Attention Plasticity and the Geometry of Long-Context Failure

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2026

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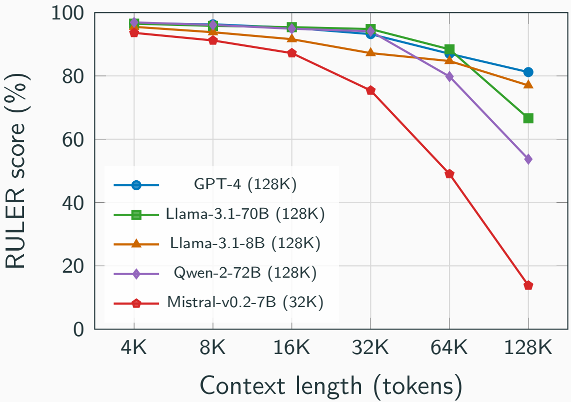
The Problem: Claimed vs. Effective Context

Models advertise 128K+ token windows, yet **effective** context falls far short.

Benchmarks detect the gap but cannot explain *why*.

This thesis: trace the gap to the geometry of attention heads.



Hypothesis: Attention as Content-Based Reranking

Attention computes a **ranking** over keys via dot-product scores:

$$\text{score}(q, k_i) = q^\top k_i = \underbrace{q_{\text{content}}^\top k_{\text{content}}}_{\text{content relevance}} + \underbrace{q_{\text{pos}}^\top k_{\text{pos}}}_{\text{positional bias}}$$

When position dominates $\Rightarrow$ ranking becomes **rigid**: the model ranks by position rather than by content relevance.

Hypothesis: This rigidity is the mechanistic pathway from architecture to effective context failure.

Research Questions

RQ1 How does position information manifest in the geometry of attention heads?

$\rightarrow$ PCA decomposition + Planar rotation model

RQ2 Does positional bias functionally constrain attention?

$\rightarrow$ Attention plasticity metric + decay theorem

RQ3 Do plasticity profiles correspond to behavioral performance?

$\rightarrow$ Cross-model benchmark correlation + training dynamics

PCA: Position Dominates Q/K Variance

qk-pca/01_pc_roles/figure.png

- PC0 captures $\sim$34% of Q+K variance (4$\times$ PC1). 23–32% of query and 9–20% of key variance is **linear in position**.
- On PC0: $|r_q| \approx 0.80 > |r_k| \approx 0.49$ — but this is a confound (PC0 mixes position with Q/K identity).

PCA: What the First Two Components Look Like

qk-pca/06_collage_example/figure.png

Rotation: Isolating the Bias Mechanism

qk-rotation/02_rq_rk_drift_bars/figure.png

- On drift axis a: **asymmetry reverses** — $|r_k^{(a)}| \approx 0.87 > |r_q^{(a)}| \approx 0.74$. Keys encode position more strongly than queries.
- Parametric bias: $\text{bias\_str} = \mu_Q^a \times \alpha_K$. 99% of 3,239 heads show recency bias; tight within families.

Attention Plasticity: The Functional Test

Key idea: Attention is a reranking mechanism. Pairwise comparison is the atomic unit of ranking.

Given a random query q and two keys k_i, k_j, does the query *content* determine which key ranks higher?

$$p = \Phi\left(\frac{\mu}{\sqrt{v}}\right), \qquad \mathrm{PP} = 4\,p\,(1-p)$$

- $\mathrm{PP} \to 1$: content determines the ranking (plastic)
- $\mathrm{PP} \to 0$: position locks the ordering (rigid)

Theorem: Under linear positional drift, AP_t decays with query position t. The *rate* of decay is diagnostic.

Experimental Setup

Cross-model study (13 models)

Family	#	Context
Ministral-3	3B/8B/14B	256K
Qwen-3	0.6B–14B	128K
Llama-3.2	1B/3B/11B	128K
Llama-3.1	8B	128K
Mistral-v0.2	7B	32K

Training dynamics

- SmolLM3-3B
- 10 checkpoints
- Pre-train → anneal → LC ext.
- 4K → 32K → 64K context

Benchmarks

- LongBench-Pro (7 models)
- RULER (2 predecessor models)

Result: Plasticity Profiles Separate Families

Family	0–20%	80–100%	$\mathrm{AP}_{\mathrm{drop}}$
Ministral-3	.655–.664	.571–.592	**.07–.08**
Qwen-3	.680–.702	.512–.540	**.16–.19**
Llama-3.2	.658–.703	.456–.489	**.17–.23**

Every model declines. The **rate** separates families:

- Ministral: gradual, near-linear
- Qwen: steeper, accelerates
- Llama: steepest overall

Result: Per-Head Strategies Differ by Family

Ministral-3-3B
Tight, homogeneous bundle

Qwen-3-4B
Large spread

Llama-3.2-3B
Steepest decline

Result: AP_{drop} Predicts LongBench-Pro

Family	AP_{drop}	LBP
Ministral	$\sim$0.07	30–40
Qwen	$\sim$0.17	31–37
Llama	0.23	16

Diagnostic outlier: Ministral-3-3B — lowest AP_{drop} (0.072), highest aggregate AP (0.622), but LBP only 30.18.

$\Rightarrow$ Context preservation is good; base capability is the bottleneck.

attention-plasticity/10_ap_drop_vs_lbp/f

Result: Training Dynamics — Three Phases

qk-rotation/10_smollm_bias_decomposition/figure.p

Phase 1 — Pre-training

Bias doubles ($0.009 \to 0.019$). $\mathrm{AP}_{\mathrm{drop}} \approx 0.06$ (mild).

Phase 2 — LC 4K to 32K

Bias collapses $6\times$ via α_K flattening. Short-context AP recovers.

Phase 3 — LC 32K to 64K

Bias halves again (total $10\times$). But $\mathrm{AP}_{\mathrm{drop}}$ **triples** to 0.16.

Result: Plasticity Trajectory

attention-plasticity/09_smollm3_trajectory/figure.

Short-context plasticity ($AP_{\text{first 20\%}}$) **recovers** during LC extension.

Long-context plasticity ($AP_{\text{last 20\%}}$) **continues to decline**.

The gap (AP_{drop}) triples despite $10\times$ bias reduction.

Limitations

- **Observational, not causal.** Associations between geometry and performance, not interventions.
- **3 families, 1 training trajectory.** Limited generalization to MoE, SSMs, or larger scales.
- **Pairwise ranking, not softmax weights.** Captures ranking quality, not weight allocation.
- **Base vs. instruct confound.** Mechanistic metrics on base models; benchmarks on instruct variants.
- **Linear drift / Gaussian assumptions.** Empirically supported but approximate; non-linear RoPE structure not captured.

Contributions

1. **Geometric framework.** PCA $\rightarrow$ Rotation $\rightarrow$ Plasticity — each resolving what the previous leaves open.

2. **Plasticity decay theorem.** Formal proof + Gaussian closed form decomposing decay into positional and content components.

3. **Cross-model validation.** $\mathrm{AP}_{\mathrm{drop}}$ separates families in benchmark order across 13 models.

4. **Bias $\neq$ context.** $10\times$ bias collapse but $\mathrm{AP}_{\mathrm{drop}}$ triples. Content signal decay is the underexplored dimension.

Future Work

Interventional Ablate low-plasticity heads, measure retrieval accuracy at distance.

Content decay Which mechanism? RoPE rotation accumulation, attention sinks, or feature drift?

Broader models MoE, SSMs, 70B+ scales.

Per-length validation Full per-length LBP/RULER correlation across all matched models.

Thank you

Questions?