

Exercises 02582
Module 7
Spring 2025

March 19, 2025

Topics: Optimal separating hyperplanes, convex optimization, support vector machines

Exercises:

- 1 Write up the analytical kernel matrix for a radial basis function and an input X with three observations (rows) and five features (columns). How big is the resulting Kernel matrix?
- 2 Apply and interpret Optimal Separating Hyperplanes (OSH):
 - (a) Compute OSH solutions for the non-overlapping 2D data in `synthetic_2D_nool_1.csv`. Play around with different choices of the kernel type (`linear`, `polynomial`, or `gaussian`) and the kernel parameter (the standard deviation for the Gaussian kernel or the degree for the polynomial kernel).
 - (b) Compute OSH solutions for the overlapping 2D data in `synthetic_2D_ol_1.csv`. Using a Gaussian kernel, try to find a value of the kernel parameter that leads to an acceptable separation between the two classes. What do you think about the generalization ability of your solution?
- 3 Apply and interpret Support Vector Machines (SVM):
 - (a) Given the data in `Ex3Data.csv`, pick the kernel type as well as values of λ and σ , which gave the best visual results. Explain your decision.
- 4 Use Lagrange multipliers to derive the optimization problem:

- (a) Ridge regression is the solution to the minimization problem $\arg \min_{\beta} \|\mathbf{y} - \mathbf{X}\beta\|^2 + \lambda \|\beta\|^2$. We found the solution during the second exercise. An equivalent formulation is $\arg \min_{\beta} \|\mathbf{y} - \mathbf{X}\beta\|^2$ subject to $\|\beta\|^2 \leq t$, a quadratic problem with a quadratic constraint. Solve this convex constrained minimization problem via the following steps:

- (i) Rewrite the constraint in the form $g(x) \leq 0$
- (ii) Move the constraint into the minimization problem using a single Lagrange multiplier.
- (iii) Differentiate w.r.t. β and set to zero.
- (iv) Solve for β .

Does the result look familiar to you?

- 5 Compute a suitable Support Vector Machine (SVM) classifier for the acute coronary syndrome data in `ACS.csv`. Use the matrix $\mathbf{X}_{truncated}$ as the data matrix and \mathbf{y} as the response variable. Find suitable parameters for a kernel SVM using 10-fold cross-validation and misclassification rate (MCR) as criterion.
- (a) You will need to preprocess data. Response should be 1 or -1 and it is a good idea to normalize data.
 - (b) Calculate the misclassification rate for the test data.
 - (c) With logistic regression you get a classification rate of 0.84. Can you do better with SVM?

Exercises 02582
Module 7
Spring 2025

March 19, 2025

Topics: Optimal separating hyperplanes, convex optimizaion, support vector machines

Resources for this exercise:

Listing 1: Resources in Matlab

```
svm = fitcsvm(X,Y, ...) % fit SVM  
contour(X1,X2,...) % plot contours  
eval_svm(X_train, y_train, X_test, y_test, ...) % calculate  
% misclassification errors
```

Listing 2: Resources in R

```
library(e1071) # SVM library  
svm.model <- svm(Y ~ .,data=Tb,degree=deg,kernel=ker,gamma=  
  gamm) # fit SVM
```

Listing 3: Resources in Python

```
from sklearn.svm import SVC # Support Vector Classifier in  
  sklearn
```

End of exercise