Exercise

Spatial representation of poses

Task 1. A point P is given, which is described by the position vector $\underline{p} = \begin{pmatrix} 4 \\ 0 \\ 10 \end{pmatrix}$.

a) Determine the coordinates of the point $P^{'}$
that results from a rotation of P by 30° about the z axis.

b) Determine the coordinates of P in a coordinate system
that has been reoriented by 30° with respect to the base coordinate system.

c) Determine the coordinates of $P^{''}$
that results from a rotation of $P^{'}$ by 60° about the x axis ($P^{'}$ was determined in a)).

d) Determine the overall rotation matrix $\underline{R}_{X,Z}$ of the subsequent rotations form the tasks a) and c)
and verify if $\underline{p}^{''} = \underline{R}_{X,Z} \cdot \underline{p}$.

e) Swap the order of the rotations and determine $\underline{R}_{Z,X}$.
What are the coordinates of $P^{''}$ after $\underline{R}_{Z,X}$ has been applied.

Task 2. Two rotation matrices are given as $\underline{Rot}\,(\underline{z}, \Theta_1)$ and $\underline{Rot}\,(\underline{z}, \Theta_2)$,
which define rotations by Θ_1 and Θ_2 about the z axis.

a) Determine the overall rotation matrix $\underline{R}_1 = \underline{Rot}\,(\underline{z}, \Theta_1) \cdot \underline{Rot}\,(\underline{z}, \Theta_2)$.

b) Determine the overall rotation matrix $\underline{R}_2 = \underline{Rot}\,(\underline{z}, \Theta_2) \cdot \underline{Rot}\,(\underline{z}, \Theta_1)$.

c) Can both rotations $\underline{R}_1$ and $\underline{R}_2$ be applied equivalently?

d) Check if the rotations $\underline{R}_1$ und $\underline{R}_2$ are equal to $\underline{Rot}\,(\underline{z}, \Theta_1 + \Theta_2)$

Task 3. Two rotation matrices are given as $\underline{Rot}\,(\underline{z}, \Theta_1)$ and $\underline{Rot}\,(\underline{y}, \Theta_2)$,
which describe rotations by Θ_1 about the z axis and
by Θ_2 about the y axis.

a) Determine the overall rotation matrix $\underline{R}_1 = \underline{Rot}\,(\underline{z}, \Theta_1) \cdot \underline{Rot}\,(\underline{y}, \Theta_2)$.

b) Determine the overall rotation matrix $\underline{R}_2 = \underline{Rot}\,(\underline{y}, \Theta_2) \cdot \underline{Rot}\,(\underline{z}, \Theta_1)$.

c) Can both rotations $\underline{R}_1$ and $\underline{R}_2$ be applied equivalently?

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