Discrete Optimization
Assignment 1

Mehdi Nadif & Viktor Hansen

October 3, 2017

Abstract

This is the first weekly assignment for the Discrete Optimization course offered at The Department of Computer Science, Uni. Copenhagen.

Theoretical part - formulation and lower bounds

1.1

1.2

We may first note that

$$\sum_{i=0}^{n} \binom{n}{i} = 2^n$$

Since 2^n is the total number of subsets of a set of size n. Since we are only looking at subsets of size between 2 and $n-2$, the number of subsets must be

$$\begin{aligned}
&2^n - \binom{n}{0} - \binom{n}{1} - \binom{n}{n-1} - \binom{n}{n} \\
&= 2^n - 1 - n - n - 1 \\
&= 2^n - 2n - 2
\end{aligned}$$

since there are as many constraints as subsets, this is also the number of constraints.

1.3

The number of constraints is equivalent to the number of combinations of $i \in V, j \in V \setminus \{1\}$. As $|V| = n$, there must therefore be $n(n-1)$ constraints.

1.4

1.5

Since v_1 is in a cycle, it will have two incident edges that are part of the cycle it is in. Let the additional edge e_{add} be the second lowest cost edge adjacent to v_1, and add it to the tree to create a cycle. If we do not include e_{add}, we have a subset of edges of G that form a tree. The optimal of such a tree is equivalent to a minimum spanning tree, which we will call $\mathcal{M}$. Now, consider a Hamiltonian tour $\mathcal{H}$. From $\mathcal{H}$, we remove the most costly of the two edges incident to v_1, which we call $e_{\max}$. Removing this edge, we now have a tree, and therefore we can safely say that

$$\text{cost}(\mathcal{H}) - \text{cost}(e_{\max}) \geq \text{cost}(\mathcal{M})$$

Since $\mathcal{M}$ is the tree with lowest possible cost. By definition, we must have that $e_{\max} \geq e_{\text{add}}$ since e_{add} was the second lowest cost of all edges and $e_{\max}$ was chosen to be the most costly among two edges. Therefore

$$\text{cost}(\mathcal{M}) + \text{cost}(e_{\text{add}}) \leq \text{cost}(\mathcal{M}) + \text{cost}(e_{\max}) \leq \text{cost}(\mathcal{H})$$

Implementation part - branch-and-bound

2.1

2.2

2.3