Solution
Homogeneous Transformations

Task 1. Determination of the homogeneous transformation according to the "'Look-At"'-specification

The direction of the view in the y-axis can be calculated by the normalized diference of the location vectors, the result is a unit vector:

$$\underline{y}_{Eye} = \frac{\underline{p}_{Ref} - \underline{p}_{Eye}}{|\underline{p}_{Ref} - \underline{p}_{Eye}|}$$

The x-axis can be calculated from the direction of view and the "'Up-Vektor"', that is the z-axis in the world coordinates:

$$\underline{x}_{Eye} = \frac{\underline{y}_{Eye} \times \underline{z}_{World}}{|\underline{y}_{Eye} \times \underline{z}_{World}|}$$

The "'Up-Vektor"' in the direction of view can be calculated from the previously calculated x- and y-axis of the eye coodinates, given a right-handed coordinate system:

$$\underline{z}_{Eye} = \frac{\underline{x}_{Eye} \times \underline{y}_{Eye}}{|\underline{x}_{Eye} \times \underline{y}_{Eye}|}$$

In order to form the transformation matrix from the coordinates, the rotation matrix from the eye to world coordinate systemcan be calculated from the calculated coordinates:

$$\underline{R} = \begin{bmatrix} \underline{x}_{Eye} \\ \underline{y}_{Eye} \\ \underline{z}_{Eye} \end{bmatrix}$$

The transformation results from the rotation vector and the translation:

$$\underline{T} = \left[\begin{array}{ccc|c} & \underline{R} & & \underline{p}_{Eye} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Task 2. Determination of a homogeneous transformation for stereo view

The homoegenous transformation to be determined based on ${}^{World}\underline{T}_{Eye_l}$ can be calculated by a series of partial transformations:

a) Counter-clockwise rotation of the left-eye coordinate system around the z-axis by α

b) Translation of the resulting coordinates by the eye distance d

c) Counter-clockwise rotation of the resulting coordinate system around the z-axis by α

$$\begin{aligned} {}^{Eye_l}\underline{T}_{Eye_r} &= \underline{Rot}(\underline{z}_l, \alpha) \cdot \underline{Trans}(\underline{x}_{l'}, d) \cdot \underline{Rot}(\underline{z}_r, \alpha) \\ &= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha & 0 & d\cos\alpha \\ \sin 2\alpha & \cos 2\alpha & 0 & d\sin\alpha \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$