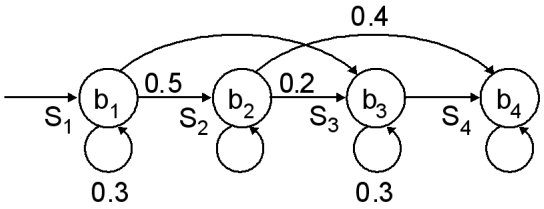
Exercise
Hidden Markov Models

Task 1. A Hidden Markov Model λ is used to predict the outcome of drawing colored balls from a series of four urns (see figure). The emissions b_i describe the distribution of red (r) and green (g) balls in the respective urn.

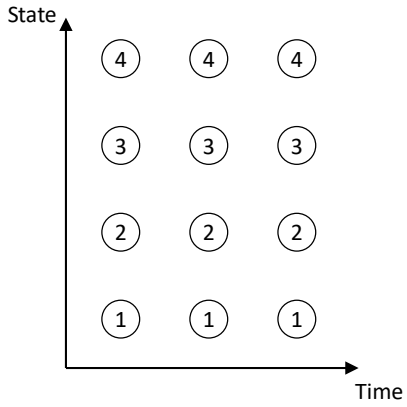


For the emission probabilities it holds:

$$B = \begin{pmatrix} b_1(r) & b_1(g) \\ b_2(r) & b_2(g) \\ b_3(r) & b_3(g) \\ b_4(r) & b_4(g) \end{pmatrix} = \begin{pmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \\ 0.5 & 0.5 \\ 0.8 & 0.2 \end{pmatrix}$$

a) Specify the missing model parameters λ!

b) Draw the possible state sequences Q_i for an observation of length 3 into the following Trellis diagram.



Consider the path $Q = \{1, 3, 4\}$ as an example and assume that the observation sequence is $O = \{r, g, r\}$.

c) What are the path probability $P(Q|\lambda)$, the observation probability $P(O|Q, \lambda)$ and the production probability $P(O, Q|\lambda)$ for this path?

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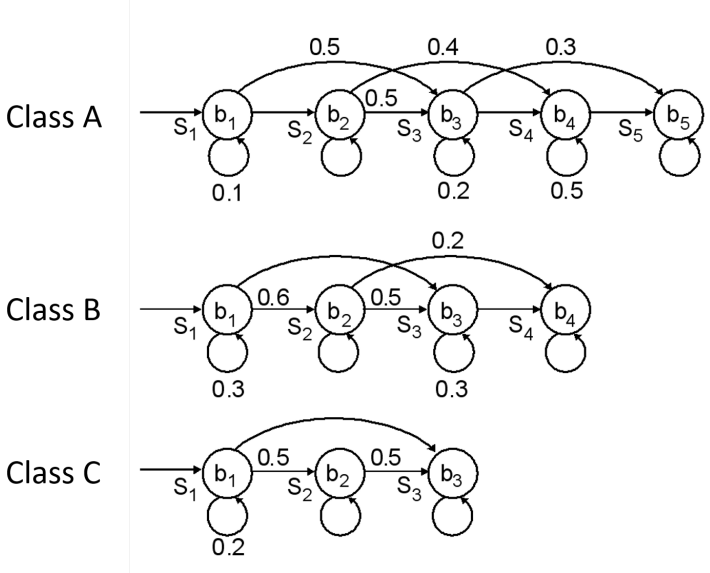
Task 2. Given a pattern recognizer with three Hidden Markov Models and an observation sequence $O = \{o_1, o_2, o_3\} = \{1.1, -0.9, 0.3\}$.



The emission functions are Gaussian distributed (see formula) with the parameters from Table 1. An observation o is a single real number. There is an **additional requirement** that the last observation must always be emitted from the last state.

$$b_i(o) = \frac{1}{\sqrt{2\pi} \cdot \sigma_i} \cdot e^{-\frac{1}{2}\left(\frac{o-\mu_i}{\sigma_i}\right)^2}$$

State	Class A	Class B	Class C
s_1	$\mu = 2, \sigma = 2$	$\mu = 1, \sigma = 1$	$\mu = 0, \sigma = 1$
s_2	$\mu = -2, \sigma = 1$	$\mu = -1, \sigma = 2$	$\mu = 0, \sigma = 3$
s_3	$\mu = 0, \sigma = 1$	$\mu = -1, \sigma = 3$	$\mu = 1, \sigma = 2$
s_4	$\mu = 1, \sigma = 3$	$\mu = 0, \sigma = 2$	
s_5	$\mu = 0, \sigma = 2$		

Table 1: Parameters of the emission functions

a) Specify the model parameters A and Π for each of the three HMMs!

b) Determine all solution paths Q for each HMM!

c) Calculate the production probability for each path for the given observation sequence O!

d) Which class is the observation sequence assigned to if only the best path (among all HMMs) is considered?

e) What is the overall production probability of the individual HMMs and why does it not make sense to use this quantity as an evaluation criterion?

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