System thinking
Lecture 11. Game theory. MNE

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Plan

1. Game theory origins and main idea
2. Solution concepts
3. Simplest games classification
4. Learning in games
5. Repeated games
6. Evolutionary games

So what is Nash equilibrium?

Definition. A Nash equilibrium is a strategy profile, when no player can increase its payoff by unilateral deviation from this profile.

Best response

For any vector $\overline{y} = (y_1, y_2, \ldots, y_n)$, denote by $\overline{y}_{-i}$ the vector

$$(y_1, y_2, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n)$$

which is the portion of y not associated with player i.

Player i best response or best reply to the strategies chosen by the other players is the strategy that yields him the s_{-i} greatest payoff that is: $u_i(s_i^*, s_{-i}) \geq u_i(s_j, s_{-i})$

for all s_i^* $s_j \neq s_i^*$

Dominant strategy definition

The strategy s_i is called a **dominant strategy** if it is a player i strictly best response to any strategies the other players might pick, in the sense that whatever strategies they pick, his payoff is highest with s_i

Dominated strategy definition

Strategy $\hat{s}_i$ is a dominated strategy, if there exists s_i such that for any possible strategies of the other players payoff from $\hat{s}_i$ is strictly less than from s_i

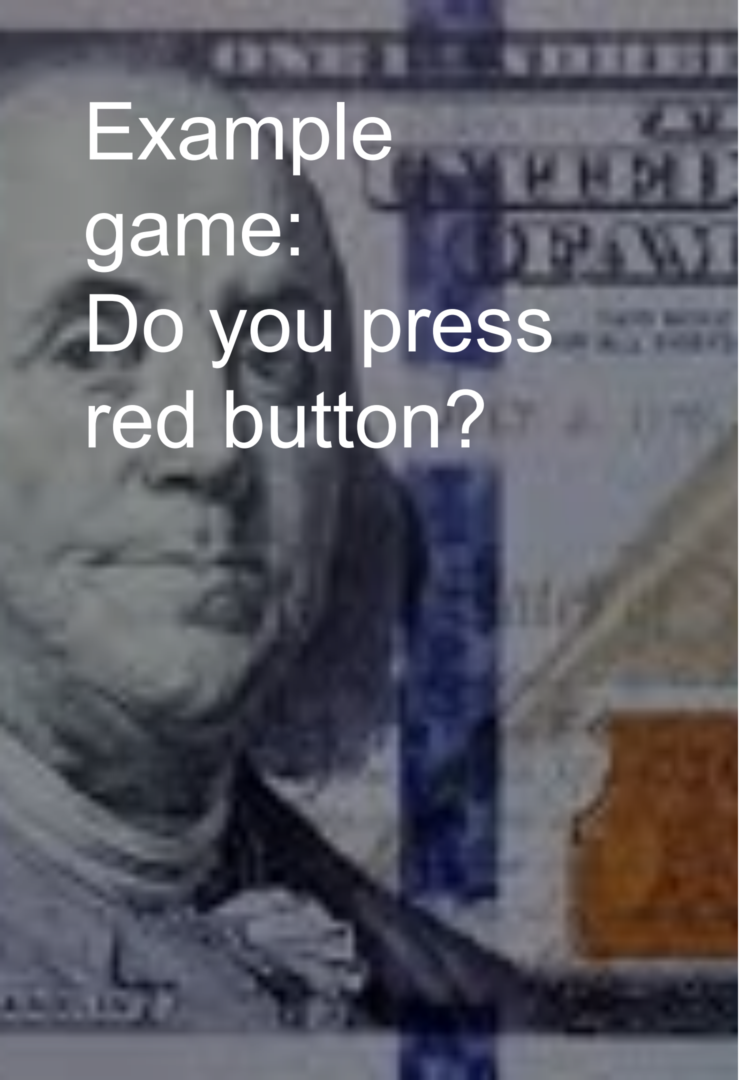
Exercise Roommates

Two roommates each need to choose to clean their apartment, and each can choose an amount of time $t_i \geq 0$ to clean. If their choices are t_i and t_j , then player i's payoff is given by

$$(10 - t_j)t_i - t_i^2$$

What is the best response correspondence of each player i?

- Which choices survive one round of IESDS?
- Which choices survive IESDS?



Example game: Do you press red button?

- So everyone has **$10** and **red button**
- If you don't press it - you do not influence others.
- If you press the button there are two effects:
  - Every other player who didn't press lose $2, every other player, who pressed button lose $1;
  - You protect yourself from others in the same way - if someone press - you lose 1 instead of two.

Payoff of player

	Nobody pressed	1 pressed	2 pressed
Press	10	9	8
Not press	10	8	6

Prisoners’ dilemma

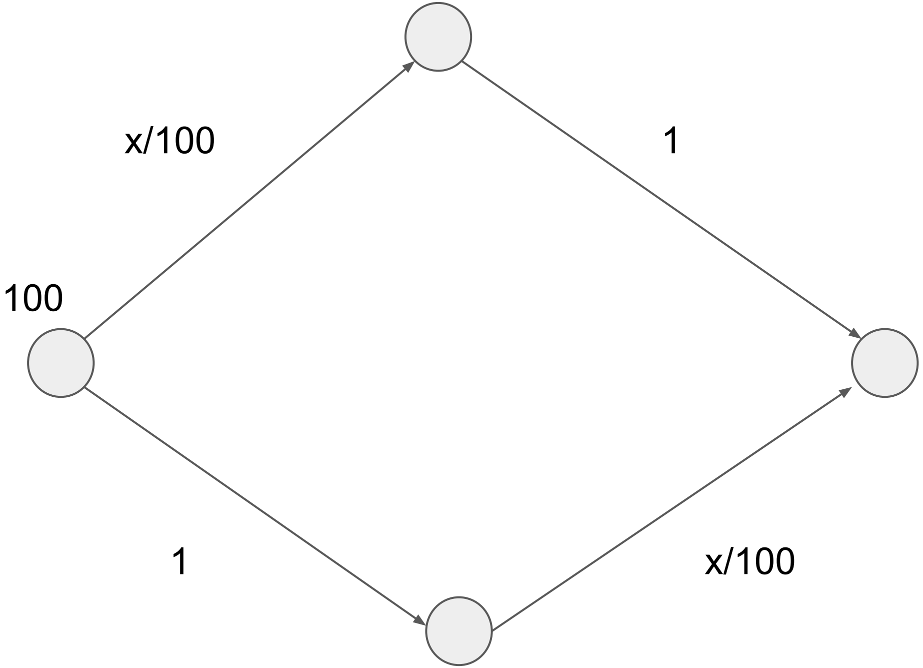
Example matrix. Prisoners' dilemma game

	Cooperate	Deflect
Cooperate	**-1** **-1**	**0** **-15**
Deflect	**-15** **0**	**-10** **-10**

Not only numbers. Braess paradox

This is also prisoners' dilemma

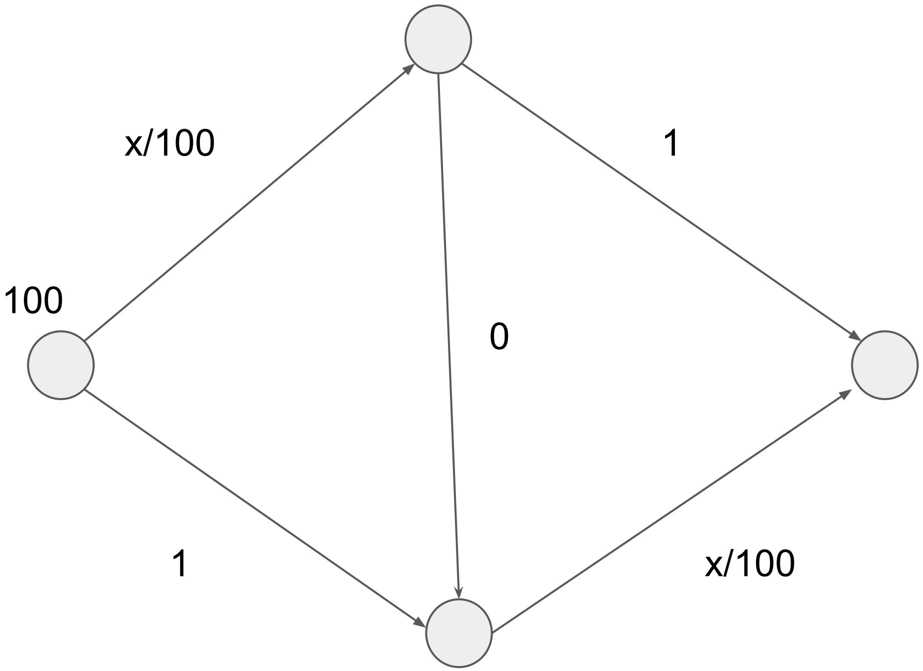
Routing game. Braess paradox



x cars. Delays are proportional to number of cars - x/100 hours.

If we have x = 100 (normalized) then best is half choose upper road, and half - down road.
This is NE. Delay is 1.5 hour

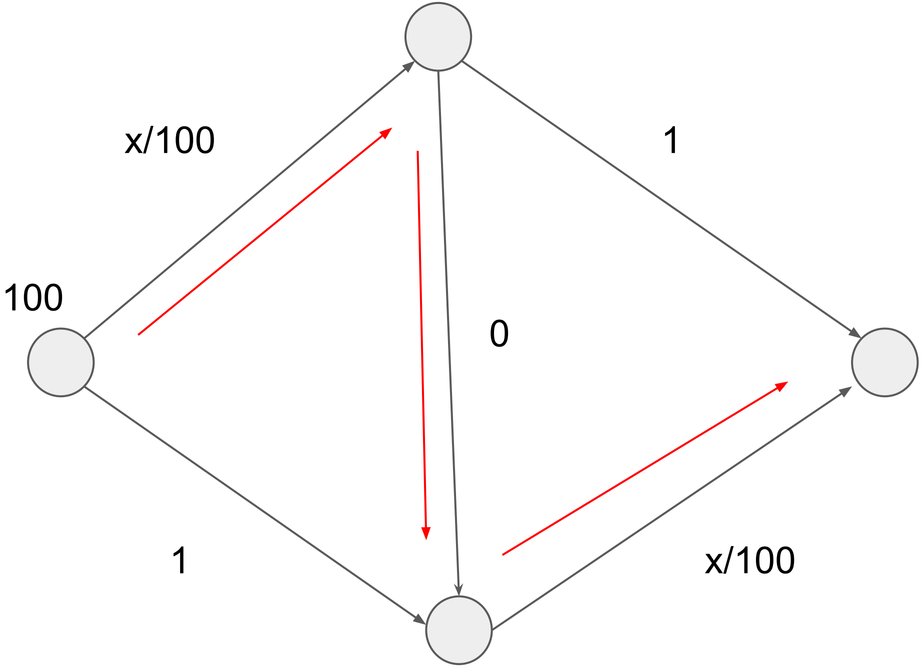
Routing game. Braess paradox



Now we add super fast line (but one-directional)

What can be wrong?

Routing game. Braess paradox



New equilibrium: 2 hours

Problem to solve

Two players. Choose simultaneously one of $1,2,3,..100. If numbers are equal they both got it. If not equal they got minimum from two numbers and player who named bigger number pays $2 to other player. What is equilibrium?

Is it realistic?

04

MORE GAMES

You can enter here the subtitle if you need it

Let’s play game № 2

1. Choose integer from 1 to 100

2. We calculate average of all numbers - **S**

3. Winner is the player with **furthest** (the most distant) number from S.


processing..

My prediction

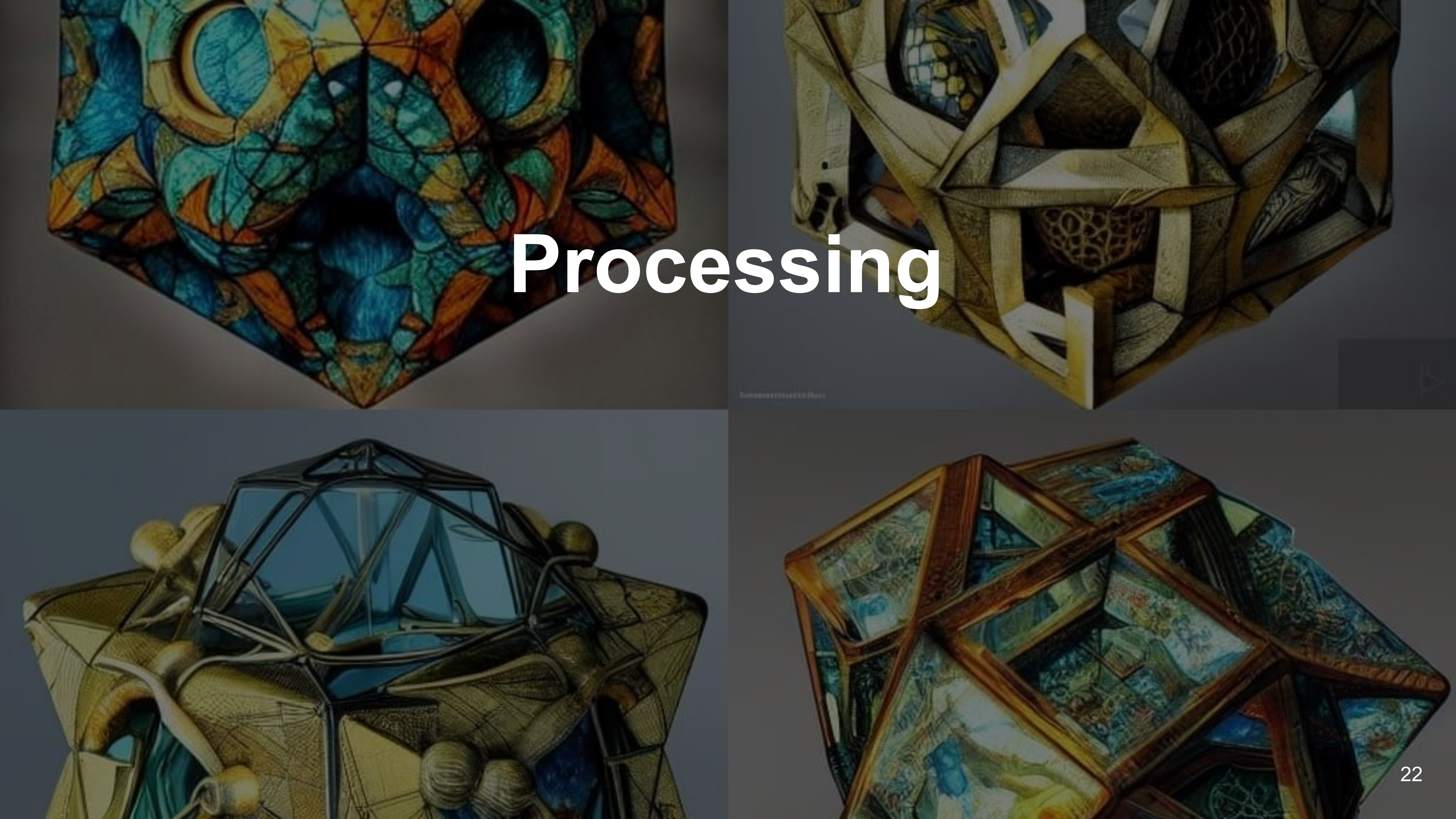
Percents of players, choosing 1 and 100 are close

Let’s play game № 3

1. Choose integer from 1 to 100

2. The number that is the choice of max amount of players - wins.

3. If two numbers has the same amount of choices - **smaller** wins.

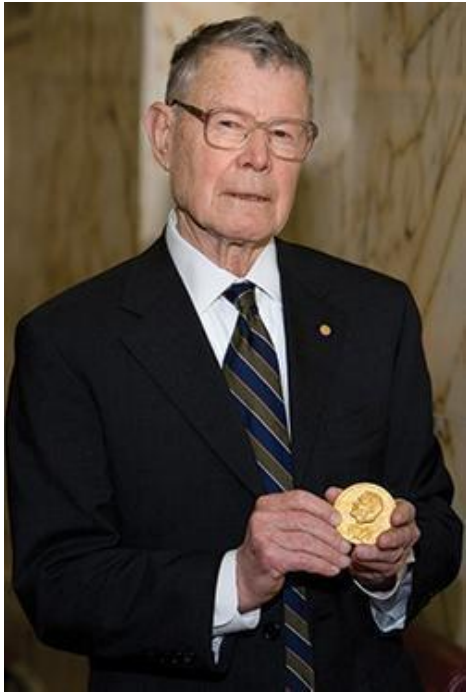


Processing

My prediction

Winner is 1

Thomas Schelling



Nobel prize with Robert Aumann (2005p.)

Meet in New York

Group of players should meet at the same place, at the same time without any arrangements.

Focal points.

Mixed strategies

Mixed strategy is probability distribution over the pure strategies.

Mixed strategy Nash equilibrium (MNE) is generalisation of pure Nash equilibrium (PNE).

Theorem (Nash, 1953). Every finite game has at least one MNE.

Consider two-player game. Expected payoffs

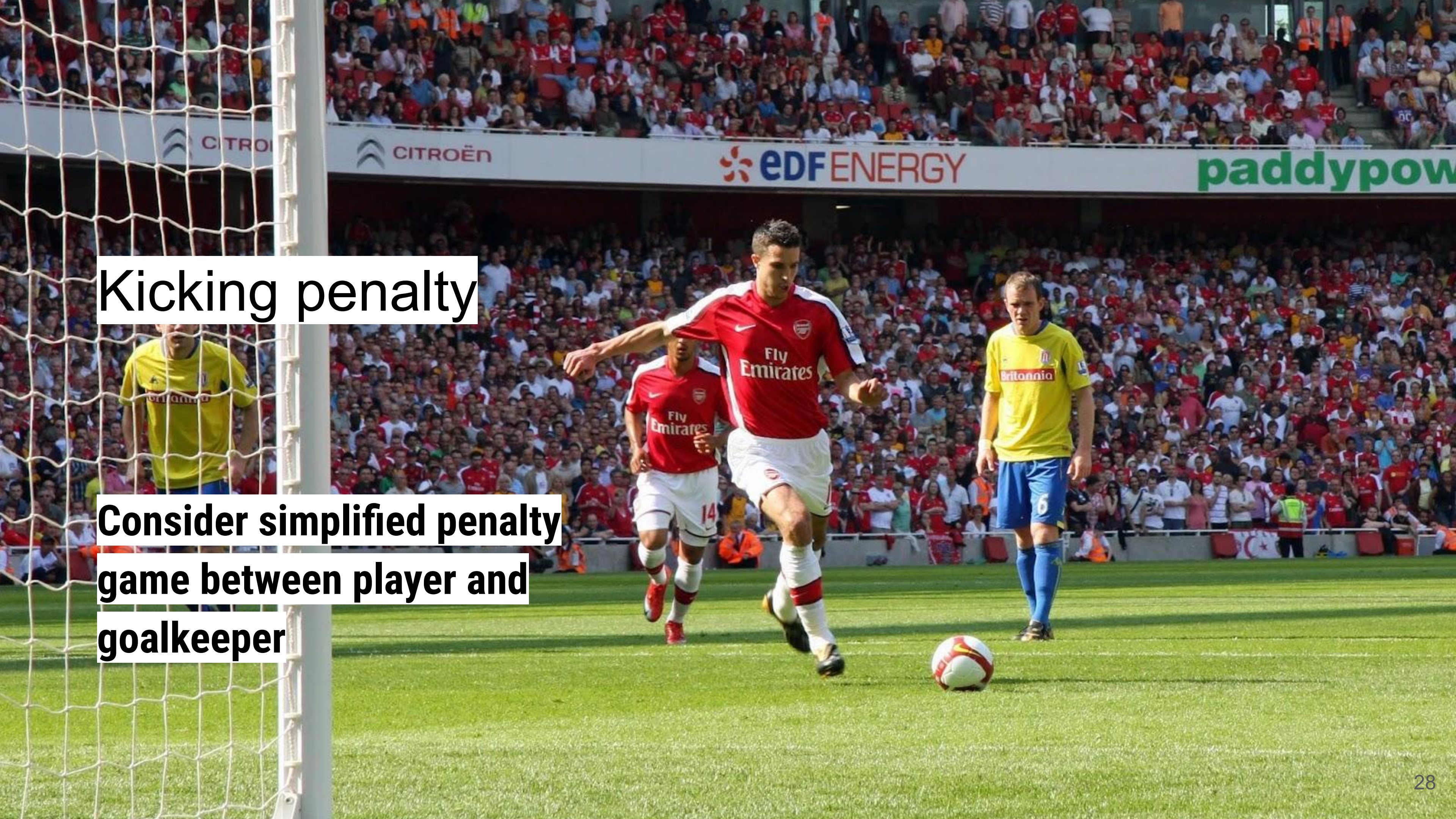
	L (q)	R (1-q)
U (p)	pq	p(1-q)
D (1-p)	(1-p)q	(1-p)(1-q)

Lets calculate expected payoffs

	L	R
U	1, 1	0, 0
D	0, 0	2, 2

q
$=$
$2(1-q)$

$q = \frac{2}{3}$
$p = \frac{2}{3}$



Kicking penalty

Consider simplified penalty game between player and goalkeeper

Penalty game (use NashPy to check)

jump

Kick

	Left	Right
Left	**1** **0**	**0** **1**
Right	**0** **1**	**1** **0**

Equilibrium:
Player
½
½
Keeper
½
½

Now question: right leg is not good. How will equilibrium change?

jump

Kick

	Left	Right
Left	1 0	0 1
Right	0.25 0.75	1 0

New equilibrium

jump

Kick

	Left	Right
Left	**0**, **1**	**1**, **0**
Right	**0.75**, **0.25**	**0**, **1**

Player:
3/7
4/7

Keeper
4/7
3/7

One more problem to show power of mixed strategies.

There are N students in the room. They all want to ask one stupid question, but afraid to look stupid. So they think - maybe someone other will ask?

	ask	wait
ask	6, 6	6, 10
wait	10, 6	0, 0

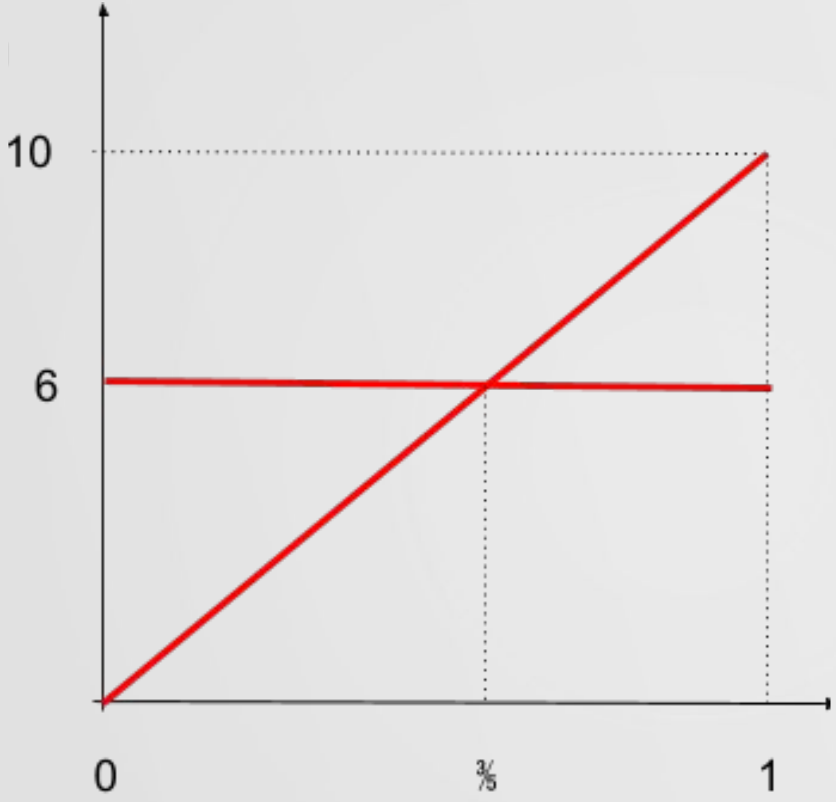
This type of game called Chicken

So the question is:

1. Find all NE’s.
2. What is probability that question will not be asked?
3. How this probability depends on the number of students?

Lets solve it for two players. Find Nash equilibriums

	ask	wait
ask	6, 6	6, 10
wait	10, 6	0, 0

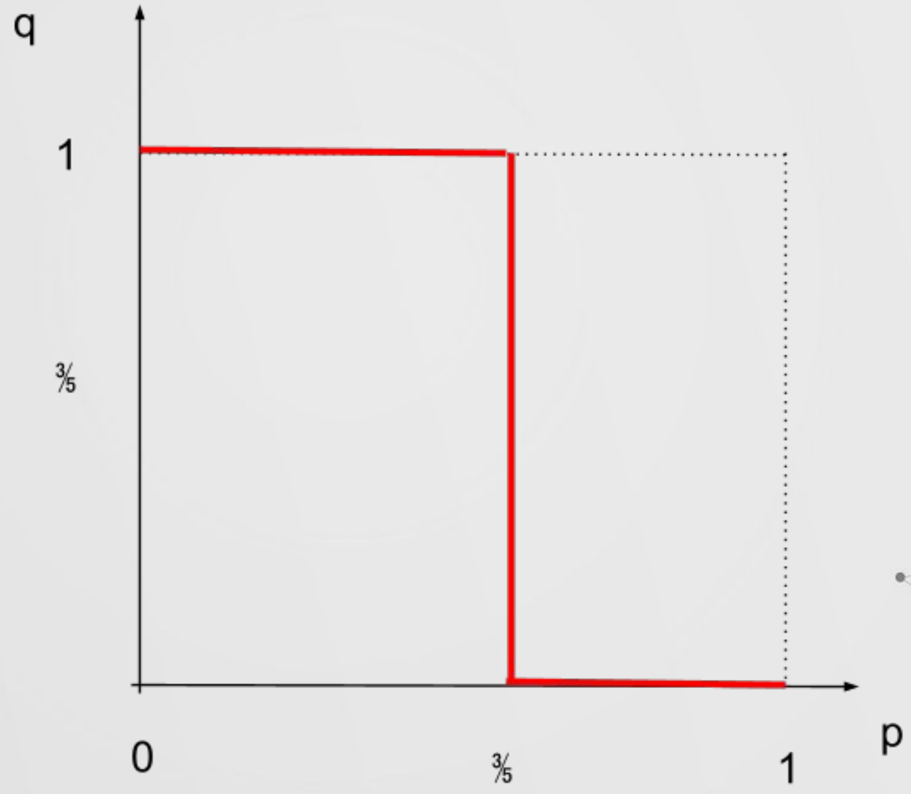


Suppose other player plays (p, 1-p)

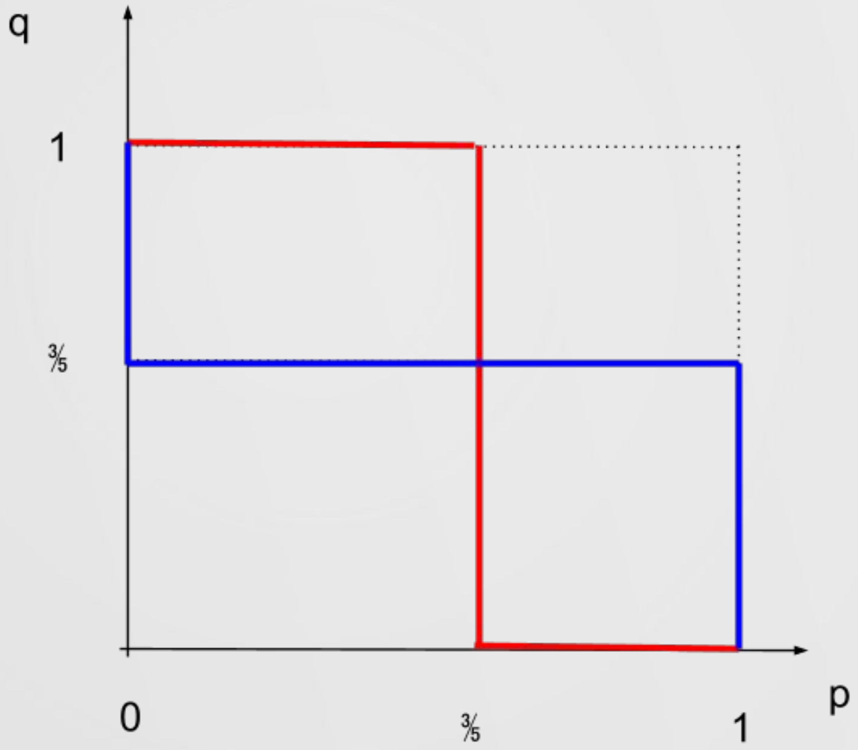
If we ask our payoff is 6

If we wait our payoff is 10p

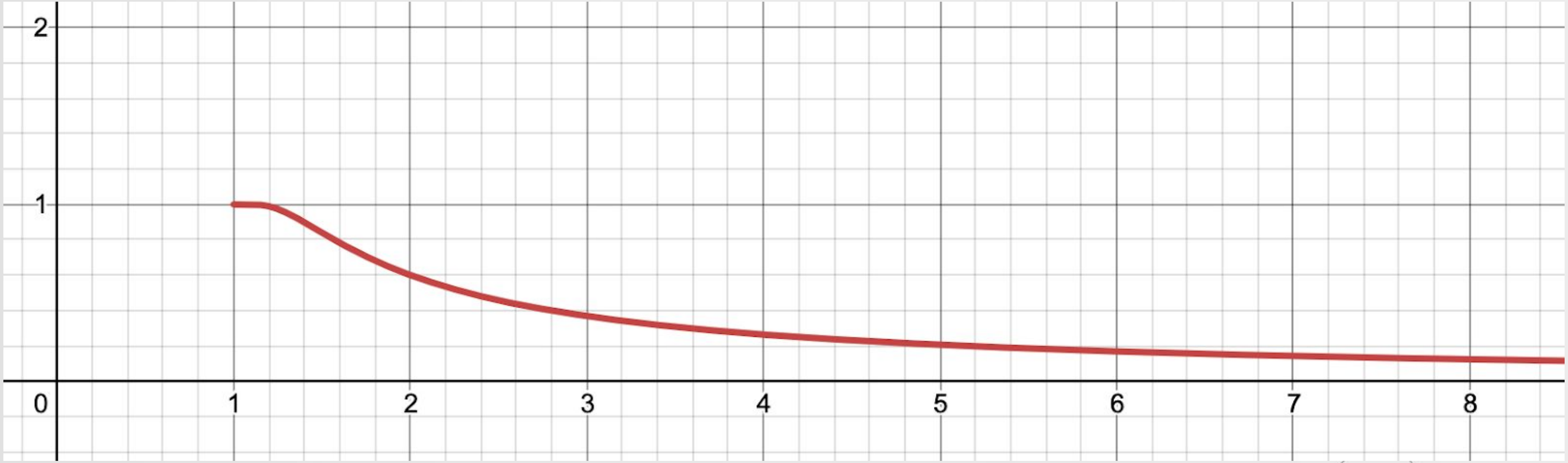
Best response correspondence



Best response correspondence intersection



Probability of asking for one student



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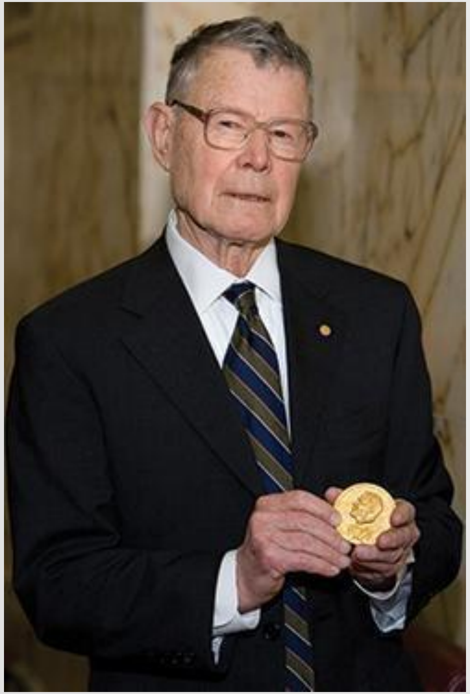
processing...

My prediction

Winner is 1

Thomas Schelling



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Focal points.

The simplest coordination game

	L	R
U	1 1	0 0
D	0 0	1 1

Coordination with assurance

	L	R
U	1 1	0 0
D	0 0	2 2

When option to burn money is useful?

	A	B
A	400, 100	0, 0
B	0, 0	100, 400

Expected payoffs in mixed strategy equilibrium? Let’s calculate

Now first player can burn $200 before (if he wants). Can it change the game?

	A	B
A	200, 100	-200, 0
B	-200, 0	-100, 400

You suggestions?

To solve this problem we need first to build tree

How many strategies each player has?

Now we need to convert game to matrix form

	AA	AB	BA	BB
B, AA	200, 100	200, 100	-200, 0	-200, 0
B, AB	200, 100	200, 100	-200, 0	-200, 0
B, BA	-200, 0	-200, 0	-100, 400	-100, 400
B, BB	-200, 0	-200, 0	-100, 400	-100, 400
NB, AA	400, 100	0, 0	400, 100	0, 0
NB, AB	0, 0	100, 400	0, 0	100, 400
NB, BA	400, 100	0, 0	400, 100	0, 0
NB, BB	0, 0	100, 400	0, 0	100, 400

Strategy profiles, surviving IEDS

NB, AA	400, 100
NB, BA	400, 100

Problems to solve.

Prove that for any a,b,c,d there is at least one PNE

	L	R
U	a, a	c, d
D	d, c	b, b

How many equilibria?

	S	T	X	Y	Z
U	**1, 3**	**1, 2**	**4, 3**	**1, 2**	**1, 2**
M	**4, 3**	**1, 3**	**0, 2**	**4, 2**	**4, 1**
D	**3, 1**	**2, 1**	**1, 2**	**1, 0**	**0, 0**

Exercise

Participants are dividing 10 $. They simultaneously choose number in [1, 10] range. If sum is less or equal to 10, they get what they claimed and the rest (if any) disappears. If sum is bigger than 10, if numbers are equal, they get 5 each, if numbers are different, one, who named smaller number get claim, and other one gets 10 - claim of other player. о хочуть а решта зникає. Build best reply correspondence.

Exercise

Player 1 chooses a number from {5,6, . . . ,20}, player 2 chooses a number from {10,11, . . . ,30}, and player 3 chooses a number from {30,31, . . . ,40}. If sum of choices is less than 50, they got -x_i. Otherwise they got 100 - x_i.

Find dominated strategies for each player. Can we solve the game removing them?

	V	W	X	Y	Z
A	4,-1	3,0	-3,1	-1,4	-2,0
B	-1,1	2,2	2,3	-1,0	2,5
C	2,1	-1,-1	0,4	4,-1	0,2
D	1,6	-3,0	-1,4	1,1	-1,4
E	0,0	1,4	-3,1	-2,3	-1,-1

Partnership game

$$u_1(s_1, s_2) = \frac{1}{2}[4(s_1 + s_2 + b\, s_1 s_2)] - s_1^2$$

$$u_2(s_1, s_2) = \frac{1}{2}[4(s_1 + s_2 + b\, s_1 s_2)] - s_2^2$$

THANKS

Does anyone have any questions?