Memory Evolutive Systems

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Chapter 1

Introduction

This is a presentation of what we call *memory evolutive systems*. We offer these as mathematical models for autonomous evolutionary systems, such as biological or social systems, and in particular, nervous systems of higher animals. Our work is rooted in category theory, a particular domain of mathematics.

1.1 Motivations

One of us, Jean-Paul Vanbremeersch, is a physician specializing in gerontology. He has long been interested in explaining the complex responses of organisms to illness or senescence. The second of us, Andrée C. Ehresmann, is a mathematician whose research areas include analysis, optimization theory, and category theory.

1.2 How Can Complexity Be Characterized?

During the late 1970s and early 1980s, researchers discussed non-linear systems, chaos theory, and fractal objects. We realized that category theory could provide tools to study concepts germane to complexity, such as:

- The *binding problem*: how do simple objects bind together to form a 'whole greater than the sum of its parts'?
- The *emergence problem*: how do properties of a complex object relate to those of its elementary parts?
- The *hierarchy problem*: how do increasingly complex objects form, from elementary particles to societies?

We initially modeled these problems using **hierarchical evolutive systems**, based on the categorical concept of *colimit* and *complexification*.

1.3 Mathematical Framework

We introduce category theory as the underlying framework for memory evolutive systems. The key mathematical concepts include:

1.3.1 Categories and Functors

A **category** $\mathcal{C}$ consists of:

- A class of objects $\mathrm{Obj}(\mathcal{C})$.
- A class of morphisms $\mathrm{Hom}(A, B)$ for each pair of objects $A, B \in \mathcal{C}$.
- A composition law satisfying associativity and identity properties.

A **functor** $F : \mathcal{C} \to \mathcal{D}$ maps objects and morphisms while preserving composition and identities.

1.3.2 Colimits and Limits

Given a diagram $D : I \to \mathcal{C}$, a *colimit* is an object C with a universal morphism $\varphi_D : D \to C$. Formally:

$$\forall X, \exists! \psi : C \to X \text{ such that } \psi \circ \varphi_D = \text{unique}. \tag{1.1}$$

Similarly, a *limit* is an object satisfying a universal property for cones over D.

1.3.3 Hierarchical Structures and Complexity

Complexification is modeled using successive colimit constructions:

$$C_0 \to C_1 \to C_2 \to \cdots \to C_n, \tag{1.2}$$

where each C_i integrates a pattern of objects and their relationships into a higher-order structure.

Chapter 2

Memory Evolutive Systems

2.1 Definition

A **memory evolutive system** (MES) is a category with:

- A **net of internal regulators** (co-regulators) responsible for system adaptation.
- A **memory** that retains past states and adapts future responses.
- A **dynamical interplay** of procedures among co-regulators.

2.2 Mathematical Model

The system configuration at time t is represented as a category $\mathcal{C}(t)$, evolving via functors $F_t : \mathcal{C}(t) \to \mathcal{C}(t+1)$.

2.3 Application to Cognition

We model cognitive processes by defining a **memory evolutive neural system** (MENS), where neurons are represented as category objects, and synaptic connections as morphisms. The semantic memory is captured by a hierarchy of categorical structures.

Chapter 3

Conclusion

Memory evolutive systems provide a mathematical framework for modeling hierarchical autonomous systems, capturing emergence and complexity using category theory. Our approach extends to cognitive science, artificial intelligence, and biological modeling.