# How losses outweight gains: study of a simple price oscillation case

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**Abstract**

Following two major cases are studied:

1. price change by the same absolute amount — gains required to cover equivalent loss,
2. price change by the same relative amount — total loss resulting from oscillation of price.

Simple equations are provided as well as reference tables, example figures and suggestions for gain/loss management.

## Disclaimer

While looking through related literature I failed to find the description of this simple yet frequently overlooked phenomenon from the mathematical point of view.

As a result — terminology and conclusions listed are of my own. Corrections are welcome.

## Introduction

Usually, when asymmetry of gains and losses is discussed — it is viewed as a psychological phenomenon of risk aversion.

My focus in this article is instead on the underlying simple mathematics.

When the price rises and falls on a certain asset — intuitively we assume that if such change happened for the same relative amount, say 10% increase followed by 10% decrease — we would end up in the same starting position. But in reality, we end up at a loss.

The following is true:

**Theorem.** *Subsequent change in price in opposite directions can be equal either by the absolute amount or relative amount (percentage), but not both.*

**Theorem.** *When comparing the result of price increase and decrease by the same relative amount — losses always overweight equivalent gains.*

# Generic case

Generic case considered in this article can be described as price change between three points $P_1$, $P_2$ and $P_3$:

$$\begin{cases} P_1 \xrightarrow{d_{12}} P_2 \xrightarrow{d_{23}} P_3, \\ P_1 \xrightarrow{d_{13}} P_3 \end{cases} \tag{1}$$

where $d_{12}$, $d_{23}$, $d_{13}$ — price deviation from 1.

$$(1 + d_{12})\dot{(}1 + d_{23}) = 1 + d_{13} \tag{2}$$

$d$ variables are not absolute, but have direction defined by its sign. Gains are positive and losses are negative.

Price deviation could be defined from prices:

$$\frac{P_2}{P_1} = 1 + d_{12}; \Rightarrow d_{12} = \frac{P_2}{P_1} - 1, \tag{3a}$$

$$d_{23} = \frac{P_3}{P_2} - 1, \tag{3b}$$

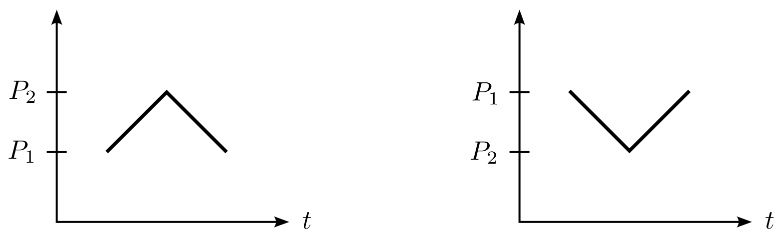
$$d_{13} = \frac{P_3}{P_1} - 1 \tag{3c}$$

Two significant corner cases exist and are described in following sections.

# Price change by the same absolute amount

The price changes from $P_1$ to $P_2$ and then returns to the original level. See the Figure 1 below.

$$\begin{cases} P_1 \xrightarrow{d_{12}} P_2 \xrightarrow{d_{23}} P_3, \\ P_1 \xrightarrow{d_{13}} P_3, \\ (1 + d_{12})\dot{(}1 + d_{23}) = 1 + d_{13}, \\ P_1 = P_3 \;\Rightarrow\; \begin{cases} d_{13} = 0, \\ d_{21} := d_{23} \end{cases} \end{cases} \Rightarrow \begin{cases} P_1 \xrightarrow{d_{12}} P_2 \xrightarrow{d_{21}} P_1, \\ (1 + d_{12})\dot{(}1 + d_{21}) = 1. \end{cases} \tag{4}$$




(a) Price rises and falls.　(b) Price falls and rises.

Figure 1: Cases for price change by the same absolute amount.

Price deviation defined from prices, based on the Equation 3:

$$\boxed{d_{12} = \frac{P_2}{P_1} - 1, \quad d_{21} = \frac{P_1}{P_2} - 1} \tag{5}$$

Price deviation defined from other deviation:

$$\boxed{d_{12} = \frac{1}{1 + d_{21}} - 1, \quad d_{21} = \frac{1}{1 + d_{12}} - 1} \tag{6}$$

Practical application for trading is suggested in Conclusions.

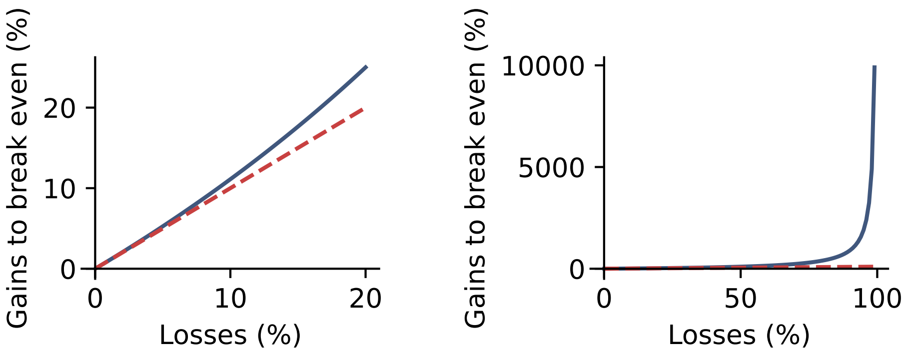
## Examples

Following examples demonstrate how bigger losses require exponentially greater gains to return to the original position.

If up to 1% this effect is negligible, then after loss of $80 \ldots 90\%$ practical recovery becomes close to impossible.

| Loss (%) | Gain to break even (%) | Gain/Loss ratio |
|---|---|---|
| 0.10 | 0.10 | 1.00 |
| 0.20 | 0.20 | 1.00 |
| 0.50 | 0.50 | 1.01 |
| 1.00 | 1.01 | 1.01 |
| 2.00 | 2.04 | 1.02 |
| 5.00 | 5.26 | 1.05 |
| 10.00 | 11.11 | 1.11 |
| 20.00 | 25.00 | 1.25 |
| 50.00 | 100.00 | 2.00 |
| 75.00 | 300.00 | 4.00 |
| 90.00 | 900.00 | 10.00 |
| 99.00 | 9900.00 | 100.00 |

Table 1: Equivalent gains and losses if price oscilates by the same absolute amount.



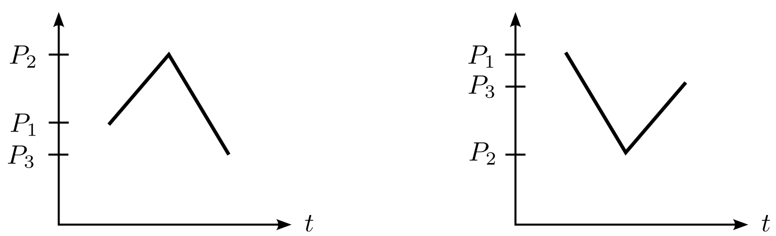

(a) Losses up to 20%. (b) Losses up to 99%.

Figure 2: Gains required to break even after a loss. Red line shows 1-to-1 Gain/Loss ratio.

## Price change by the same relative amount

The price changes from $P_1$ to $P_2$ to $P_3$, but it is known that the price deviation is the same in both cases. and then returns to the original level. See the Figure 3 below.

$$\begin{cases} P_1 \xrightarrow{d_{12}} P_2 \xrightarrow{d_{23}} P_3, \\ P_1 \xrightarrow{d_{13}} P_3, \\ d_{12} = -d_{23} \ \Rightarrow \begin{cases} d_{13} \neq 0, \\ P_1 \neq P_2 \neq P_3. \end{cases} \end{cases} \tag{7}$$




(a) Price rises and falls. (b) Price falls and rises.

Figure 3: Cases for price change by the same relative amount.

Price deviation defined from prices is the same as in the Equation 3.
Price deviation defined from other deviation:

$$\begin{cases} d_{12} = -d_{23}, \\ (1 + d_{12})(1 + d_{23}) = 1 + d_{13} \end{cases} \Rightarrow \begin{cases} d_{12} = -d_{23}, \\ \boxed{d_{13} = -d_{12}^2 = -d_{23}^2} \end{cases} \tag{8}$$

If such oscillation continues, *total loss* after a number of repeatative iterations could be calculated from the Equation 9:

$$\boxed{d_{1n} = (1 - d_{13}^2)^n} \tag{9}$$

where $d_{1n}$ — total price deviation from the initial 1 after $n$ consecutive price oscillations by $d_{13}$ each, as defined in the Equation 8.

Practical application for trading is suggested in Conclusions.

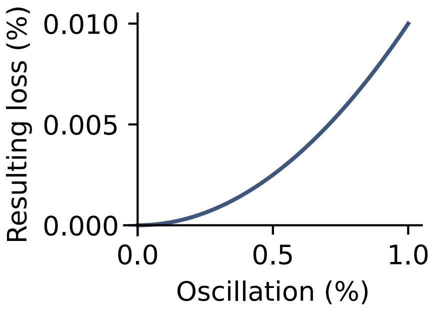
## Examples

In the Table 2 calculated from the Equation 8 we can see, that price oscillation by the same relative value brings exponentially bigger losses, the bigger is the oscillation amplitude.

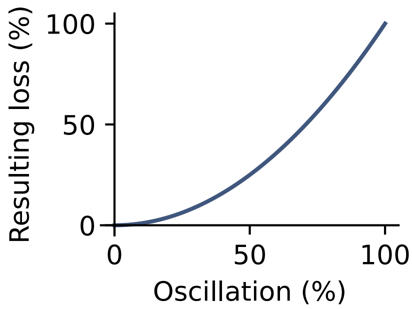
This is a counter-intuitive effect, where we would expect no change to the total price after oscillation.

| Price oscillation (%) | Resulting loss (%) |
|---|---|
| 0.10 | 0.00 |
| 0.20 | 0.00 |
| 0.50 | 0.00 |
| 1.00 | 0.01 |
| 2.00 | 0.04 |
| 5.00 | 0.25 |
| 10.00 | 1.00 |
| 20.00 | 4.00 |
| 50.00 | 25.00 |
| 75.00 | 56.25 |
| 90.00 | 81.00 |
| 99.00 | 98.01 |

Table 2: Total losses if price oscilates by the same relative value.




(a) Losses up to 1%.

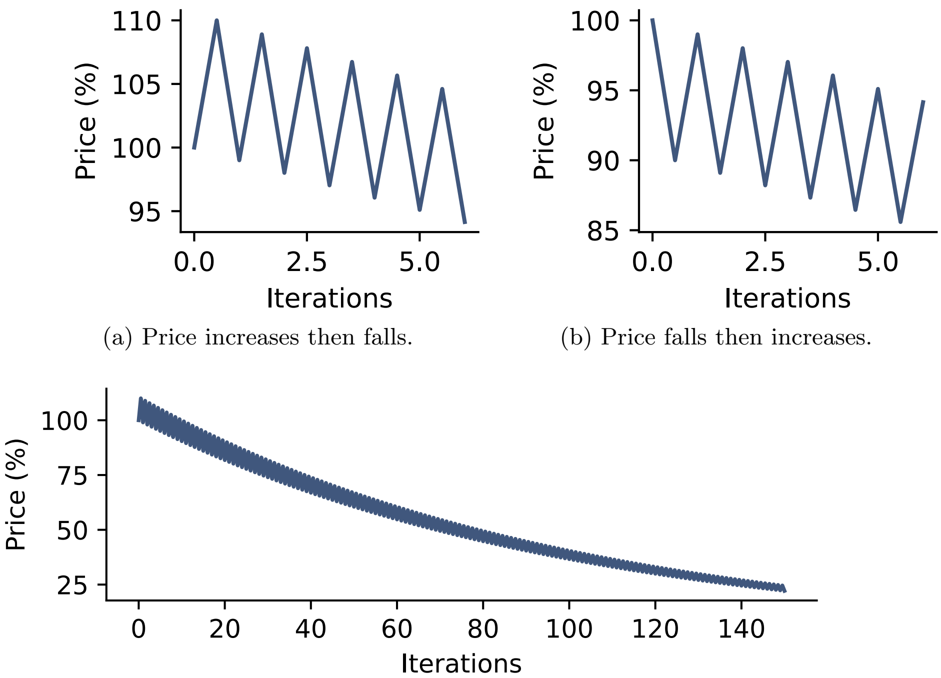



(b) Losses up to 100%.

Figure 4: Total losses if price oscilates by the same relative value.

Figure 4 demonstrates no difference to the resulting price if oscillaiton begins with the price increase or decrease.

As expected, such effect would result in accumulating losses over longer period, as displayed in the Table 5.




(a) Price increases then falls.

(b) Price falls then increases.

(c) Oscillation over longer period.

Figure 5: Oscillations of price by 10% at every iteration.

## Conclusions

When tracking relative price change, to not end up in a loss, ensure (from the Equation 6):

$$\boxed{\begin{aligned} d_{rise} &\geq \frac{1}{1 - |d_{previous\ fall}|} - 1, \quad (10a)\\ |d_{fall}| &\leq \left| \frac{1}{1 + d_{previous\ rise}} - 1 \right| \quad (10b) \end{aligned}}$$

It is wise to control losses. As seen from the examples provided in this article, gains required to break even after a loss grow exponentially, eventually culminating in such catastrophic losses that recovery from them becomes practically impossible.

Also, if you had a price change sequence for the same relative amount, you would always end up at a loss equal to:

$$\boxed{d_{loss} = -d_{rise}^2 = -d_{fall}^2} \tag{11}$$

The loss grows exponentially relative to the oscillation amplitude.

There is no differene if price first rises or falls (see Equation 8).

As a result, after a period of trading with equal relative gains and losses you would end up at a growing total loss, the longer the trading period and the bigger the price oscillation amplitude.

Total loss after a period of $n$ such oscillations can be calculated as:

$$\boxed{d_{total\ loss} = (1 - d_{loss}^2)^n} \tag{12}$$

# Appendix

All source files for this article available at:

- https://github.com/viktorsboroviks/viktorsboroviks.github.io
- tag: v1.3

# Revision History

| Revision | Date | Author(s) | Description |
|---|---|---|---|
| 1.0 | 19 Feb 2023 | VB | Initial version. |
| 1.1 | 23 Feb 2023 | VB | Minor text corrections; added total loss equation. |