

Exercise

Introduction

Task 1. Trigonometry

Solve the following equation for the range $0 \leq x \leq 2\pi$:

a) $6 \cos^2(x) - \sin(x) - 4 = 0$

b) $\sin^2(2x) - 2 \sin^2(x) + \cos(4x) - 2 \cos^2(x) + 1 = 0$

Task 2. Vectors and Matrices

a) Given are the vectors $\underline{v}_1 = \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}$ und $\underline{v}_2 = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}$

1. Calculate the lengths of the vectors \underline{v}_1 and \underline{v}_2
2. Normalise the vectors \underline{v}_1 and \underline{v}_2
3. Calculate $3\underline{v}_1 + 2\underline{v}_2$, $\underline{v}_1 \cdot \underline{v}_2$ and $\underline{v}_1 \times \underline{v}_2$
4. Show that $\underline{v}_1 \times \underline{v}_2$ is orthogonal to \underline{v}_1 and \underline{v}_2
5. Calculate the angle between \underline{v}_1 and \underline{v}_2

b) Given is the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$

1. Calculate the determinant $|\underline{A}|$
2. Calculate the transposed matrix \underline{A}^T
3. Calculate the inverse matrix \underline{A}^{-1}
4. Is \underline{A} an orthonormal matrix? Justify your answer.

Task 3. Laplace Transform

a) Determine the Laplace transform $F(s) = L\{f(t)\}$ for

1. $f(t) = t$
2. $f(t) = e^{-\alpha t}$
3. $f(t) = e^{-\alpha t} \sin(\omega t)$

b) Determine the inverse Laplace transform $f(t) = L^{-1}\{F(s)\}$ for

1. $F(s) = \frac{4}{s^2 + 6s + 9}$
2. $F(s) = \frac{4}{s^2 + 8}$
3. $F(s) = \frac{10s + 8}{s(s^2 + 3s + 2)}$

c) Solve the following differential equations

1. $\ddot{y}(t) + 2\dot{y}(t) + 2y(t) = 5 \sin(t)$
with $y(0) = -2$
and $\dot{y}(0) = 0$
2. $\dot{x}(t) + 2\dot{y}(t) - 3y(t) = 3e^t$
 $\dot{x}(t) - \dot{y}(t) - 6x(t) = 6$
with $x(0) = y(0) = 0$