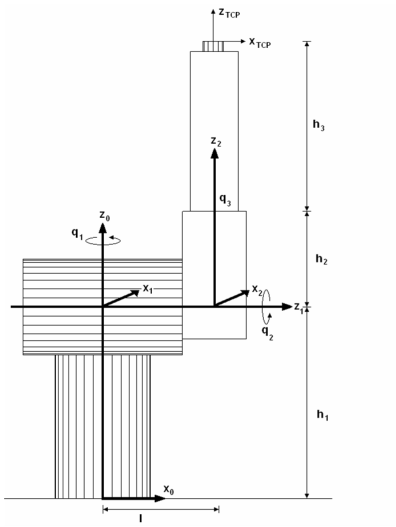
Solution
Forward Transformation

Task 1.

a) DH-compliant coordinate systems:



b) A solution for the Denavit-Hartenberg parameters is composed:

i	Θ_i	d_i	a_i	α_i
1	$q_1 + \frac{\pi}{2}$	h_1	0	$+\frac{\pi}{2}$
2	q_2	l	0	$-\frac{\pi}{2}$
3	$-\frac{\pi}{2}$	$q_3 + h_2 + h_3$	0	0

where $K_3 = K_{TCP}$.

c) The forward transformation ${}^0\underline{T}_3$ of the manipulator is composed by multiplication of all partial transformations:

${}^0\underline{T}_3 =^0 \underline{T}_1 \cdot^1 \underline{T}_2 \cdot^2 \underline{T}_3$

Each of the partial transformation results from a DH parameter set:

${}^{i-1}\underline{T}_i = \underline{Red}\left(\underline{z}_{i-1}, \Theta_i\right) \cdot \underline{Trans}\left(0, 0, d_i\right) \cdot \underline{Trans}\left(a_i, 0, 0\right) \cdot \underline{Red}\left(\underline{x}_i, \alpha_i\right)$

With the selected DH parameters the partial transformations are valid:

$$
{}^0\underline{T}_1 = \begin{bmatrix} cos\left(q_1 + \frac{\pi}{2}\right) & -cos\left(\frac{\pi}{2}\right) sin\left(q_1 + \frac{\pi}{2}\right) & sin\left(\frac{\pi}{2}\right) sin\left(q_1 + \frac{\pi}{2}\right) & 0 \\ sin(q_1 + \frac{\pi}{2}) & cos\left(\frac{\pi}{2}\right) sin\left(q_1 + \frac{\pi}{2}\right) & -sin\left(\frac{\pi}{2}\right) cos\left(q_1 + \frac{\pi}{2}\right) & 0 \\ 0 & sin\left(\frac{\pi}{2}\right) & cos\left(\frac{\pi}{2}\right) & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -S_{q_1} & 0 & C_{q_1} & 0 \\ C_{q_1} & 0 & S_{q_1} & 0 \\ 0 & 1 & 0 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1\underline{T}_2 = \begin{bmatrix} cos(q_2) & -cos\left(-\frac{\pi}{2}\right) sin(q_2) & sin\left(-\frac{\pi}{2}\right) sin(q_2) & 0 \\ sin(q_2) & cos\left(-\frac{\pi}{2}\right) sin(q_2) & -sin\left(-\frac{\pi}{2}\right) cos(q_2) & 0 \\ 0 & sin\left(-\frac{\pi}{2}\right) & cos\left(-\frac{\pi}{2}\right) & l \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{q_2} & 0 & -S_{q_2} & 0 \\ S_{q_2} & 0 & C_{q_2} & 0 \\ 0 & -1 & 0 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2\underline{T}_3 = \begin{bmatrix} cos\left(-\frac{\pi}{2}\right) & -sin\left(-\frac{\pi}{2}\right) & 0 & 0 \\ sin\left(-\frac{\pi}{2}\right) & cos\left(-\frac{\pi}{2}\right) & 0 & 0 \\ 0 & 0 & 1 & h_2 + h_3 + q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & h_2 + h_3 + q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Using $sin(\alpha) = -cos\left(\alpha + \frac{\pi}{2}\right) = cos\left(\alpha - \frac{\pi}{2}\right)$.

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This results in the solution of the forward kinematics:

$$
{}^0\underline{T}_3 = \begin{bmatrix} -S_{q_1} & 0 & C_{q_1} & 0 \\ C_{q_1} & 0 & S_{q_1} & 0 \\ 0 & 1 & 0 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C_{q_2} & 0 & -S_{q_2} & 0 \\ S_{q_2} & 0 & C_{q_2} & 0 \\ 0 & -1 & 0 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot {}^2\underline{T}_3
$$

$$
{}^0\underline{T}_3 = \begin{bmatrix} -S_{q_1}C_{q_2} & -C_{q_1} & S_{q_1}S_{q_2} & lC_{q_1} \\ C_{q_1}C_{q_2} & -S_{q_1} & -C_{q_1}S_{q_1} & lS_{q_1} \\ S_{q_1} & 0 & C_{q_1} & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & h_2 + h_3 + q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0\underline{T}_3 = \begin{bmatrix} C_{q_1} & -S_{q_1}C_{q_2} & S_{q_1}S_{q_2} & lC_{q_1} + (h_2 + h_3 + q_3)\, S_{q_1}S_{q_2} \\ S_{q_1} & C_{q_1}C_{q_2} & -C_{q_1}S_{q_2} & lS_{q_1} - (h_2 + h_3 + q_3)\, C_{q_1}S_{q_2} \\ 0 & S_{q_2} & C_{q_2} & h_1 + (h_2 + h_3 + q_3)\, C_{q_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$