Hints
Representations of Orientation

Task 1.

a) $\underline{R} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$

$\Theta = atan2\left[\sqrt{3}, -1\right] = \frac{2\pi}{3}, k_x = -\frac{1}{\sqrt{3}}, k_y = \frac{1}{\sqrt{3}}, k_z = \frac{1}{\sqrt{3}}$
Keep in mind: vector $\underline{k}$ has to be normalised!

Task 2.

a) $\underline{R} = \begin{bmatrix} 0 & 0 & +1 \\ 0 & +1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$\cos\beta = 0, \beta$ chosen as $\frac{\pi}{2} = 90°, \alpha - \gamma = atan2\left(-R_{12}, R_{22}\right) = 0$
Solution is ambiguous, as the coupling of α and γ cannot be resolved!

Task 3.

a) If the incremental change of orientation for multiple partial rotations n is supposed to lead to a continuous interpolation, a linearity $\left(f\left(a+b\right) \equiv f\left(a\right) + f\left(b\right)\right)$ would have to be valid for each step of the interpolation:

$\underline{RPY}\left(\alpha_i + \Delta\alpha, \beta_i + \Delta\beta, \gamma_i + \Delta\gamma\right) \equiv \underline{RPY}\left(\alpha_i, \beta_i, \gamma_i\right) \cdot \underline{RPY}\left(\Delta\alpha, \Delta\beta, \Delta\gamma\right)$

$\left[\underline{RPY}\left(\alpha_i + \Delta\alpha, \beta_i + \Delta\beta, \gamma_i + \Delta\gamma\right)\right]_{3,1} = -\sin\beta_i\cos\Delta\beta - \cos\beta_i\sin\Delta\beta$

$\left[\underline{RPY}\left(\alpha_i, \beta_i, \gamma_i\right) \cdot \underline{RPY}\left(\Delta\alpha, \Delta\beta, \Delta\gamma\right)\right]_{3,1} = -\sin\beta_i\cos\Delta\alpha\cos\Delta\beta + \cos\beta_i\sin\gamma_i\sin\Delta\alpha\cos\Delta\beta - \cos\beta_i\cos\gamma_i\sin\Delta\beta$

b) $\Delta\alpha = \frac{1}{2}\cdot\left(0° + 90°\right) = +45°$, $\Delta\beta = \frac{1}{2}\cdot\left(0° - 90°\right) = -45°$, $\Delta\gamma = \frac{1}{2}\cdot\left(0° - 0°\right) = 0°$

$\left[\underline{RPY}\left(-90° + 45°, +90° - 45°, 0° + 0°\right)\right]_{3,1} = -\frac{\sqrt{2}}{2}$

$\left[\underline{RPY}\left(-90°, +90°, 0°\right) \cdot \underline{RPY}\left(+45°, -45°, 0°\right)\right]_{3,1} = -\frac{1}{2}$

Task 4.

a) For the conversion quaternion $\rightarrow$ rotation matrix see p. 23 in the lecture notes.

b) Rotation axis: $\underline{n} := \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, angle: φ

Rotation axis has to be normalised!

$\rightarrow \underline{n}_{norm} = \underline{n} \cdot \frac{1}{\|\underline{n}\|}$

Result: $Q_{rot} = cos\left(\frac{\varphi}{2}\right) + sin\left(\frac{\varphi}{2}\right) \cdot \frac{1}{\sqrt{14}}\left(i1 + j2 + k3\right)$

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c) With $\varphi = 45° = 0.7854\ rad$

$Q_{rot} = cos\left(\frac{0.7854}{2}\right) + sin\left(\frac{0.7854}{2}\right) \cdot \frac{1}{\sqrt{14}}\left(i1 + j2 + k3\right) =: \mathbf{q}$

Given is the position vector $\underline{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

With lecture notes p. 22, eq. (2.40) for the definition of the inverse and with the rules thereafter, the result is:

$\underline{v}' = \mathbf{q}\underline{v}\mathbf{q}^{-1} = \mathbf{q}\underline{v}\frac{\mathbf{q}^*}{\mathbf{q}\mathbf{q}^*} = \begin{pmatrix} 0.6437 \\ 1.3361 \\ 0.8947 \end{pmatrix}$