LXXII. *The Heartbeat considered as a Relaxation Oscillation, and an Electrical Model of the Heart.* By BALTH. VAN DER POL, *D.Sc.*, *and* J. VAN DER MARK *.

[Plates X.–XII.]

1. *Relaxation Oscillations.*

THE equation

$$\ddot{v} - \alpha(1-v^2)\,\dot{v} + \omega^2 v = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is representative of an oscillatory system of which the resistance is a function of the elongation. When α is a positive quantity the system has a resistance which for a small amplitude is negative. Therefore, the position

$$v = 0$$

is unstable. When, further,

$$\alpha^2 >> \omega^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

it is obvious that as long as

$$v^2 << 1,$$

the variable v will initially leave the value $v=0$ in an aperiodic way, but when later

$$v^2 > 1,$$

the resistance has changed its sign and has become positive and, therefore, v will have the tendency to go back again towards $v=0$. The possibility of (1) even with the condition (2) having still a purely periodic solution is made plausible by the above considerations, and a full description of the solutions of (1) was given by one of the present authors some years ago †.

It followed from the research mentioned that the fundamental period T_{rel} of the solution of (1) with the condition (2) is

$$T_{rel} = 1{\cdot}61\frac{\alpha}{\omega^2}.$$

* Communicated by the Authors. A more detailed account of these considerations will appear in the next issue of *L'Onde Electrique.*

† Balth. van der Pol, Phil. Mag. ii. p. 978 (1926); *Jarhb. d. dr. Tel.* (*Zs. f. Hochfreq. Technik*) xxviii. p. 178 (1926), xxix. p. 114 (1927).

If (1) is taken to represent an electrical oscillation

$$\alpha = \frac{R}{L},$$

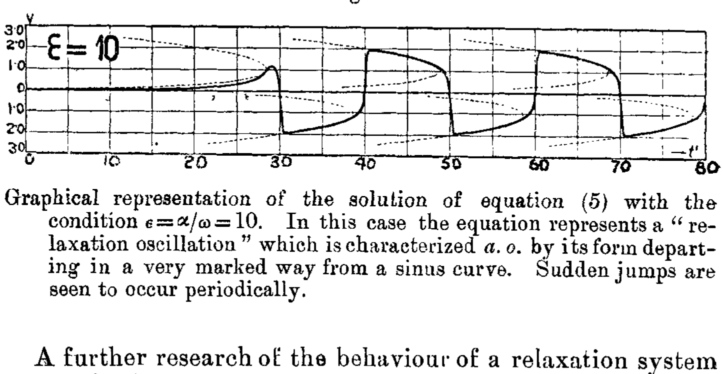
$$\omega^2 = \frac{1}{LC},$$

and thus

$$T_{rel} = 1 \cdot 61 \; RC,$$

and therefore the fundamental period of the new oscillations is, apart from the numerical constant, given by a time constant or relaxation time, and for this reason the name *relaxation oscillation* was suggested. In fig. 1 (taken from the paper mentioned above) the solution of (1) is depicted, and it is seen that the "wave form" deviates very considerably from the sinus function.

Fig. 1.



Graphical representation of the solution of equation (5) with the condition $\epsilon = \alpha/\omega = 10$. In this case the equation represents a "relaxation oscillation" which is characterized *a. o.* by its form departing in a very marked way from a sinus curve. Sudden jumps are seen to occur periodically.

A further research of the behaviour of a relaxation system under the influence of an impressed periodic electromotive force of small amplitude revealed very remarkable synchronization properties *.

In fact it was found that, when this E.M.F. was near resonance, the *free* relaxation period of the system could be varied over a wide range (of the order of an octave), the system continuing to vibrate with the impressed period. Also the system was found very easily to synchronize on a *subharmonic* of the impressed E.M.F., *i. e.*, when the frequency of the latter was ω_0, the fundamental frequency of the "forced" oscillation in the system was ω_0/n, n being any whole number up to 100 or 200. These relaxation

* Balth. van der Pol and J. van der Mark, "Frequency Demultiplication," 'Nature,' Sept. 10th, 1927.

oscillations therefore enable us to realise a *frequency demultiplication*. The experiment showed that the amplitude of the relaxation oscillation could not considerably be influenced by the external E.M.F. Therefore, the frequency of a relaxation oscillation can easily be influenced by an external periodic E.M.F., while the amplitude is quite "rigid." Exactly the reverse is the case with systems obeying (1) but with the condition

$$\alpha^2 << \omega^2,$$

as, *e.g.*, a triode oscillator under the influence of an impressed E.M.F. *

Summarizing the properties of relaxation oscillations we have :

(*a*) their *time period* is determined by a time constant or relaxation time ;

(*b*) their *wave form* deviates considerably from a sinus, and, as very steep parts occur, many higher harmonics of pronounced amplitude are present ;

(*c*) a small impressed periodic force can easily force the relaxation system in step with it (*automatic synchronization* even on subharmonics) while under these circumstances,

(*d*) the amplitude is hardly influenced at all.

Though relaxation oscillations were originally derived in the way outlined above, it will be clear that they are to be found in many realms of nature, for there are many different types of relaxation time. We are mostly used to find a decay phenomenon to occur only once in a special experiment, and it is typical in the case of relaxation oscillations to find such an asymptotic occurrence to repeat itself periodically. Obviously this automatic periodic re-occurrence of such a typical aperiodic phenomenon is closely related to the presence of some form of energy source which is to be found behind the negativity of the resistance of (1).

Some instances of typical relaxation oscillations are : the æolian harp, a pneumatic hammer, the scratching noise of a knife on a plate, the waving of a flag in the wind, the humming noise sometimes made by a water-tap, the squeaking of a door, the multivibrator of Abraham and Bloch †, the

* This synchronisation property of a triode oscillator was first found by W. H. Eccles and J. H. Vincent, British Patent Spec. clxiii. p. 462, application date Febr. 17th, 1920. The theory of this phenomenon is to be found in Balth. van. der Pol, Phil. Mag. iii. p. 65 (1927).

† Abraham & Bloch, *Ann. de Physique*, xii. p. 237 (1919).

tetrode multivibrator *, the periodic sparks obtained from a Wimshurst machine, the Wehnelt interrupter, the intermittent discharge of a condenser through a neon tube, the periodic re-occurrence of epidemics and of economical crises, the periodic density of an even number of species of animals living together, and the one species serving as food for the other †, the sleeping of flowers, the periodic re-occurrence of showers behind a depression, the shivering from cold, menstruation, and, finally, the beating of the heart ‡.

In all these examples the frequency of these periodic phenomena is not determined by the product of an elasticity and a mass but by some form of relaxation time.

That the frequency of these periodic phenomena is not rigidly constant is due to the fact that a relaxation time is determined *a. o.* by some form of resistance, and it is a well-known fact that outer circumstances may much easier influence a resistance than a mass or elasticity.

2. *Schematic Dynamical Representation of the Heart.*

In applying the theory of relaxation oscillations to the beating of the heart we will consider the heart as a system of three degrees of freedom: the *sinus*, the *atrium* (*auricle*), and the *ventricle*. As normally the two auricles beat in exact synchronism and the same can be said of the two ventricles, we will further speak of *the* auricle and *the* ventricle.

When we consider the heart as having three degrees of freedom only, it will be obvious that we exclude at the outset those movements of the heart which can only be described by partial differential equations. For instance, we cannot find back the finite velocity of propagation of contraction over the wall of the heart. We therefore shall not consider flutter or fibrillation, as these phenomena are directly connected with progressing and standing waves.

Returning to the view of the heart having the above-mentioned three degrees of freedom only, we consider each of them to be able to perform a relaxation oscillation by itself, each of the three having its own natural period. Moreover, a coupling exists between the sinus and the auricle, the former acting on the latter. Another coupling

* Phil. Mag. li. p. 991 (1926).

† Volterra, *Accad. Lincei, Atti* (1926).

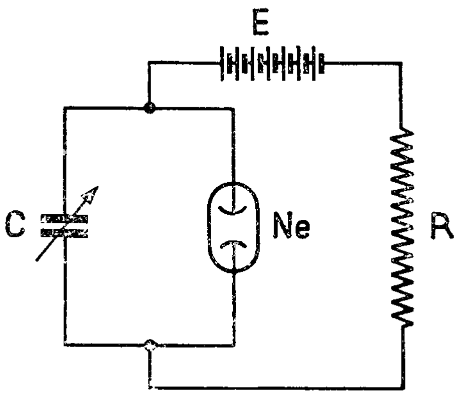
‡ The view that heart beats belong to the type of relaxation oscillations was first expressed two years ago by Balth. van der Pol (Phil. Mag. ii. p. 992 (1926)).

exists between the auricle and the ventricle, which coupling is realized through the existence of the bundle of His.

In a normal heart both the couplings mentioned have the peculiarity that they transmit a stimulus in one direction only, *i.e.*, from the sinus to the auricle, and from the auricle to the ventricle, respectively. These couplings are therefore in the normal heart of a unidirective character. In an electrical model to be described later on, these two couplings are therefore represented by two triodes, which do not amplify at all, but which were simply inserted in order to provide a unidirective instrument.

The purpose of this paper is to give a general connected view of the heartbeats considered as relaxation oscillations. The properties of these oscillations, which have lately been studied,

Fig. 2.



A system capable of producing relaxation oscillations. It consists of a neon lamp Ne, a condenser C of approximately 1 microfarad, a resistance R of the order of 1 megohm, and a battery of about 180 volt.

enable us to consider the rhythm of the heart from a new general point of view, giving a logical connected account of the heart-rhythm. According to this new theory some anomalies of the heart-rhythm can be predicted, which, so far as the authors are aware, have not yet been observed or recognized in the human heart.

3. *Electrical Model of the Heart.*

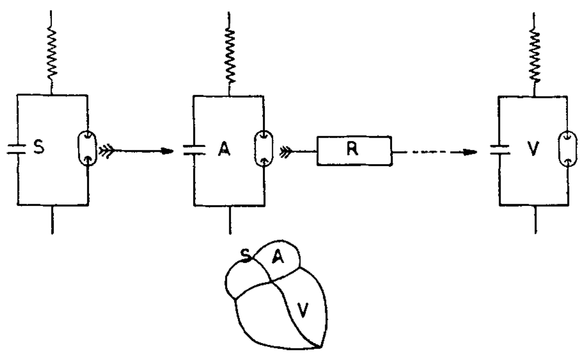
In constructing a model of the heart according to the theory expounded above, various systems capable of producing relaxation oscillations could be chosen. As a very practical example, which is also well suited for demonstrations, an intermittent discharge through a neon tube can be made use of. Such a system is depicted in fig. 2, where E is a battery of say 150 to 200 volts, R a resistance of the order of 1 megohm, and C a capacity of the order of one microfarad.

As the charging-up time of the condenser is given by the product of the capacity C and the resistance R, the time period T_{rel} of this relaxation oscillation will be about

$$T_{rel} \doteqdot CR = 10^{-6} \,.\, 10^{+6} = 1 \text{ sec.}$$

We therefore see the neon lamp to give a short flash once about every second.

Fig. 3.



Schematic representation of the heart by three relaxation systems: S (=Sinus), A (=Auriculum), and V (=Ventriculum). R is a retardation system representing in the model the finite time necessary for a stimulus to be transmitted through the A-V bundle.

In fig. 3 the heart is represented by three such systems, where the first one S stands for the sinus, A for the atrium, and V for the ventricle. Between A and V a rectangle R is drawn representing a retardation system imitating the finite time taken for a stimulus to be transmitted from the atrium through the atrio-ventricular-bundle to the ventricle. In our electrical model this retardation is brought about by the action of a fourth neon tube, which therefore takes care that the ventricular systole sets in somewhat later than the corresponding auricular one. However, any other retardation system could be chosen, and we want to stress the fact that the working of our model is independent of the type of system used for this retardation.

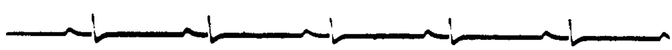
A photo of the complete instrument is given in fig. 4 (Pl. X.), where it is seen that the three neon tubes S, A, and V of fig. 3 are brought to the front of the instrument, each flash corresponding to the activity of the respective part of the heart.

At the back of the instrument three keys are mounted, with which a short electrical impulse can be given to the systems S, A, and V of fig. 3, thus causing extrasystolæ of the sinus, auricle, and ventricle respectively. Moreover, the coupling between A and V (the auricle and ventricle) can be varied at will, thus imitating the beautiful experiments of Erlanger of gradually clamping the bundle of His.

The whole model consists of resistances and capacities only, and no intended inductances are introduced in the system.

Electrocardiograms could be taken from this artificial heart by adding the current impulses in the auricle and

Fig. 5.



Typical electrocardiogram of the artificial heart. The P top and the QRS complex are clearly visible. The T top however is missing, due to insufficient definite data at hand as regards its origin.

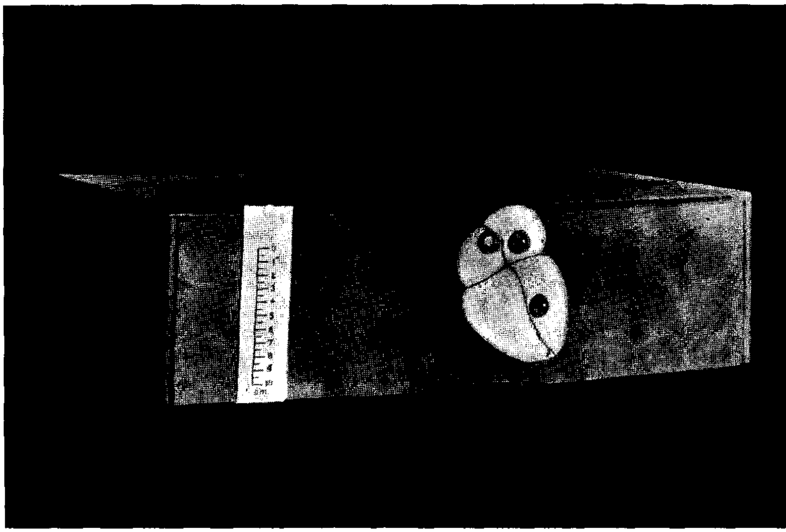
in the ventricle; a further filtering circuit was used to remodel somewhat the form of the impulses, which were registered, after amplification, with a Cambridge oscillograph. Obviously this filter circuit had no influence on the working of the model, and is not used for ordinary demonstration purposes.

A typical electrocardiogram of our artificial heart, working in a normal way, is shown in fig. 5, where the P top and QRS complex is plainly visible, while the T top is missing. As the origin of the T top in the electrocardiogram of the human heart is not quite certain yet, we could not insert a representing mechanism for it.

4. *Observations and Measurements made with the Electrical Model.*

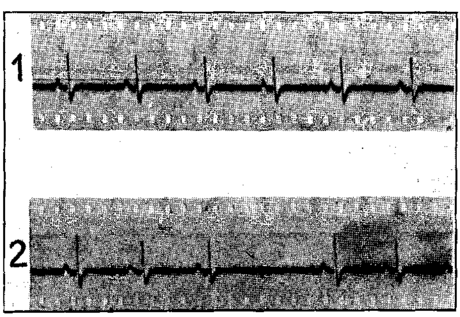
4*a*. *Heart block.*—In fig. 6 we give nine films taken with our model giving the effect of gradually decreasing the coupling between auricle and ventricle, thus imitating the experiments of Erlanger on clamping the bundle of His.

Fig. 4.



Outside view of model.

Fig. 7.

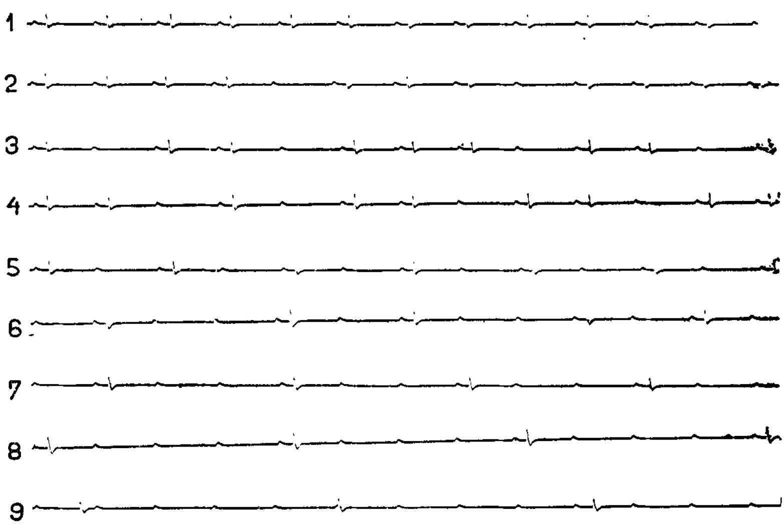


1. Normal heart beat.
2. Sino-auricular block.

Film No. 1 gives the normal heart beat ; No. 2 shows an occasional failure of the ventricle ; in No. 3 these failures become more frequent ; in No. 4 we approach the case of 2 : 1 *heart block*, which stage is reached in film No. 5. In No. 6 we have alternatively 2 : 1 and 3 : 1 block. Further, we see in Nos. 7 and 8, respectively, 3 : 1 and 4 : 1 block, till at last in film No. 9 the coupling has become zero, which gives *complete heart block.*

A typical case of *sino-auricular block* is given in fig. 7 (Pl. X.), where the first film gives the normal heart beat and the second shows clearly the block just mentioned.

Fig. 6.



Electrocardiograms from the artificial heart obtained by gradually reducing the coupling between the A and V system (clamping the bundle of His). The development of 2 : 1, 3 : 1, and 4 : 1, as well as complete heart block, is clearly shown.

4 *b. Extra systolæ.*—When our heart model beats in the normal way we can impress a small electrical impulse on the ventricle. If this is done directly after a ventricular systole we find that nothing happens. The ventricle therefore is still in its *refractory period* : the condenser of the V system is not yet charged up to such a potential that the extra E.M.F., superimposed upon it causes the total potential to reach the flashing potential of the neon lamp. When we repeat the same experiment slightly later,

after a ventricular systole, we find a flash of the V system to occur; the ventricle has now passed its refractory state. When this flash occurs and we have therefore excited a ventricular extrasystole, the condenser discharges through its neon tube in a normal way down again to the potential where the gas discharge breaks, and thus we find back the famous law of "*all or nothing*": a stimulus has either no effect at all or it causes a complete systole to occur. When we investigate into the magnitude of the stimulus necessary to cause a ventricular extrasystole as a function of the phase of the ventricular cycle, we find this stimulus to decrease exponentially with an increasing phase, as was to be expected. The magnitude of the stimulus E necessary to produce an extra systole as a function of the time t, counted from the moment of a former systole, is therefore represented by the formula

$$E = Ae^{-\frac{t}{CR}} - B,$$

which follows directly from the manner in which a condenser is charged through a resistance. No doubt a similar law applies to the real heart.

Consider fig. 8 (Pl. XI.), where the film No. 1 again represents the electrocardiogram of our artificial heart beating in the normal way. Film No. 2, however, shows a *ventricular extrasystole* given shortly before an impulse from the atrium arrives. It is clearly seen that the next following auricular systole finds the ventricle still in the refractory period, so that it does not cause a normal ventricular systole. When, as in No. 3, the ventricular extrasystole is caused to occur earlier in the ventricular cycle, the next coming auricular systole *does* have effect and causes the normal ventricular systole, so that here we have the case of an "*interpolated*" *ventricular systole.*

Again in fig. 9 (Pl. XI.), film 1 represents the normal working of the heart. In film 2 an *auricular extrasystole* is caused. Here, again, the auricular extrasystole is followed after the normal time by a ventricular systole, but a *compensatory period* is found after this auricular extrasystole, so that the next one comes at the normal time rigidly determined by the frequency of the sinus. Film 3 represents the same case, but with the difference that the auricular extrasystole was excited a little earlier than in film 2. Therefore the auricular extrasystole finds the ventriculum still in its refractory period, and under these

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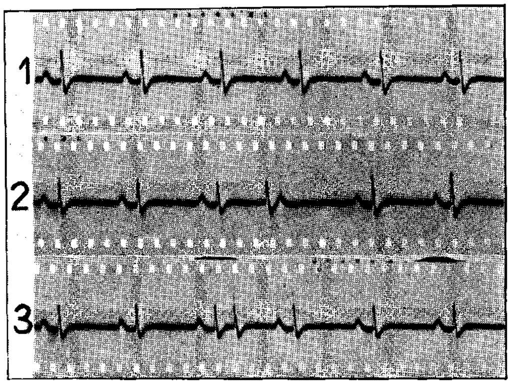
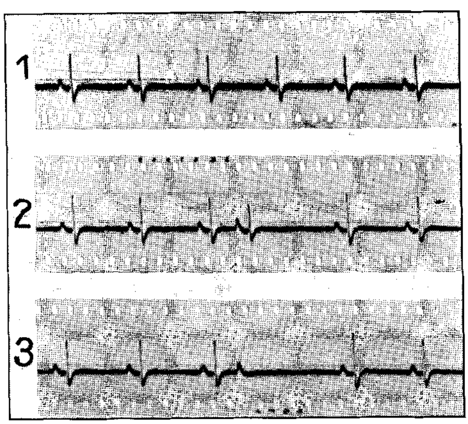
FIG. 8.



Ventricular extrasystolæ :—1, normal heart beat; 2, late ventricular extrasystole resulting in the ventricle being in the refractory state when the next following normal stimulus arrives from the auricle; 3, early ventricular extrasystole; here the ventricle is *not* any more in the refractory period when the next following normal stimulus arrives from the auricle and thus an *interpolated ventricular* systole is obtained.

FIG. 9.



1. Normal heart beat.
2. Auricular extrasystole (with the ventricle responding).
3. Auricular extrasystole (ventricle still in refractory period).

Note the increased amplitude of the following normal ventricular systole.

circumstances therefore the auricular extrasystole is *not followed* by a ventricular one.

Fig. 10 (Pl. XII.), No. 1, gives the normal heart beat. Film No. 2 represents a case of a *sinus extrasystole.* It is clearly seen that the original rhythm has been lost, as is the case in the human heart.

Finally, we reproduce in fig. 11 (Pl. XII.) an electro-cardiogram of the human heart, taken in three normal positions 1, 2, and 3, with the same special amplifier used in the experiments with the artificial heart and also with the same Cambridge oscillograph, which has a shorter natural time period and therefore responds quicker than the string galvanometer.

This amplifier, which had the property of amplifying the very low frequencies of say one half period per second or less, as well as the higher frequencies, enabled us to make visible the real human heart beat as periodic flashes of a neon tube. This neon tube flashed twice in each heart cycle, when it was connected to the output side of the amplifier, the input side being connected to the two hands of the patient.

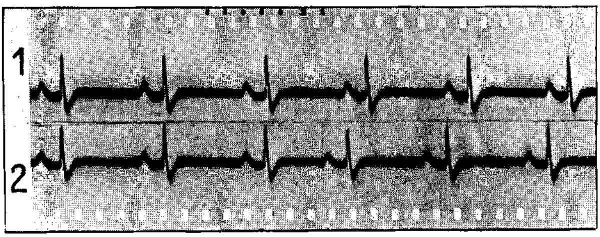
5. *Final Considerations and Suggestions of new Possibilities.*

The very close analogy between the working of our model and the beating of the mammalian heart leaves no doubt that the view expressed in the former paragraphs of regarding the heart beat as a relaxation oscillation is correct. Therefore, without going into the detailed nervous and physical-chemical action of the heart, it can safely be concluded that what ultimately determines the period of the heart is a *diffusion time* (a relaxation time). As mentioned above, the model described represents a first approximation only to the action of the heart, and it could be extended in several directions, thus assigning to the heart more than three degrees of freedom only.

Moreover, a reduction of the electromotive force of the battery connected to the model reduces the "tonus," and soon a point is reached where our electrical heart does not beat any more. The system under these conditions closely shows, so far as the response to an external stimulus is concerned, the behaviour of an ordinary muscle. In fact, a cross-striated *muscle* can be represented by what may be called a "*relaxation cable,*" about which we hope to report in another communication.

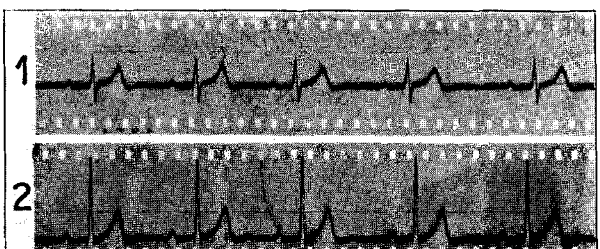
In conclusion, we give some further possible disorders mainly obtained mathematically and which were all verified

FIG. 10.



1. Normal heart beat.
2. Sinus extrasystole disturbing the whole heart rhythm.

FIG. 11.



Electrocardiogram of the real heart taken in the standard positions 1, 2, and 3 with the aid of a special very low frequency amplifier and a Cambridge oscillograph.

with the aid of our model. Possibly these anomalies either have or will be found in the human heart as well.

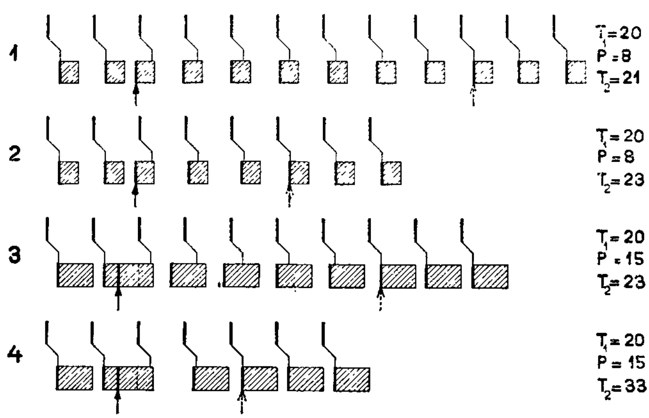
It follows from the considerations expounded above that there are two possibilities that may cause a partial heart block:

(*a*) the amplitude of the stimulus arriving through the A–V bundle at the ventricle is smaller than normal;

(*b*) the natural relaxation period of the ventricle is prolonged.

Both causes may result in exactly the same working of the heart, though means may be found to distinguish between the two.

Fig. 12.



Transient periods elapsing after a ventricular extrasystole is given at the moment indicated by the first arrow. Only at the moment indicated by the second (dotted) arrow is the normal periodic action of the heart resumed. This figure gives the course of events calculated and verified experimentally with the aid of the model, and with the condition that the free natural relaxation period T_2 of the ventricle is larger than that (T_1) of the auricle.

But also an acceleration of the natural free ventricular period may result in anomalies, which bear some resemblance to a combination of heart block with ventricular extrasystolæ. Especially these anomalies occur when the free relaxation period of the ventricle is slightly greater or slightly smaller than the period of the sinus and when the conduction through the A–V bundle is reduced. Let, *e.g.*, fig. 12

represent the auricular and ventricular systolæ. The shaded portions following the ventricular systolæ represent the duration of the refractory period as determined *a.o.* by the amplitude of the stimulus arriving from the A–V bundle. Calling:

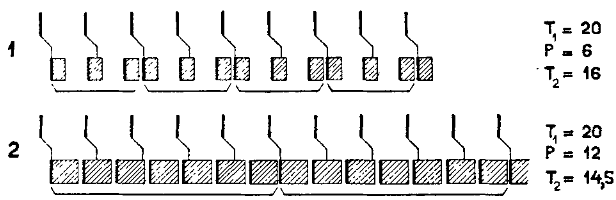
T_1 = period of the stimulus arriving from the A–V bundle.

T_2 = free relaxation period of the ventricle.

P = length refractory period.

Fig. 12, Nos. 1, 2, 3, and 4 represent different effects of extrasystolæ. The durations of T_1, T_2, and P are given at the side of the figures in arbitrary units. In this fig. 12 $T_1 < T_2$. Under these circumstances a ventricular extrasystole may temporarily disturb the rhythm (mathematically: transients). The extrasystole is everywhere indicated by

Fig. 13.



Some possibilities (derived theoretically and verified with the aid of the model) when the free natural relaxation period T_2 of the ventricle is slightly smaller than the auricle period T_1. The shaded portions indicate (like in fig. 12) the refractory period of the ventricle. Especially No. 1 shows the possibility that the fundamental period of the rhythm in this case is *not* equal to the period of the auricle, but is twice as long. Here *three* ventricular systolæ occur in the time of only *two* auricular systole. The last ventricular systole in each fundamental period resembles somewhat to cause a retrograde beat of the auricle.

an arrow, while the dotted arrows give the beginning of normal working of the heart again. The anomalies of figs. 12 and 13 have been derived on the basis of a strict unidirectional conduction through the A–V bundle, and it will be clear from these figures that some of the anomalies resemble somewhat "retrograde beats."

It is seen from fig. 12, No 1, that with the numbers as given in the figure, it takes the time of 7 auricular systolæ after the extrasytole before the normal rhythm is restored again. For the special instant of the extrasystole of No. 2 and for the corresponding T_1, T_2, and P, the transient period lasts about three auricular beats. Again, in Nos. 3 and 4, it takes respectively about 5·5 and 2·5 auricular periods before the normal action is restored again.

Finally, in fig. 13, two possible anomalies are depicted. Here, $T_1 > T_2$, *i.e.*, the natural ventricular free period is shorter than the auricular period. Thus the possibility arrives that the fundamental period of the complete system consists of a whole number of auricular systolæ, *i.e.*, the phenomenon repeats itself exactly only after, *e.g.*, 4 or 5 or 6 auricular beats. This fundamental period is indicated by the brackets underneath each drawing. We therefore find the possibility of a, *e.g.*, 2 : 3 block and a 5 : 7 block. We are not aware whether these phenomena occur in the real heart.

From the fact that, when

$$T_1 > T_2,$$

a forced ventricular beat can only occur when the stimulus from the A–V bundle arrives at the ventricle when the latter is outside its refractory period, it follows that the fundamental period T, where

$$T = nT_1,$$

is determined by the following two *Diophantic inequalities*

$$R < (nT_1 - mT_2) < T_2,$$

where both n and m are whole numbers and n the smallest possible one satisfying the inequalities.

Eindhoven, Natuurkundig Laboratorium
der N. V. Philips' Gloeilampen-fabrieken.
May 1928.

LXXIII. *On the Adsorption of the Alkali Metals on a Mercury-Vacuum Interface.* *By* R. Stevenson Bradley, *B.A.**.

THE adsorption of vapours on the surface of mercury has recently excited considerable interest; the corresponding case of adsorption of metals dissolved in mercury has not yet been studied.

The most interesting amalgams, and those for which most results are available, are those of the alkali metals. Both

* Communicated by Prof. R. Whytlaw-Gray.