Hints
Universal transformation

Task 1.

a) **Axis 1 is a rotary axis**

With ${}^0\underline{d}_3$ as position part of the forward transformation ${}^0\underline{T}_3$ in world coordinates:

$$\underline{r}_0 = {}^0\underline{d}_3 = \begin{bmatrix} l_1 C_{q1} + (h_2 + h_3 + q_3)\, S_{q1} S_{q2} \\ l_1 S_{q1} - (h_2 + h_3 + q_3)\, C_{q1} S_{q2} \\ h_1 + (h_2 + h_3 + q_3)\, C_{q2} \end{bmatrix}$$

$$\underline{z}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{J}_1 = \begin{bmatrix} \underline{z}_0 \times \underline{r}_0 \\ \underline{z}_0 \end{bmatrix} = \begin{bmatrix} -l_1 S_{q1} + (h_2 + h_3 + q_3)\, C_{q1} S_{q2} \\ l_1 C_{q1} + (h_2 + h_3 + q_3)\, S_{q1} S_{q2} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Axis 2 is a rotary axis

With ${}^1\underline{d}_3$ as position part of the forward transformation ${}^1\underline{T}_3$ in world coordinates:

$$\underline{r}_1 = {}^1\underline{d}_3 = {}^0\underline{d}_3 - {}^0\underline{d}_1 = {}^0\underline{d}_3 - \begin{bmatrix} 0 \\ 0 \\ h_1 \end{bmatrix} = \begin{bmatrix} l_1 C_{q1} + (h_2 + h_3 + q_3)\, S_{q1} S_{q2} \\ l_1 S_{q1} - (h_2 + h_3 + q_3)\, C_{q1} S_{q2} \\ (h_2 + h_3 + q_3)\, C_{q2} \end{bmatrix}$$

From the orientation part of the forward transformation ${}^0\underline{T}_1$:

$$\underline{z}_1 = {}^0\underline{R}_1^z = \begin{bmatrix} C_{q1} \\ S_{q1} \\ 0 \end{bmatrix}$$

$$\underline{J}_2 = \begin{bmatrix} \underline{z}_1 \times \underline{r}_1 \\ \underline{z}_1 \end{bmatrix} = \begin{bmatrix} (h_2 + h_3 + q_3)\, S_{q1} C_{q2} \\ -(h_2 + h_3 + q_3)\, C_{q1} C_{q2} \\ -(h_2 + h_3 + q_3)\, S_{q2} \\ C_{q1} \\ S_{q1} \\ 0 \end{bmatrix}$$

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Axis 3 is a translational axis

From the orientation part of the forward transformation ${}^{0}\underline{T}_{2} = {}^{0}\underline{T}_{1} \cdot {}^{1}\underline{T}_{2}$:

$$\underline{z}_2 = {}^{0}\underline{R}_2^{z} = \left[{}^{0}\underline{R}_1 \cdot {}^{1}\underline{R}_2\right]^{z} = \begin{bmatrix} S_{q1}S_{q2} \\ -C_{q1}S_{q2} \\ C_{q2} \end{bmatrix}$$

$$\underline{J}_3 = \begin{bmatrix} \underline{z}_2 \\ \underline{0} \end{bmatrix} = \begin{bmatrix} S_{q1}S_{q2} \\ -C_{q1}S_{q2} \\ C_{q2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

b) The singularities of the robot coincide with the singularities of the Jacobian matrix. For 6-axis industrial robots, the zeros of the Jacobian determinant can be used to infer the singularities. Here we have a Jacobian matrix for a 3-axis manipulator, which is not quadratic, so that no determinant can be determined. There are no general procedures to determine the points at which such a Jacobian matrix loses rank; however, the literature gives practical hints [M. W. Spong, M. Vidyasagar; "‘Robot Dynamics and Control"’; 1989].

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