Memory Evolutive Systems: A Categorical Framework for Complex Systems

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1 Introduction

This paper presents a categorical framework for complex systems based on Memory Evolutive Systems.

2 Basic Structure

The hierarchical structure of a Memory Evolutive System is represented by:

$$C_0 \xrightarrow{F_1} C_1 \xrightarrow{F_2} C_2$$

where each level represents increasing complexity and abstraction.

3 Colimit Construction

The binding process is represented by colimits:

$$\begin{array}{ccc} \text{Transaction} & \xrightarrow{\text{credit}} & \text{CreditAccount} \\ \downarrow_{\text{debit}} & & \downarrow_{\text{balance}} \\ \text{DebitAccount} & \xrightarrow{\text{balance}} & \text{Money} \end{array}$$

4 Complex Patterns

Complex patterns emerge through natural transformations:

$$\begin{array}{ccc} C(t) & \xrightarrow{F_t} & C(t+1) \\ \downarrow_{\alpha_t} & & \downarrow_{\alpha_{t+1}} \\ D(t) & \xrightarrow{G_t} & D(t+1) \end{array}$$

5 Balance Sheet Functor

The balance sheet functor is defined as:

$$
\begin{aligned}
&B(\text{Account}) \\
&= \mathbb{R} \text{ (account balance)} \\
&B(f : A \to B) \\
&= (+f_{\text{amount}}) \text{ (transaction amount)}
\end{aligned}
$$