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System thinking
Lecture 12. MNE and Repeated games

Plan

1. Mixed Nash equilibrium
2. Repeated games

Mixed strategies

Mixed strategy is probability distribution over the pure strategies.

Mixed strategy Nash equilibrium (MNE) is generalisation of pure Nash equilibrium (PNE).

Theorem (Nash, 1953). Every finite game has at least one MNE.

Consider two-player game. Expected payoffs

	L (q)	R ($1-q$)
U (p)	pq	$p(1-q)$
D ($1-p$)	$(1-p)q$	$(1-p)(1-q)$

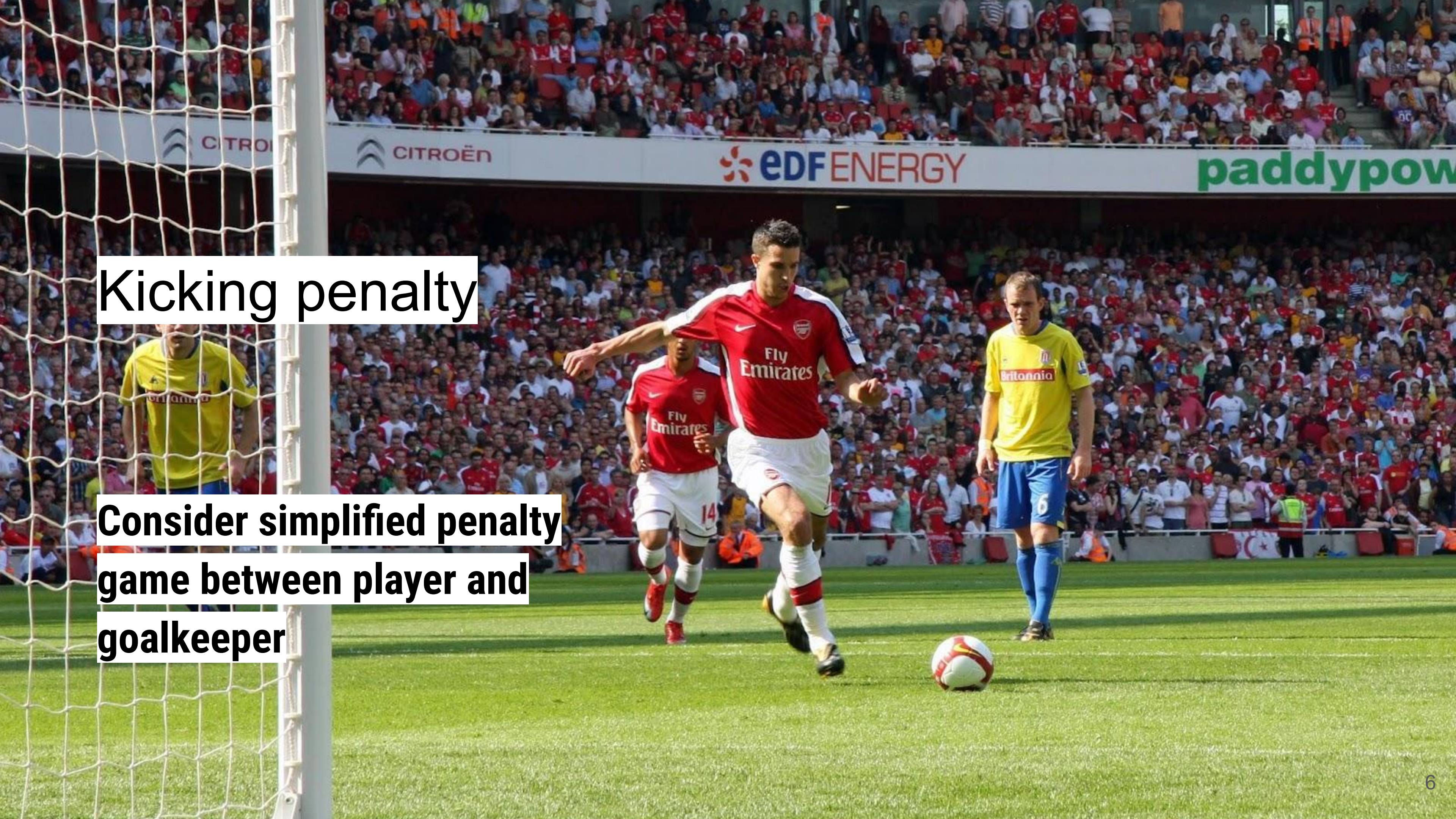
Lets calculate expected payoffs

	L	R
U	1 1	0 0
D	0 0	2 2

$q = 2(1-q)$

$q = 2/3$

$p = 2/3$



Kicking penalty

Consider simplified penalty game between player and goalkeeper

Penalty game

jump

Kick

	Left	Right
Left	**1** **0**	**0** **1**
Right	**0** **1**	**1** **0**

Equilibrium:
Player
½
½
Keeper
½
½

Now question: right leg is not good. How will equilibrium change?

jump

Kick

	Left	Right
Left	1 0	0 1
Right	0.25 0.75	1 0

New equilibrium

		jump	
		Left	Right
Kick	Left	**1** **0**	**0** **1**
	Right	**0.25** **0.75**	**1** **0**

Player:
3/7
4/7

Keeper
4/7
3/7

One more problem to show power of mixed strategies.

There are N students in the room. They all want to ask one stupid question, but afraid to look stupid. So they think - maybe someone other will ask?

	ask	wait
ask	6, 6	6, 10
wait	10, 6	0, 0

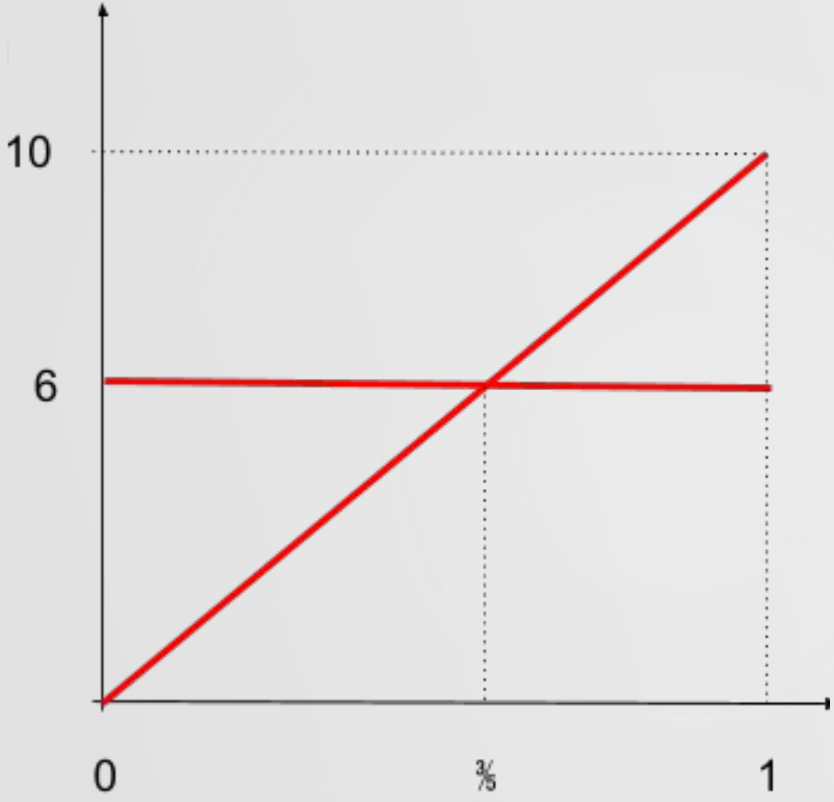
This type of game called Chicken

So the question is:

1. Find all NE’s.
2. What is probability that question will not be asked?
3. How this probability depends on the number of students?

Lets solve it for two players. Find Nash equilibriums

	ask	wait
ask	6, 6	6, 10
wait	10, 6	0, 0

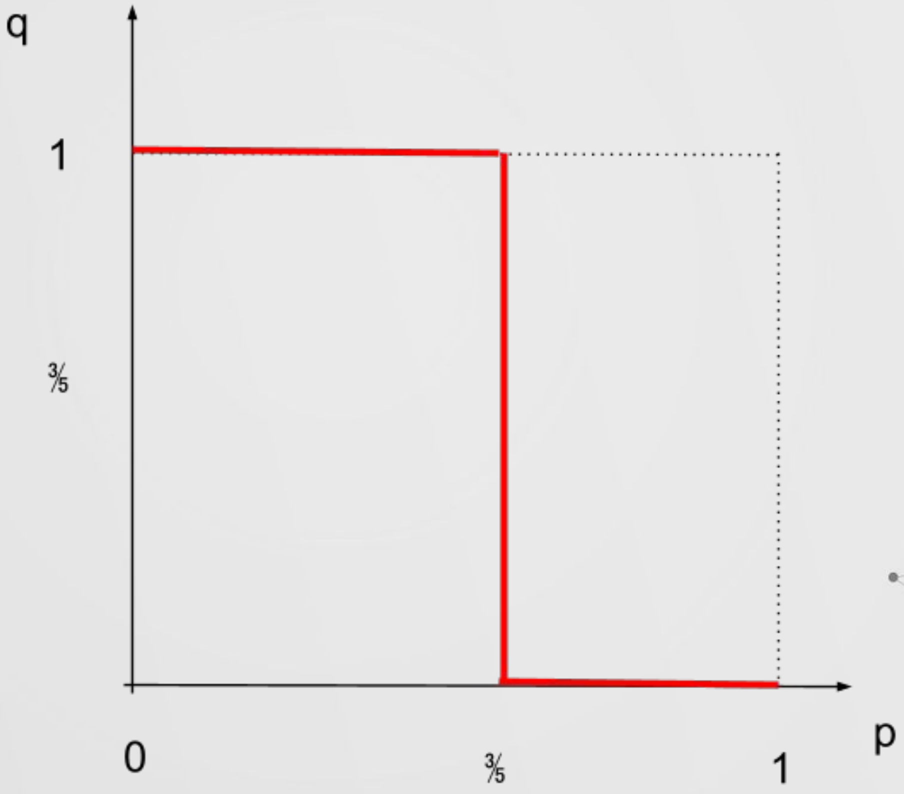


Suppose other player plays (p, 1-p)

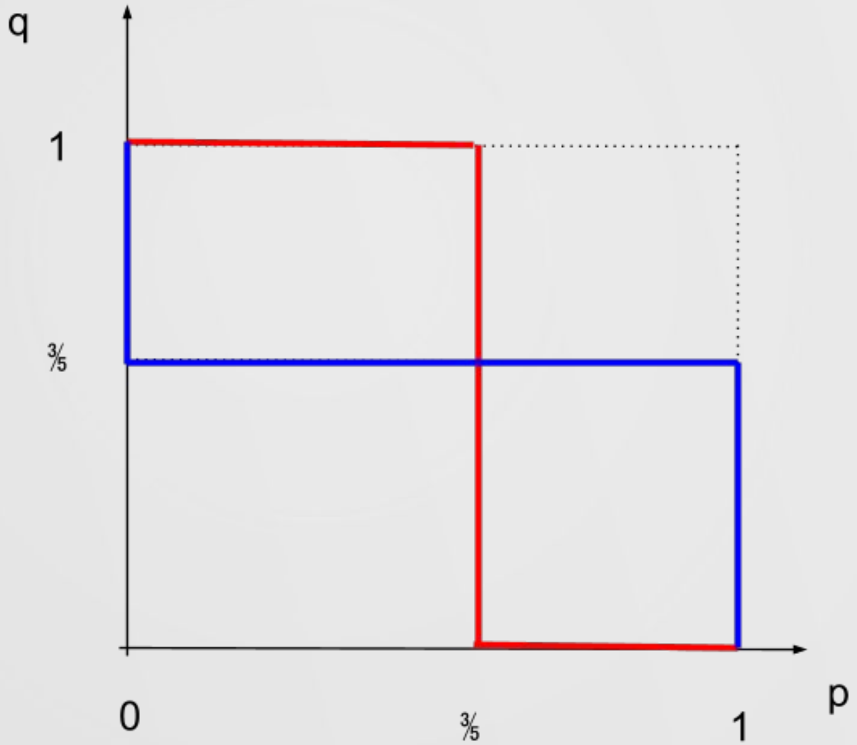
If we ask our payoff is 6

If we wait our payoff is 10p

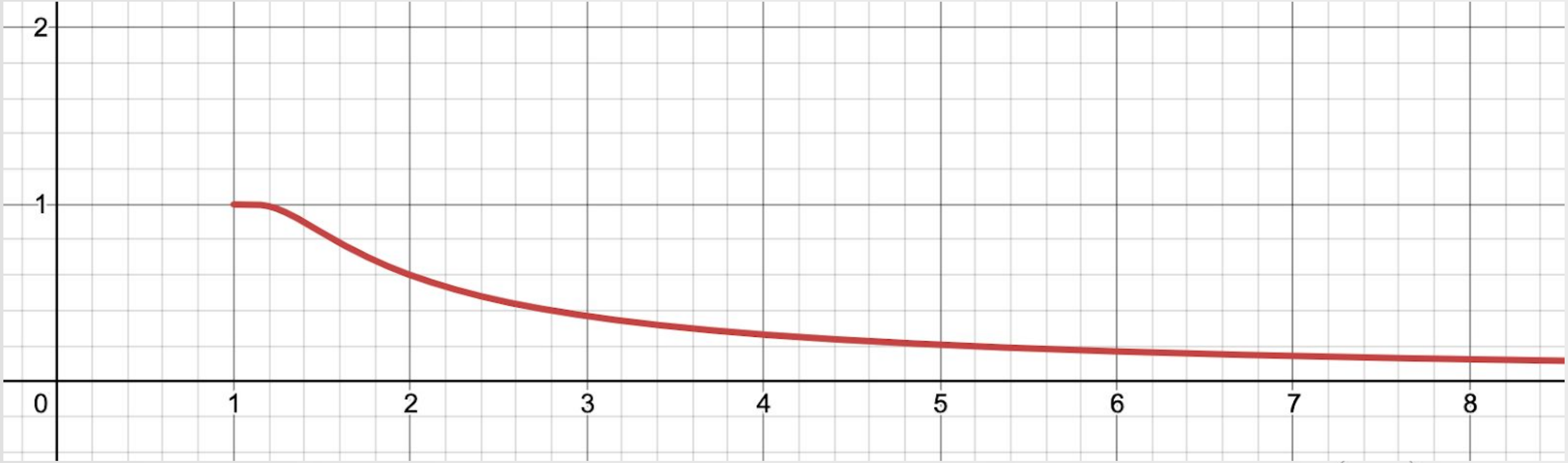
Best response correspondence



Best response correspondence intersection



Probability of asking for one student



Let’s play game № 3

1. Choose integer from 1 to 100

2. The number that is the choice of max amount of players - wins.

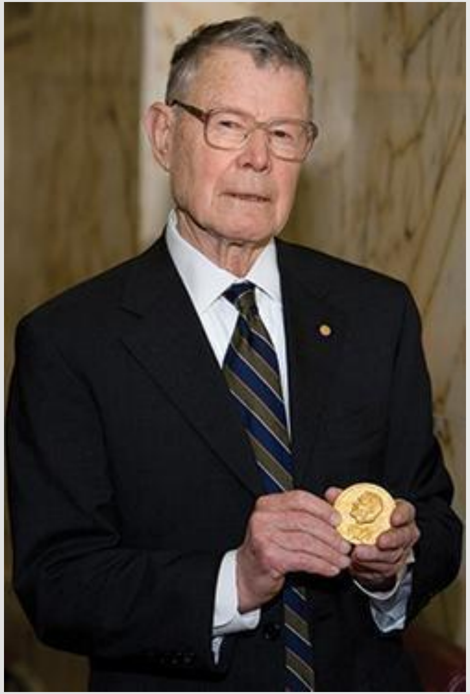
3. If two numbers has the same amount of choices - **smaller** wins.


processing...

My prediction

Winner is 1

Thomas Schelling



Nobel prize with Robert Aumann (2005p.)

Meet in New York

Group of players should meet at the same place, at the same time without any arrangements.

Focal points.

The simplest coordination game

	L	R
U	1 1	0 0
D	0 0	1 1

Repeated games. Finite and infinite

Mathematics is a great motivator for all. Its career starts with zero and it never ends (infinity)

What about two-round game

	L	M	R
U	4 3	0 0	1 4
D	0 0	2 1	0 0

Let’s find Nash equilibria (pure strategies)

	L	M	R
U	**4** 3	0 0	**1** **4**
D	0 0	**2** **1**	0 0

Consider two rounds game

	L	M	R
U	4 3	0 0	1 4
D	0 0	2 1	0 0

Payoffs from equilibrium play:

First player can obtain 2 + 2 = 4 or 1 + 2 = 3 or 1 + 1 = 2

Second 4 + 4 = 8 or 1 + 1 = 2 or 1 + 4 = 5

Now the problem with rationality

	L	M	R
U	4 3	0 0	1 4
D	0 0	2 1	0 0

1. Pareto optimal solution is U, L
2. But this is not equilibrium
3. Quite large game tree - this is producing new window of opportunity

New idea - strategy for all rounds - retaliation

	L	M	R
U	4 3	0 0	1 4
D	0 0	2 1	0 0

Consider strategy for first player:

Play U until second player did not play R, after that play always D.

Calculate payoffs for two rounds game

	LL	LR	RR	RM
Payoffs	8 6	5 7	1 4	3 2

Calculate payoffs for three rounds game

	LLL	LLR	RMM
Payoffs	12 9	9 10	5 6

Retaliation crystalized

	L	M	R
U	4 3	0 0	1 4
D	0 0	2 1	0 0
T	-1 0	2 3	-1 0

Consider strategy for first player:

Play U until second player did not play R, after that play always D.

If all round before last U, L was played - on last round play T

Otherwise - D

Still there is strange thing - betraying in last round

Two ideas:

1. Infinite games (with discount)
2. Infinitely repeated games

Prisoner Dilemma

	C	D
C	**3** **3**	**5** **0**
D	**0** **5**	**1** **1**

Strategic problem

Players can't cooperate to get better result.

Shubik’s dollar auction

Rules:

Highest bid wins, but second player also pays

Unusual behavior (but it is now hard to replicate)

Fool’s Lottery

Lottery 1 mln

You can send any amount of tickets.

All tickets take part in lottery, but prize is value / number

For any repeated finite game

1. **Let’s start solving from the end.**
2. **On last step (D,D) is the only NE.**
3. **Eliminate last round, now apply this for pre-last round.**
4. **Always (D,D) is SPNE**

Infinite game

- To calculate expected payoffs we need to add estimation of future games
- The main idea is that 3 today is not the same as 3 tomorrow. In future game can stop unexpectedly, so we need to discount future reward.
- Discount $\boldsymbol{\delta} \in (0,1)$.

Let’s calculate reward of strategy always C

- $3 + 3\delta + 3\delta^2 + \ldots = 3/(1 - \delta)$
- And for always Deflect $1 + \delta + \delta^2 + \ldots = 1/(1 - \delta)$
-

Principle of one-step deviation

- Allow to analyze infinite games
- Strategy is SPNE if no one-step deviation can increase payoff.
- Strategy GrimTrigger

We need to check two conditions

1. If someone Deflect is it profitable to keep Deflecting?
2. If noone Deflect is it profitable to to Deflect at some point.

	C	D
C	3 3	5 0
D	0 5	1 1

Condition of cooperation

Payoffs from cooperation in equilibrium:

$3 + 3\delta + 3\delta^2 + \dots = 3 / (1 - \delta)$

If first player deviates at first step his payoff is

$5 + \delta + \delta^2 + \dots = 5 + \delta / (1 - \delta)$

So we have condition

$5 + \delta / (1 - \delta) < 3 / (1 - \delta)$

$5(1 - \delta) + \delta < 2$

$3 - 4\delta < 0$

$\delta > 3/4$

Conclusions

1. Cooperation arises when players start to think about the future and there is uncertainty about it.
2. Strange that for **any** finite game it is not working.

Tit-for-tat

Strategy GrimTrigger is very severe. One wrong step and no come back.

Also there is indication from experiments that humans usually use more mild strategy Tit-for-Tat

THANKS

Does anyone have any questions?

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