Exercise
Representations of Orientation

Roll-Pitch-Yaw Orientation in space can be described using the three RPY angles $\underline{RPY}(\alpha, \beta, \gamma)$. The angles α, β and γ represent three consecutive rotations:

1: rotation about the z-axis (roll) by angle α

2: rotation of the already rotated Systems about the new y-axis (pitch) by angle β

3: rotation of the twice rotated Systems about the new x-axis (yaw) by angle γ

$$\begin{aligned} & \underline{RPY}(\alpha, \beta, \gamma) \\ = & \underline{Rot}(\underline{z}, \alpha) \cdot \underline{Rot}(\underline{y}, \beta) \cdot \underline{Rot}(\underline{x}, \gamma) \\ = & \begin{bmatrix} \cos\alpha \cdot \cos\beta & -\sin\alpha \cdot \cos\gamma + \cos\alpha \cdot \sin\beta \cdot \sin\gamma & \cos\alpha \cdot \sin\beta \cdot \cos\gamma + \sin\alpha \cdot \sin\gamma \\ \sin\alpha \cdot \cos\beta & \cos\alpha \cdot \cos\gamma + \sin\alpha \cdot \sin\beta \cdot \sin\gamma & \sin\alpha \cdot \sin\beta \cdot \cos\gamma - \cos\alpha \cdot \sin\gamma \\ -\sin\beta & \cos\beta \cdot \sin\gamma & \cos\beta \cdot \cos\gamma \end{bmatrix} \end{aligned}$$

Task 1. Given is the RPY rotation $\underline{RPY}_1 = \underline{RPY}(90°, 90°, 0°)$.

a) Determine the corresponding rotation vector $\underline{k}$ and the rotation angle Θ and sketch the source and target coordinate systems.

Task 2. Given is the rotation vector $\underline{k} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and the rotation angle $\Theta = 90°$.

a) Determine the orientation representation in RPY angles and draw the source and target coordinate systems.

Path interpolation To move a robot on a straight line from point P_0 to point P_n, the path is divided into n sections according to n scan times. For this purpose the vector $\overline{P_0 P_n}$ is split into n equally long sub-vectors $\frac{1}{n}\overline{P_0 P_n}$ to interpolate the path.

Task 3. It shall be investigated whether a path interpolation of a robot can be achieved with RPY angles. An initial orientation $\underline{RPY}_0 = \underline{RPY}(\alpha_0, \beta_0, \gamma_0)$ and a final orientation $\underline{RPY}_n = \underline{RPY}(\alpha_n, \beta_n, \gamma_n)$ are given. Using n partial rotations with $\Delta\alpha = \frac{1}{n} \cdot (\alpha_n - \alpha_o)$, $\Delta\beta = \frac{1}{n} \cdot (\beta_n - \beta_o)$, $\Delta\gamma = \frac{1}{n} \cdot (\gamma_n - \gamma_o)$ should lead to a continuous interpolation of the orientation.

a) Verify if continuous orientation change is maintained by determining the element (3,1) of the rotation matrix.

b) Illustrate the result from a) by using $\underline{RPY}_0 = \underline{RPY}(-90°, +90°, 0°)$, $\underline{RPY}_n = \underline{RPY}(0°, 0°, 0°)$ for the start and end position, respectively (with $n = 2$).

Task 4. This task illustrates rotation using quaternions.

a) Specify the quaternion which represents a rotation about the x-axis by angle φ. Convert the quaternion representation into a rotation matrix.

b) Determine the quaternion which represents a rotation about the rotation vector $\underline{n} = (1, 2, 3)^T$ by angle φ.

c) Rotate the location vector $\underline{v} = (1, 1, 1)^T$ by an angle of 45° about the given axis using the quaternion from part b) and specify its new coordinates.

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