Report Template
FYS-STK3155 - Project 1*

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Regression methods are central to statistical modeling, yet their practical behavior depends on approximation properties and optimization procedures. Using the Runge function as a test case, this project implements and compares Ordinary Least Squares (OLS), Ridge, and Lasso regression. The study examines polynomial fitting and highlights Runge's phenomenon as a challenge in high-degree models. Bias–variance tradeoffs are investigated through resampling and cross-validation, and gradient descent is applied as an alternative optimization approach. The results illustrate how regularization and optimization choices shape model performance in applied regression tasks.

I. INTRODUCTION

Regression analysis forms a cornerstone of statistical modeling and machine learning. Introductory texts present it as the basic framework for relating features and responses, typically through simple linear models on small, well-behaved datasets [1]. In applied settings, however, the behavior of regression methods is shaped by mathematical nuances in approximation and optimization.

Polynomial functions often serve as standard examples for exploring such nuances. The Runge function, named after Carl Runge and first studied in 1901 [Runge, 1901], is a smooth rational function that exhibits oscillatory behavior when approximated by high-degree polynomials, a phenomenon now known as Runge's phenomenon.

In this article, we implement and compare OLS, Ridge, and Lasso regression on the Runge function. We explore the bias-variance tradeoff through resampling and cross-validation techniques. In conjunction, gradient descent optimization is explored to ...

Citing some central ideas or problems in the literature is a good idea here. [2][3][1, 4]

II. METHODS

A. Regression methods

1. *Ordinary Least Squares*

Make sure that figures [1] and tables contain enough information in their captions, axis labels etc. so that an eventual reader can gain a good impression of your work by studying figures and tables only. And some more text here

*https://github.com/viktorbgulbrandsen/fysstk3155/tree/main/Project1

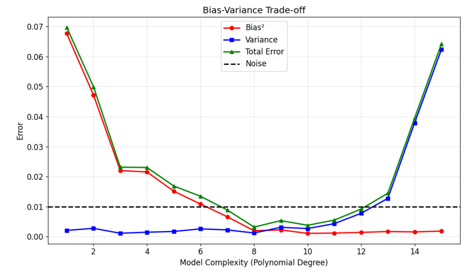

Figure 1: Bias–variance trade-off for polynomial regression estimated with bootstrap resampling. The curves show bias squared, variance, and total error (MSE) as functions of polynomial degree, with the dashed line indicating the noise level.

2. *Ridge Regression*

3. *Lasso Regression*

B. Optimization methods

Make sure that figures [2] and tables contain enough information in their captions, axis labels etc. so that an eventual reader can gain a good impression of your work by studying figures and tables only. And some more text

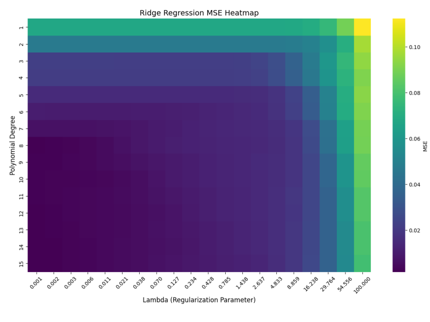

Figure 2: Heatmap of test mean squared error (MSE) for Ridge regression as a function of polynomial degree (rows) and regularization parameter λ (columns). The plot illustrates how model complexity and regularization jointly affect predictive performance.

here.

1. Closed-form solutions

2. Iterative optimization

3. Automatic differentiation

C. Validation methods

1. Bias-variance decomposition

2. Resampling techniques

3. Cross-validation

D. Implementation

- Explain how you implemented the methods and also say something about the structure of your algorithm and present very central parts of your code, not more than 10 lines
- You should plug in some calculations to demonstrate your code, such as selected runs used to validate and verify your results. A reader needs to understand that your code reproduces selected benchmarks and reproduces previous results, either numerical and/or well-known closed form expressions.

E. Use of AI tools

ChatGPT was used as a coding assistant for debugging, generating boilerplate code (e.g. function templates, Git commands) and for clarifying statistical concepts and more intricate algorithms. It also helped with structure planning and word choices. But analysis, implementaiton and final decisions were done by the student

III. RESULTS AND DISCUSSION

- Present your results
- Give a critical discussion of your work and place it in the correct context.
- Relate your work to other calculations/studies
- An eventual reader should be able to reproduce your calculations if she/he wants to do so. All input variables should be explained properly.
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IV. CONCLUSION

- State your main findings and interpretations
- Try to discuss the pros and cons of the methods and possible improvements
- State limitations of the study
- Try as far as possible to present perspectives for future work

[1] T. Hastie, R.Tibshirani, and J.Friedman, *The Elements of Statistical Learning: Data Mining, Inference, and Prediction, Second Edition. Springer Series in Statistics* (Springer, New York, 2009), URL `https://link.springer.com/book/10.1007%2F978-0-387-84858-7`.

[2] M. Hjorth-Jensen, *Computational Physics Lecture Notes 2015* (Department of Physics, University of Oslo, Norway, 2015), URL `https://github.com/CompPhysics/ComputationalPhysics/blob/master/doc/Lectures/lectures2015.pdf`.

[3] H. J. T. Zhang Yi, Yan Fu, Computers and Mathematics with Applications **47**, 1155 (2004), URL `https://doi.org/10.1016/S0898-1221(04)90110-1`.

[4] K. B. Hein, *Data Analysis and Machine Learning: Using Neural networks to solve ODEs and PDEs* (Department of Informatics, University of Oslo, Norway, 2018), URL `https://compphysics.github.io/MachineLearning/doc/pub/odenn/html/._odenn-bs000.html`.