$\therefore$ The concatenated transformation matrix

$$[T] = [T_r]\,[R_\theta]\,[T_r]^{-1}$$

$$= \begin{bmatrix} \cdots & \cdots & \cdots \\ 0 & 1 & 0 \\ 1.2 & -5.9 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.656 & 0.7547 & 0 \\ -0.7547 & 0.656 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1.2 & 5.9 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.656 & 0.7547 & 0 \\ -0.7547 & 0.656 & 0 \\ 5.2399 & -2.9648 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1.2 & 5.9 & 1 \end{bmatrix}$$

$$\therefore \quad [T] = \begin{bmatrix} 0.656 & 0.7547 & 0 \\ -0.7547 & 0.656 & 0 \\ 4.0399 & 2.9352 & 1 \end{bmatrix}$$

Applying [T] on line segment $AB = \begin{bmatrix} -10 & 1.2 \\ 11 & 17.1 \end{bmatrix}$

and using homogeneous coordinates of AB, the rotated line segment AB

$$= \begin{bmatrix} -10 & 1.2 & 1 \\ 11 & 17.1 & 1 \end{bmatrix} \begin{bmatrix} 0.656 & 0.7547 & 0 \\ -0.7547 & 0.656 & 0 \\ 4.0399 & 2.9352 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3.4257 & -3.8245 & 1 \\ -1.6495 & 22.4545 & 1 \end{bmatrix}$$

Ex. 1.19.1

Find concatenated transformation matrix for reflection through line y = – 4 and apply it on position vector [– 1 2].

☑ **Soln. :**

The equation of given line is $y = -4$ $(y = k)$

As given line is parallel to X-axis we,

(i) Translate the point $P = [0\ -4] = [x_1,\ y_1]$ to origin so that line coincides with X axis using transformation matrix,

$$[T_r] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_1 & -y_1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

followed by,

(ii) Reflection through X axis using transformation matrix,

$$[R_l] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ followed by,

(iii) Back translation of the line, using inverse translation matrix

$$[T_r]^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{bmatrix}$$

∴ The concatenated transformation matrix is,

$$[T] = [T_r]\,[R_1]\,[T_r]^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \ldots \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -8 & 1 \end{bmatrix}$$

Applying it on position vector $[-1 \quad 2]$, using homogeneous co-ordinates,

$$\text{Reflected position vector} = [-1 \ 2 \ 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -8 & 1 \end{bmatrix}$$

$$= [-1 \quad -10 \quad 1]$$

$\therefore$ Reflected position vector $= [-1 \quad -10]$

Ex. 1.19.2

Reflect point P $[x \quad y]$ through line $x = 4$.

☑ **Soln. :**

The equation of given line is $x = 4$. $\qquad (x = k)$

Ex. 1.19.2

Reflect point P [x y] through line x = 4.

☑ **Soln. :**

The equation of given line is x = 4. (x = k)

As the given line is parallel to Y axis, we,

(i) Translate the line (i.e. point [4 0]) to origin so that line coincides with Y axis using transformation matrix.

$$[T_r] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -x_1 & -y_1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \text{ followed by,}$$

(ii) Reflection through Y axis using transformation matrix

$$[R_1] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ followed by,}$$

(iii) Back translation of the line using inverse translation matrix

$$[T_r]^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

$$\therefore \ [T] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 8 & 0 & 1 \end{bmatrix}$$

Applying [T] on point P = [x y], using homogenous coordinates of point P.

The transformal point,

$$P^* = P\,[T]$$

$$= [x \quad y \quad 1] \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 8 & 0 & 1 \end{bmatrix}$$

$$= [-x+8 \quad y \quad 1] = [-x+8 \quad y]$$

Ex. 1.19.4

Reflect Δ ABC $= \begin{bmatrix} 4 & 6 \\ 0 & 0 \\ 0 & 5 \end{bmatrix}$ through line $y = \frac{1}{2}(x+4)$

☑ **Soln. :**

The given line is

$$y = \frac{1}{2}(x+4)$$

i.e. $$y = \frac{1}{2}x + 2 \quad (y = mx + k)$$

We reflect Δ ABC through given line as follows:

(i) Translation of point P [0 2] to origin O [0 0] usi translation matrix.

$$[T_r] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_1 & -y_1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

So that transformed line passes through origi followed by,

(i) Rotation of line through angle θ so that it coincide with X axis where,

$$\alpha = \tan^{-1}(m) = \tan^{-1}\left(\frac{1}{2}\right) = 26°33'$$

and $$\theta = -\alpha = -26°33'$$

using transformation matri

$$\alpha = \tan^{-1}(m) = \tan^{-1}\left(\frac{1}{2}\right) = 26°33'$$

and $\theta = -\alpha = -26°33'$

using transformation matrix

$$[R\theta] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8944 & -0.4472 & 0 \\ 0.4472 & 0.8944 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ followed by,}$$

(iii) Reflection of the object through X axis using transformation matrix

$$[R_l] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ followed by,}$$

(iv) Inverse rotation of the line using inverse rotation matrix,

$$[R_\theta]^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8944 & 0.4472 & 0 \\ -0.4472 & 0.8944 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ followed by, .}$$

(v) Inverse translation of the line using,

$$[T_r]^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

∴ The required concatenated transformation matrix.

$$[T] = [T_r]\,[R_\theta]\,[R_1]\,[R_\theta]^{-1}\,[T_r]^{-1}$$

$$\therefore [T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0.8944 & -0.4472 & 0 \\ 0.4472 & 0.8944 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.8944 & 0.4472 & 0 \\ -0.4472 & 0.8944 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8944 & -0.4472 & 0 \\ 0.4472 & 0.8944 & 0 \\ -0.8944 & -1.7888 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8944 & -0.4472 & 0 \\ 0.4472 & 0.8944 & 0 \\ -0.8944 & -1.7888 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.8944 & 0.4472 & 0 \\ -0.4472 & 0.8944 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8944 & 0.4472 & 0 \\ 0.4472 & -0.8944 & 0 \\ -0.8944 & 1.7888 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.8944 & 0.4472 & 0 \\ -0.4472 & 0.8944 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5999 & 0.7999 & 0 \\ 0.7999 & -0.5999 & 0 \\ -1.5999 & 1.1999 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} 0.5999 & 0.7999 & 0 \\ 0.7999 & -0.5999 & 0 \\ -1.5999 & 3.1999 & 1 \end{bmatrix}$$

Now, applying [T] on object [X],

$$[X] = \begin{bmatrix} A \\ B \\ C \end{bmatrix}.$$

And using homogenous coordinates, the transformed object.

$$[X^*] = [X]\,[T]$$

$$= \begin{bmatrix} 4 & 6 & 1 \\ 0 & 0 & 1 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 0.5999 & 0.7999 & 0 \\ 0.7999 & -0.5999 & 0 \\ -1.5999 & 3.1999 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5.5991 & 2.8001 & 1 \\ -1.5999 & 3.1999 & 1 \\ 2.3996 & 0.2004 & 1 \end{bmatrix}$$

∴ Reflected Δ ABC

$$= \begin{bmatrix} A^* \\ B^* \\ C^* \end{bmatrix} = \begin{bmatrix} 5.599 & 2.800 \\ -1.599 & 3.199 \\ 2.399 & 0.200 \end{bmatrix}$$

1.20 Two Dimensional Projection

1.21 Overall Scaling

- Overall scaling is produced by entry s where,

$$P^* = P[T] = [x \ y \ 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{bmatrix}$$

$$= [x \ y \ s] \qquad h = s$$

$$= \begin{bmatrix} \frac{x}{s} & \frac{y}{s} & 1 \end{bmatrix} \qquad h = 1$$

$$= \begin{bmatrix} \frac{x}{s} & \frac{y}{s} \end{bmatrix}$$

- Note that effect of overall scaling by s units is uniform scaling by $\frac{1}{s}$ units.
- $\therefore$ If $s > 1$, it is a contraction of the object
- If $0 < s < 1$, it is expansion of the object.

Ex. 1.21.1

Find concatenated transformation matrix for

(i) Reflection through $y = -x$ line following by

(ii) Overall scaling by 2 units followed by

(iii) $[T_3] = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

Apply it on $\Delta ABC = \begin{bmatrix} 2 & 4 \\ -2 & 5 \\ -1 & -1 \end{bmatrix}$

(i) Transformation matrix for reflection through $y = -x$ line is

$$\therefore \quad [T_1] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(ii) Transformation matrix for overall scaling by 2 units is

$$[T_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(iii) Given

$$[T_3] = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ The concatenated transformation matrix.

$$[T] = [T_1]\,[T_2]\,[T_3]$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad [T] = \begin{bmatrix} 0 & -1 & -3 \\ -1 & 0 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

$\therefore$ Tran

Ex. 1.21.2

Transform Δ

Δ ABC =

Soln. :

Transf

[T

Apply

coordinate

Δ AB

Ex. 1.21.2

Transform Δ ABC using overall scaling by 2 units where,

$$\Delta ABC = \begin{bmatrix} -2 & 4 \\ -6 & -4 \\ -2 & -9 \end{bmatrix}$$

☑ **Soln. :**

Transformation matrix for overall scaling by 2 units is,

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Applying it on Δ ABC and using homogenous coordinates of Δ ABC transformed

$$\Delta ABC = \begin{bmatrix} -2 & 4 & 1 \\ -6 & -4 & 1 \\ -2 & -9 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 & 2 \\ -6 & -4 & 2 \\ -2 & -9 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ -3 & -2 & 1 \\ -1 & -\frac{9}{2} & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -3 & -2 \\ -1 & -\frac{9}{2} \end{bmatrix}$$

What is the effect of the transformation matrix $[T] = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ on the two-dimensional object.

Soln. :

Given $[T] = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

It produces shearing in x-direction by '– 2' units.

▶ Ex. 1.22.25 SPPU - Oct. 2006, 1 Mark

Write the 2 × 2 transformation matrix for shearing in x-direction by – 2 units.

Soln. :

$[T]$ = Transformation matrix for shearing in x-direction by – 2 units

$$= \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

▶ Ex. 1.22.26 SPPU - Oct. 2007, 1 Mark

What is the effect of the transformation matrix $[T] = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ on a two-dimensional object.

Soln. :

$$[T] = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

It produces shearing in x-direction by factor '– 2'

▶ Ex. 1.22.27 SPPU - March 2009, 1 Mark

Find matrix of transformation to transform segment y + x = 1 in 2 dimensional structure into the line y – x = 1.

Soln. :

Given lines are

L : Untransformed : $y + x = 1$ i.e. $y = -x + 1$ $(m = -1)$

L*: Transformed : $y - x = 1$ i.e. $y = x + 1$ $(m = 1)$

Observe L* is reflection of L through y-axis.

$\therefore$ Required transformation matrix, $[T] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

▶ **Ex. 1.22.28** **SPPU - Oct. 2004, 1 Mark**

Scale the unit square at origin uniformly by a factor 2.

☑ **Soln. :**

Let $[X]$ = Unit square at origin $= \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$

$[T]$ = Uniform scaling matrix by factor 2

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Now, we apply [T] on [X]

$$\therefore\ [X^*] = [X]\,[T] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 0 \\ 2 & 2 \\ 0 & 2 \end{bmatrix}$$

Which is scaled object.

▶ **Ex. 1.22.29** **SPPU - April 2007, 1 Mark**

The circle of area 10 cm^2 is scaled uniformly by factor 2 then what is the area of transformed figure.

☑ **Soln. :**

Let A_i = Area of untransformed circle, $= 10\ cm^2$

Let $[T]$ = Uniform transformation matrix by factor 2

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$\therefore$ det $[T] = 4$

Now, Area A_t of transformed figure,

$$A_t = A_i \cdot \det[T] = 10 \times 4$$

$$= 40\ cm^2$$

Ex. 1.22.20 **SPPU - Oct. 2005, 1 Mark**

What is the determinant of the inverse of any pure rotation matrix ?

Soln. :

We have rotation matrix,

$$[T] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Now inverse,

$$[T]^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ +\sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore \det [T]^{-1} = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$

$$= \cos^2\theta + \sin^2\theta = 1$$

Hence $\det [T]^{-1} = 1$

Ex. 1.22.21 **SPPU - Oct. 2006, 1 Mark**

What is the effect of the transformation matrix $[T] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ on a two-dimensional object ?

Soln. :

It rotates a two-dimensional object about origin through angle θ.

... of the transformation matrix $[T] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ on a two-dimensional object ?

☑ **Soln. :**

It rotates a two-dimensional object about origin through angle θ.

▶ Ex. 1.22.22 SPPU - Oct. 2005, 1 Mark

Write the rotation matrix required to rotate the line $y = 2x$ so that it is coincident with x-axis.

☑ **Soln. :**

Given line :

$$y = 2x$$

$$\therefore \text{ Slope, } m = 2 \qquad \text{i.e. } \tan\theta = 2$$

$$\therefore \theta = \tan^{-1}(2) \qquad \text{(inclination)}$$

Now we rotate the line in **clockwise** direction about origin through

$$\theta = \tan^{-1}(2) = +63.43°$$

$\therefore$ Rotation matrix,

$$[T] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(63.43)° & -\sin(63.43)° \\ \sin(63.43)° & \cos(63.43)° \end{bmatrix}$$

▶ Ex. 1.22.23 SPPU - Oct. 2008, 1 Mark

$$T = \begin{bmatrix} 1/5 & 0 \\ 0 & 3 \end{bmatrix}$$

▶ **Ex. 1.22.32** **SPPU - March 2010, 5 Marks**

Rotate triangle ABC about its centroid through an angle 60°, where $A = [2\ -4]$, $B = [3\ \ 0]$, $C = [-2\ \ 1]$.

☑ **Soln. :**

Given that

$$A = [x_1\ \ y_1] = [2\ -4]$$

$$B = [x_2\ \ y_2] = [3\ \ 0]$$

$$C = [x_3\ \ y_3] = [-2\ \ 1]$$

By centroid formula, centroid of Δ ABC is P

$$\therefore\ [P] = [x\ \ y] = \left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right]$$

$$= \left[\frac{2+3-2}{3}\ \frac{-4+0+1}{3}\right] = [1\ -1]$$

Now we rotate ΔABC about centroid P $[1\ -1]$.

Let $[T_1]$ = Transformation matrix for translation in X and Y direction by -1 and 1 units respectively.

$$\therefore\ [T_1] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$[T_2]$ = Rotation about origin through $\theta = 60^\circ$

$$= \begin{bmatrix} \cos 60 & & \\ -\sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & \sqrt{3}/2 & 0 \\ -\sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0.8660 & 0 \\ -0.8660 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$[T_3]$ = Inverse translation used in $[T_1] = [T_1]^{-1}$

$$[T_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Now to perform given rotation the required combined transformation is

$$[T] = [T_1]\,[T_2]\,[T_3]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0.866 & 0 \\ -0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} 0.5 & 0.866 & 0 \\ -0.866 & 0.5 & 0 \\ -0.366 & -1.366 & 1 \end{bmatrix}$$

Now we apply [T] on Δ ABC say [X] to get $[X^*]$

$$\therefore \quad [X^*] = [X]\,[T] = \begin{bmatrix} 2 & -4 & 1 \\ 3 & 0 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0.866 & 0 \\ -0.866 & 0.5 & 0 \\ -0.366 & -1.366 & 1 \end{bmatrix}$$

$$\therefore \quad [X^*] = \begin{bmatrix} 4.098 & -1.634 & 1 \\ 1.134 & 1.232 & 1 \\ -2.232 & 0.500 & 1 \end{bmatrix}$$

▶ **Ex. 1.22.40** **SPPU - April 2011, 5 Marks**

Find the transformation matrix under which a circle with radius 4 is transformed to an ellipse with semi-major axis 4 and semi-minor axis 2.

☑ **Soln. :**

Consider P(x, y) be point on circle $x^2 + y^2 = 4^2$ ...(1)

And P* (x*, y*) be point on ellipse with semi-major axis 4 and semi-minor axis 2.

$$\frac{x^*}{4^2} + \frac{y^*}{2^2} = 1 \Rightarrow \frac{x^{*2}}{16} + \frac{y^{*2}}{4} = 1$$

$$\Rightarrow \quad x^{*2} + 4y^{*2} = 16 \qquad \text{...(2)}$$

From (1) and (2) $\quad x^2 + y^2 = 16 = x^{*2} + 4y^{*2}$

$$x^2 + y^2 = x^{*2} + 4y^{*2}$$

Comparing we get $x^* = x$ and $4y^{*2} = y^2$

$$2y^* = y$$

$$y^* = \frac{y}{2}$$

∴ we get know change in X-co-ordinate and scaling in Y-co-ordinate by factor $\frac{1}{2}$

$[T] = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$ this is required transformation matr

$\therefore$ A* [4 3], B* [0 5] and C* [2 4]

▶ **Ex. 1.22.44**

SPPU - April 2006, 5 Marks, Oct. 2011, 1 Mark

If the 2×2 transformation matrix [T] transforms the line segment AB to the line segment A* B*, find [T] where = [1 1] B = [1 3] A* = [8 4] B* [20 6].

☑ **Soln. :**

Let the untransformed line segment be,

$$[X] = \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

and the transformed line segment be,

$$[X^*] = \begin{bmatrix} A^* \\ B^* \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 20 & 6 \end{bmatrix}$$

Also, if the 2×2 transformation matrix $[T] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then we have,

$$[X^*] = [X]\,[T]$$

$$\begin{bmatrix} 8 & 4 \\ 20 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+c & b+d \\ a+3c & b+3d \end{bmatrix}$$

Equating the entries,

$$a + c = 8 \qquad \text{...(i)}$$

$$b + d = 4 \qquad \text{...(ii)}$$

$$a + 3c = 20 \qquad \text{...(iii)}$$

$$b + 3d = 6 \qquad \text{...(iv)}$$

Solving Equation (i) and (iii) simultaneously we get,

$$a = 2$$

$$c = 6$$

$$a + c = 8$$

$$a + 3c = 20$$

$$- \quad - \quad -$$

$$-2c = -12$$

$$c = 6 \text{ and } a = 8 - 6 = 2$$