Simulating magnetic hysteresis in the Stoner-Wohlfarth Model using Fortran 90

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Outline

The Stoner-Wohlfarth model is a simple model for a magnetic hysteresis, involving a single-domain particle with magnetcrystalline anisotropy in an external magnetic field.

Goal:

- Correctly set-up the energy functional for the Stoner-Wohlfarth particle
- Implement a reliable energy minimizer
- Calculate the hysteresis and other relevant quantities, such as the energy profiles and the Stoner-Wohlfarth Astroid

The inspiration for this topic was taken from a python simulation of the SW model I did in the course "VU Micromagnetics and Spintronics: Models and Simulation".

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- Ferromagnetic materials form so-called domains, clusters in which many magnetic moments align parallel. Under the influence of an external magnetic field, these domains quickly all orientate themselves in the direction of the field.

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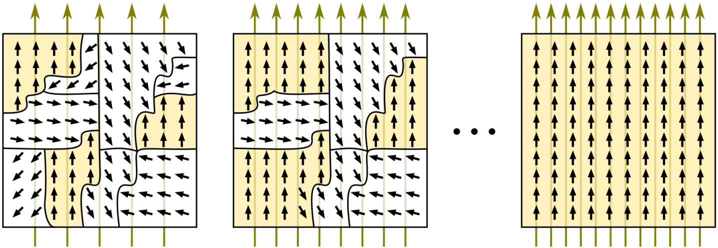

Figure: Magnetic domains in a ferromagnet and their response to an applied field.

Source: Wikimedia commons, 'Growing magnetic domains', 2020. Author: MikeRun,
https://commons.wikimedia.org/wiki/File:Growing-magnetic-domains.svg

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- The particle possesses magnetocrystalline anisotropy along its easy axis, i.e. the magnetization 'likes' to orient itself along this direction, as this minimizes its total energy.
- Despite not being accurate enough to model hysteretic behavior on a large scale, it is a very good model for small magnetic particles used in magnetic storage, magnetic sensors, nanotechnology and biomagnetism. Furthermore, it is an extremely useful model for understanding magnetic anisotropy.

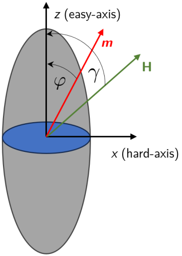

Figure: A sketch of a stoner-Wohlfarth particle.

- The energy per volume unit $e = E/V$ of the SW particle can be derived as

$$e = e_{\mathrm{aniso}} + e_{\mathrm{ext}},$$

and these energetic contributions (derivation skipped) lead to an energy functional dependent only on the angle φ,

$$e(\varphi) = k \cdot \sin^2 \varphi - \mu_0 M_s (H_{\|} \cos \varphi + H_{\perp} \sin \varphi). \qquad (1)$$

SW Model

- In equation 1, $H_{||} = H\cos\gamma$ and $H_{\perp} = H\sin\gamma$ are the components of the external field **H** parallel and perpendicular to the easy axis, respectively.
- Often, one is interested in the critical 'switching field', i.e. the strength of the field **H** required to reorient the magnetization in its direction. If we consider the energy, this switching field is given by the saddle points of the functional, i.e one must find $\frac{\partial e}{\partial \varphi} = 0$ and $\frac{\partial^2 e}{\partial \varphi^2} = 0$.

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What is a good choice for a minimizer? Secant method or derivative-based method?

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- The exact algorithm used is the Fletcher-Reeves algorithm, which calculates mutually conjugate search directions δ_j in each iteration,

$$\delta_{j+1} = \begin{cases} -\nabla f(x_j), & \text{if } j = 0 \\ -\nabla f(x_j) + x_j \delta_j, & \text{if } j = 1, 2, \ldots, n-1. \end{cases} \tag{2}$$

In the above equation,

$$x_j = x_{j-1} + \alpha_j \delta_j, \tag{3}$$

where α_j is the jth step length. x_j in Eq. 2 is given by,

$$x_j = \frac{\|\nabla f(x_j)\|^2}{\|\nabla f(x_{j-1})\|^2}. \tag{4}$$