Manipal University Jaipur

Department of Mathematics and Statistics

B.Tech. First Year

Assignment – I, MAS1001_Calculus and Matrices

Topics: Matrix Algebra: Rank, Inverse of a matrix, and solution of linear simultaneous equations. Eigenvalues and Eigenvectors of a matrix, Cayley-Hamilton theorem. Curvatures.

Q. No.	Questions Script	Answer
1.	Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ by using elementary row transformations.	
2.	Find the inverse of the matrix $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ by using elementary row transformations.	
3.	Find the value of b such that the rank of the matrix A is 3. $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$	b = 2, -6
4.	Find the rank of matrix $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$	Rank = 2
5.	Find the rank of a matrix $A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix}$	Rank =2
6.	Discuss the consistency of the system and if consistent, solve the equations: x+y+z=6 x+2y+3z=14 2x+4y+7z=30	x=0, y=4, z=2
7.	Discuss the consistency of the system and if consistent, solve the equations: 2x-3y+7z=5 3x+y-3z=13 2x+19y-47z=32	No Solution.

8.	Investigate that for what value of λ & μ, the system of equations: $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$, have (i) No Solution (ii) a Unique Solution (iii) an infinite solution.	(a) No Solution for $\lambda=3$, $\mu\neq 10$. (b) Unique Solution for $\lambda\neq 3$ and for any value of μ. (c) Infinite Solution for $\lambda=3$, $\mu=10$.
9.	Using Loop Current Method in an electric circuit, the following equations are obtained: $7i_1 - i_2 = 10$, $i_1 - 6i_2 + 3i_3 = 0$, $3i_2 - 13i_3 = 20$. Test the consistency of the system and if possible, find the values of the current i_1, i_2 & i_3.	System is consistent and have unique solution: i_1=-1.68amp, i_2=-0.62 amp & i_3=1.34 amp
10.	Test for consistency of the following system of equations 2x-3y+5z=1, 3x+y-z=2, x+4y-6z=1 and, if consistent, solve the system.	Ans. x=7-2t/11, y=1+17t/11, z=t, where t is a parameter
11.	Find the eigen values and eigen vectors for the matrix A: $$A=\begin{bmatrix}8&-6&2\\-6&7&-4\\2&-4&3\end{bmatrix}$$	$\lambda=0, 3, 15$ $\begin{bmatrix}1\\2\\2\end{bmatrix}\begin{bmatrix}2\\1\\-2\end{bmatrix}\begin{bmatrix}2\\-2\\1\end{bmatrix}$
12.	Find the eigen values and eigen vectors for the matrix A: $$A=\begin{bmatrix}2&1&1\\1&2&1\\0&0&1\end{bmatrix}$$	$\lambda=1, 1, 3$ $\begin{bmatrix}1\\0\\1\end{bmatrix}\begin{bmatrix}1\\-1\\0\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix}$
13.	Verify Cayley-Hamilton theorem for the matrix A: $$A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$$ Hence express $B = A^8-11A^7-4A^6+A^5+A^4-11A^3-3A^2+2A+I$ as a quadratic polynomial in A, also find B.	$B=A^2+A+I$ $B=\begin{bmatrix}16&27&34\\27&50&61\\34&61&77\end{bmatrix}$
14.	Verify the Cayley-Hamilton theorem for the matrix A and, hence, find A^{-1} $$A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$$	$A^{-1}=\frac{1}{4}\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}$
15.	Find the radius of curvature at the point $(\frac{3a}{2},\frac{3a}{2})$ of the curve $x^3+y^3=3axy$.	$\frac{3a}{8\sqrt{2}}$ in magnitude
16.	Show that the radius of curvature for the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$ is $\frac{a}{\sqrt{2}}$.	
17.	Find the circle of curvature of the curve $2xy+x+y=4$ at (1,1)	$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\frac{9}{2}$
18.	Find the circle of curvature at origin for the curve $y=\frac{x(1-x)}{1+x^2}$	$(x-1)^2+(y+1)^2=2$

19.	If ρ be the radius of curvature at any point P on the parabola $y^2 = 4ax$ and S be its focus, then show that ρ^2 varies as $(SP)^3$.	
20.	Show that, for the curve $x = a(3\cos t - \cos 3t)$; $y = a(3\sin t - \sin 3t)$, the radius of curvature at any point 't' is $3a \sin t$.	
21.	Find the radius of curvature at any point of the cycloid $x = a(\theta + sin\theta)$, $y = a(1 - cos\theta)$.	4a cos $(^{\theta}/_{2})$
22.	If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2 = 4ax$, then show that $(\rho_1)^{\frac{-2}{3}} + (\rho_2)^{\frac{-2}{3}} = (2a)^{\frac{-2}{3}}$.	
23.	Find the centre and circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$.	$\left(\frac{3a}{4}, \frac{3a}{4}\right)$ $\left(x - \frac{3a}{4}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = \frac{a^2}{2}$
24.	Find the centre of the curve $y = x^3 - 6x^2 + 3x + 1$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$.	$\left(-36, \frac{-43}{6}\right)$