Simulating magnetic hysteresis in the Stoner-Wohlfarth Model using Fortran 90

Viktor Knežević

VU First steps in model development
Institute for Meteorology and Geophysics, University of Vienna

June 27, 2024

Outline

The Stoner-Wohlfarth model is a simple model for a magnetic hysteresis, involving a single-domain particle with magnetcrystalline anisotropy in an external magnetic field.

Goal:

- Correctly set-up the energy functional for the Stoner-Wohlfarth particle
- Implement a reliable energy minimizer
- Calculate the hysteresis and other relevant quantities, such as the energy profiles and the Stoner-Wohlfarth Astroid

The inspiration for this topic was taken from a python simulation of the SW model I did in the course "VU Micromagnetics and Spintronics: Models and Simulation".

A (very) brief recap of macroscopic magnetism

- Magnetic materials consist of a large number of so-called 'elementary magnets', i.e. magnetic dipole moments of each atom which interact with each other and external magnetic fields.
- In electrodynamics of continua, we define the magnetization field $\mathbf{M}$ as

$$\mathbf{M} = \frac{\mathrm{d}\mathbf{m}}{\mathrm{d}V},$$

  where $\mathbf{m}$ is a magnetic moment and $\mathrm{d}V$ is the infinitesimal volume element.
- A magnetic hysteresis curve describes how the magnetic moments of a material behave in response to an external magnetic field, in general, this depends on the nature of a magnetic material (dia-, para-, and ferromagnetic) and other properties.
- Ferromagnetic materials form so-called domains, clusters in which many magnetic moments align parallel. Under the influence of an external magnetic field, these domains quickly all orientate themselves in the direction of the field.

A (very) brief recap of macroscopic magnetism

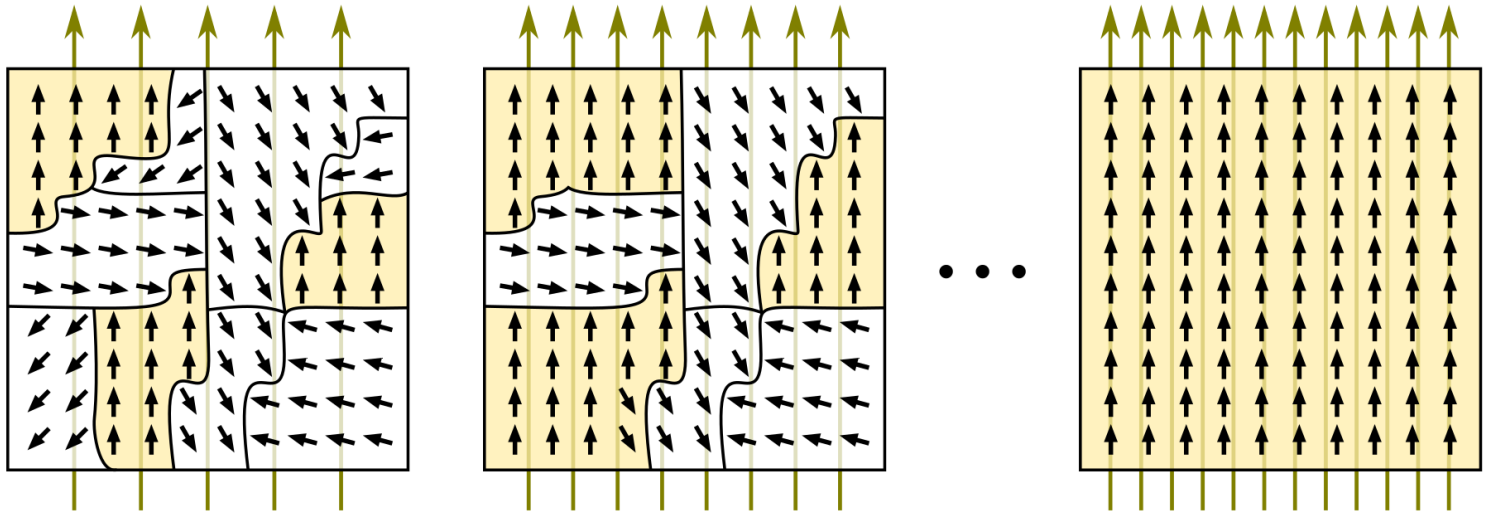

Figure: Magnetic domains in a ferromagnet and their response to an applied field.

Source: Wikimedia commons, 'Growing magnetic domains', 2020. Author: MikeRun, `https://commons.wikimedia.org/wiki/File:Growing-magnetic-domains.svg`

SW Model

- Describing the hysteresis of a ferromagnetic material is theoretically very complex and involves various energetic contributions, and a simulation requires complex numerical treatment to accurately resolve all effects $\rightarrow$ *micromagnetic* theory and simulations
- We can turn to a simpler model, which is the Stoner-Wohlfarth model, describing a single-domain ellipsoid magnetic particle.
- The particle possesses magnetocrystalline anisotropy along its easy axis, i.e. the magnetization 'likes' to orient itself along this direction, as this minimizes its total energy.
- Despite not being accurate enough to model hysteretic behavior on a large scale, it is a very good model for small magnetic particles used in magnetic storage, magnetic sensors, nanotechnology and biomagnetism. Furthermore, it is an extremely useful model for understanding magnetic anisotropy.

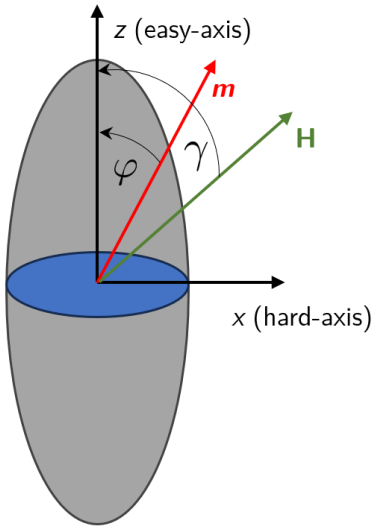

Figure: A sketch of a stoner-Wohlfarth particle.

- The energy per volume unit $e = E/V$ of the SW particle can be derived as

$$e = e_{\text{aniso}} + e_{\text{ext}},$$

and these energetic contributions (derivation skipped) lead to an energy functional dependent only on the angle φ,

$$e(\varphi) = k \cdot \sin^2 \varphi - \mu_0 M_s (H_{\parallel} \cos \varphi + H_{\perp} \sin \varphi). \tag{1}$$

SW Model

- In equation 1, $H_{\parallel} = H\cos\gamma$ and $H_{\perp} = H\sin\gamma$ are the components of the external field **H** parallel and perpendicular to the easy axis, respectively.
- Often, one is interested in the critical 'switching field', i.e. the strength of the field **H** required to reorient the magnetization in its direction. If we consider the energy, this switching field is given by the saddle points of the functional, i.e one must find $\frac{\partial e}{\partial \varphi} = 0$ and $\frac{\partial^2 e}{\partial \varphi^2} = 0$.
- The hysteresis itself can be simulated by minimizing the energy as a function of α for a fixed angle γ and a range of field strengths $H \rightarrow$ the obtained α can be used to determine the magnetization orientation at the energetic minimum (magnetization in field direction $\mathbf{m}_H = \cos\alpha$).

What is a good choice for a minimizer? Secant method or derivative-based method?

Minimization

We go for an iterative derivative-based minimizer, precisely, the conjugate gradient method.

- Maybe a bit 'overkill'/unnecessary in a one-dimensional problem, but provides very stable results, which is not always the case with simple gradient descent. Also requires less input arguments than secant methods, which makes code more simple
- The exact algorithm used is the Fletcher-Reeves algorithm, which calculates mutually conjugate search directions δ_j in each iteration,

$$\delta_{j+1} = \begin{cases} -\nabla f(x_j), & \text{if } j = 0 \\ -\nabla f(x_j) + x_j \delta_j, & \text{if } j = 1, 2, \ldots, n-1. \end{cases} \tag{2}$$

In the above equation,

$$x_j = x_{j-1} + \alpha_j \delta_j, \tag{3}$$

where α_j is the jth step length. x_j in Eq. 2 is given by,

$$x_j = \frac{\|\nabla f(x_j)\|^2}{\|\nabla f(x_{j-1})\|^2}. \tag{4}$$